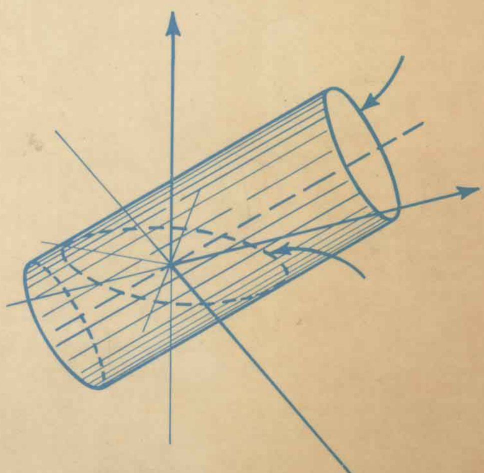
STRENGTH OF MATERIALS AND STRUCTURES

An introduction to the mechanics of solids and structures

JOHN CASE AND A. H. CHILVER

Second Edition



Strength of materials and structures

An introduction to the mechanics of solids and structures

(SI units)

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Strength of materials and structures

Preface

The first edition of this book was published in 1959, and since then the book has had a number of reprintings. All these reprintings have shown that the book is widely used as an introductory text to the field of the strength of materials and structures, and it is hoped that this new edition will ensure the book's continuing usefulness. The first edition was published under the title of *Strength of Materials;* the book is used in fact as a general introduction to both the strength of materials and structures, and in the second edition this broader title has been chosen. As an elementary text, the book of course gives an introduction to the application of basic ideas in solid and structural mechanics to engineering problems.

The content covers most of the requirements of an engineering undergraduate in his first and second years, and in some cases for the whole of his course. For more advanced studies, the authors' Advanced Strength of Materials will cover the requirements for final honours degree courses and for post-graduate studies.

The book begins with a simple discussion of stresses and strains in materials and structural components, and the forms these take in tension, compression and shear; in Chapter 5 some simple general properties of stress and strain are first introduced. These basic properties are then applied to a wide range of problems, including shells, beams and shafts; plastic as well as elastic problems are treated. In Chapter 17 a simple introduction is given to the important principle of virtual work, and two special forms of this—leading to strain energy and complementary energy—are dealt with in Chapter 18. The final chapters are devoted, respectively, to buckling, vibrations and impact stresses.

Both worked examples and unsolved problems are given in the text, and all these are treated in SI units. Some of the examples, and many of the additional problems, are based on questions set by various examining bodies; the sources of these questions are shown in the text.

This new edition was begun by both authors, but because of Mr John Case's death in 1969, the new edition was completed by Dr Chilver. During the life of the first edition, many useful comments and corrections were suggested by readers; corrections and amendments based on these have been incorporated in this second edition; but readers' comments will still be most welcome.

Principal notation

a length
b breadth
c wave velocity, distance
d diameter
e eccentricity
h depth
j number of joints
l length
m mass, modular ratio, number of members
n frequency, load factor, distance
p pressure
q shearing force per unit length
r radius
s distance
t thickness
u displacement
v displacement, velocity
w displacement, load intensity, force
x coordinate
y coordinate
z coordinate

A area
C complementary energy
D diameter
E Young's modulus
F shearing force
G shearing modulus
H force
I second moment of area
J torsion constant
K bulk modulus
L length
M bending moment
P force
Q force
R force, radius
S force
T torque
U strain energy
V force, volume, velocity
W work done, force
X force
Y force
Z section modulus

α coefficient of linear expansion
γ shearing strain
δ deflection
ϵ direct strain
η efficiency
θ temperature, angle of twist
ν Poisson's ratio

ρ density
σ direct stress
τ shearing stress
ω angular velocity
Δ deflection
Φ step-function

Note on SI units

The units used throughout the book are those of the Système Internationale d'Unités; this is usually referred to as the SI system. In the field of the strength of materials and structures we are concerned with the following basic units of the SI system:

length	metre (m)
mass	kilogramme (kg)
time	second (s)
temperature	kelvin (K)

There are two further basic units of the SI system—electric current and luminous intensity—which we need not consider for our present purposes, since these do not enter the field of the strength of materials and structures. For temperatures we shall use conventional degrees centigrade (° C), since we shall be concerned with temperature changes rather than absolute temperatures. The units which we derive from the basic SI units, and which are relevant to our field of study, are:

force	newton (N)	$kg.m.s^{-2}$
work, energy	joule (J)	$kg.m^2.s^{-2} = Nm$
power	watt (W)	$kg.m^2.s^{-3} = Js^{-1}$
frequency	hertz (Hz)	cycle per second

The acceleration due to gravity is taken as:

$$g = 9{\cdot}81 \text{ ms}^{-2}$$

Linear distances are expressed in metres and multiples or divisions of 10^3 of metres, i.e.

kilometre (km)	10^3 m
metre (m)	1 m
millimetre (mm)	10^{-3} m

In many problems of stress analysis these are not convenient units, and others, such as the centimetre (cm), which is 10^{-2} m, are more appropriate.

The unit of force, the newton (N), is the force required to give unit acceleration (ms^{-2}) to unit mass (kg). In terms of newtons the common force units in the foot-pound-second system (with $g = 9{\cdot}81 \text{ ms}^{-2}$) are

$$1 \text{ lb.wt} = 4{\cdot}45 \text{ newtons (N)}$$

$$1 \text{ ton.wt} = 9{\cdot}96 \times 10^3 \text{ newtons (N)}$$

In general, decimal multiples in the SI system are taken in units of 10^3. The prefixes we make most use of are:

kilo	k	10^3
mega	M	10^6
giga	G	10^9

Thus:

$$1 \text{ ton.wt} = 9{\cdot}96 \text{ kN}$$

The unit of force, the newton (N), is used for external loads and internal forces, such as shearing forces. Torques and bending moments are expressed in newton-metres (Nm).

An important unit in the strength of materials and structures is stress. In the foot-pound-second system, stresses are commonly expressed in lb.wt/in^2, and tons/in^2. In the SI system these take the values:

$$1 \text{ lb.wt/in}^2 = 6{\cdot}89 \times 10^3 \text{ N/m}^2 = 6{\cdot}89 \text{ kN/m}^2$$

$$1 \text{ ton.wt/in}^2 = 15{\cdot}42 \times 10^6 \text{ N/m}^2 = 15{\cdot}42 \text{ MN/m}^2$$

Yield stresses of the common metallic materials are in the range:

$$200 \text{ MN/m}^2 \quad \text{to} \quad 750 \text{ MN/m}^2$$

Again, Young's modulus for steel becomes:

$$E_{\text{steel}} = 30 \times 10^6 \text{ lb.wt/in}^2 = 207 \text{ GN/m}^2$$

Thus, working and yield stresses will be expressed in MN/m^2 units, while Young's modulus will be given in GN/m^2 units.

Contents

1 Tension and compression: direct stresses

1.1 Introduction

The strength of a material, whatever its nature, is defined largely by the internal stresses, or intensities of force, in the material. A knowledge of these stresses is essential to the safe design of a machine, aircraft, or any type of structure. Most practical structures consist of complex arrangements of many component members; an aircraft fuselage, for example, is an elaborate system of interconnected sheeting, longitudinal stringers, and transverse rings. The detailed stress analysis of such a structure is a difficult task, even when the loading conditions are simple. The problem is complicated further because the loads experienced by a structure are variable and sometimes unpredictable. We shall be concerned mainly with stresses in materials under relatively simple loading conditions; we begin with a discussion of the behaviour of a stretched wire, and introduce the concepts of direct stress and strain.

1.2 Stretching of a steel wire

One of the simplest loading conditions of a material is that of *tension*, in which the fibres of the material are stretched. Consider, for example, a long steel wire held rigidly at its upper end, Fig. 1.1, and loaded by a mass hung from the lower end. If vertical movements of the lower end are observed during loading it will be found that the wire is stretched by a small, but measurable, amount from its original unloaded length. The material of the wire is composed of a large number of small crystals which are only visible under microscopic study; these crystals have irregularly shaped boundaries, and largely random orientations with respect to each other; as loads are applied to the wire, the crystal structure of the metal is distorted.

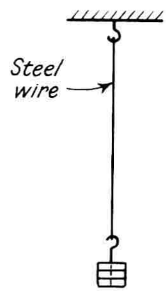


Fig. 1.1 Stretching of a steel wire under end load.

For small loads it is found that the extension of the wire is roughly proportional to the applied load, Fig. 1.2. This linear relationship between load and extension was discovered by Robert Hooke in 1678; a material showing this characteristic is said to obey *Hooke's law*.

As the tensile load in the wire is increased, a stage is reached where the material ceases to show this linear characteristic; the corresponding point on the load-extension curve of Fig. 1.2 is known as the *limit of proportionality*. If the wire is made of a high-strength steel then the load-extension curve up to the *breaking point* has the form shown in Fig. 1.2. Beyond the limit of proportionality the extension of the wire increases non-linearly up to the breaking point.

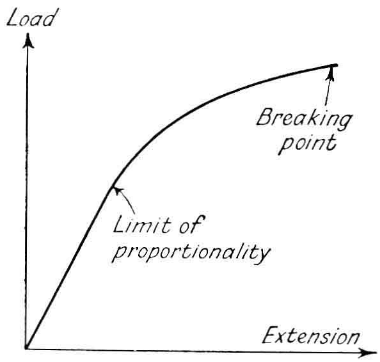


Fig. 1.2 Load-extension curve for a steel wire, showing the limit of linear-elastic behaviour (or limit of proportionality) and the breaking point.

The limit of proportionality is important because it divides the load-extension curve into two regions. For loads up to the limit of proportionality the wire returns to its original unstretched length on removal of the loads; this property of a material to recover its original form on removal of the loads is known as *elasticity*; the steel wire behaves, in fact, as a stiff elastic spring. When loads are applied above the limit of proportionality, and are then removed, it is found that the wire recovers only part of its extension and is stretched permanently; in this condition the wire is said to have undergone an *inelastic*, or *plastic*, extension.

In the case of elastic extensions, work performed in stretching the wire is stored as *strain energy* in the material; this energy is recovered when the loads are removed. During inelastic extensions work is performed in making permanent changes in the internal structure of the material; not all the work performed during an inelastic extension is recoverable on removal of the loads; this energy reappears in other forms, mainly as heat.

The load-extension curve of Fig. 1.2 is not typical of all materials; it is reasonably typical, however, of the behaviour of *brittle* materials, which are discussed more fully in §1.5. An important feature of most engineering materials is that they behave elastically up to the limit of proportionality, that is, all extensions are recoverable for loads up to this limit. The concepts of linearity and elasticity* form the basis of the theory of small deformations in stressed materials.

1.3 Tensile and compressive stresses

The wire of Fig. 1.1 was pulled by the action of a mass attached to the lower end; in this condition the wire is in *tension.* Consider a cylindrical bar *ab*, Fig. 1.3, which has

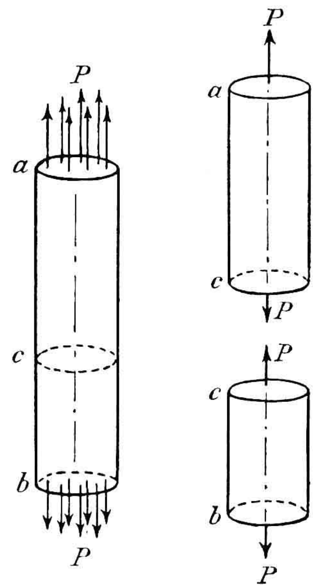


Fig. 1.3 Cylindrical bar under uniform tensile stress; there is a similar state of tensile stress over any imaginary normal cross-section.

a uniform cross-section throughout its length. Suppose that at each end of the bar the cross-section is divided into small elements of equal area; the cross-sections are taken normal to the longitudinal axis of the bar. To each of these elemental areas an equal tensile load is applied normal to the cross-section and parallel to the longitudinal axis of the bar. The bar is then uniformly stressed in tension.

Suppose the total load on the end cross-sections is P; if an imaginary break is made perpendicular to the axis of the bar at the section c, Fig. 1.3, then equal forces P are required at the section c to maintain equilibrium of the lengths ac and cb. This is equally true for any section across the bar, and hence on any imaginary section perpendicular to the axis of the bar there is a total force P.

* The definition of elasticity requires only that the extensions are recoverable on removal of the loads; this does not preclude the possibility of a non-linear relation between load and extension, although no such non-linear elastic relationships are known for materials in common use in engineering.

When tensile tests are carried out on steel wires of the same material, but of different cross-sectional areas, the breaking loads are found to be proportional approximately to the respective areas of the wires. This is so because the tensile strength is governed by the *intensity* of force on a normal cross-section of a wire, and not by the total force. This intensity of force is known as *stress*; in Fig. 1.3 the *tensile stress* σ at any normal cross-section of the bar is

$$\sigma = \frac{P}{A} \tag{1.1}$$

where P is the total force on a cross-section, and A is the area of the cross-section.

In Fig. 1.3 uniform stressing of the bar was ensured by applying equal loads to equal small areas at the ends of the bar. In general we are not dealing with equal force intensities of this type, and a more precise definition of stress is required. Suppose δA is an element of area of the cross-section of the bar, Fig. 1.4; if the normal force acting

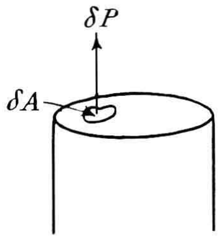


Fig. 1.4 Normal load on an element of area of the cross-section.

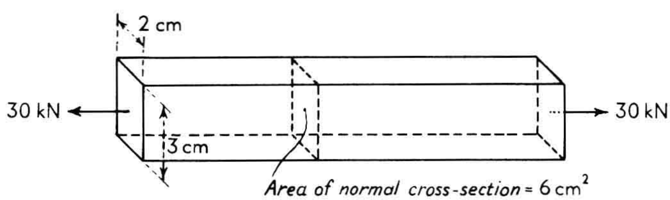
on this element is δP, then the tensile stress at this point of the cross-section is defined as the limiting value of the ratio $(\delta P/\delta A)$ as δA becomes infinitesimally small. Thus

$$\sigma = \operatorname*{Limit}_{\delta A \to 0} \frac{\delta P}{\delta A} = \frac{dP}{dA} \tag{1.2}$$

This definition of stress is used in studying problems of non-uniform stress distribution in materials.

When the forces P in Fig. 1.3 are reversed in direction at each end of the bar they tend to *compress* the bar; the loads then give rise to *compressive stresses.* Tensile and compressive stresses are together referred to as *direct stresses.*

Problem 1.1: A steel bar of rectangular cross-section, 3 cm by 2 cm, carries an axial load of 30 kN. Estimate the average tensile stress over a normal cross-section of the bar.



Solution

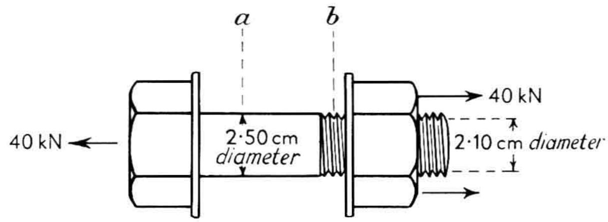
The area of a normal cross-section of the bar is

$$A = 0{\cdot}03 \times 0{\cdot}02 = 0{\cdot}6 \times 10^{-3}\ \text{m}^2$$

The average tensile stress over this cross-section is then

$$\sigma = \frac{P}{A} = \frac{30 \times 10^3}{0{\cdot}6 \times 10^{-3}} = 50\ \text{MN/m}^2$$

Problem 1.2: A steel bolt, 2·50 cm in diameter, carries a tensile load of 40 kN. Estimate the average tensile stress at the section *a* and at the screwed section *b*, where the diameter at the root of the thread is 2·10 cm.



Solution

The cross-sectional area of the bolt at the section *a* is

$$A_a = \frac{\pi}{4}(0{\cdot}025)^2 = 0{\cdot}491 \times 10^{-3}\ \text{m}^2$$

The average tensile stress at A is then

$$\sigma_a = \frac{P}{A_a} = \frac{40 \times 10^3}{0{\cdot}491 \times 10^{-3}} = 81{\cdot}4\ \text{MN/m}^2$$

The cross-sectional area at the root of the thread, section *b*, is

$$A_b = \frac{\pi}{4}(0{\cdot}021)^2 = 0{\cdot}346 \times 10^{-3}\ \text{m}^2$$

The average tensile stress over this section is

$$\sigma_b = \frac{P}{A_b} = \frac{40 \times 10^3}{0{\cdot}346 \times 10^{-3}} = 115{\cdot}6\ \text{MN/m}^2$$

1.4 Tensile and compressive strains

In the steel wire experiment of Fig. 1.1 we discussed the extension of the whole wire. If we measure the extension of, say, the lowest quarter-length of the wire we find that for a given load it is equal to a quarter of the extension of the whole wire. In general we find that, at a given load, the ratio of the extension of any length to that length is constant for all parts of the wire; this ratio is known as the *tensile strain.*