College Trigonometry

David E. Stevens

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PREFACE

Intent

A course in college trigonometry must meet the needs of students with diverse mathematical backgrounds and goals. Science and engineering students enroll in this course before beginning a traditional three-semester calculus sequence. Business and social science students enroll in this course before taking courses in finite mathematics, statistics, or introductory calculus. Other students enroll in this course because it is a prerequisite for elementary courses in astronomy, biology, physics, chemistry, computer science, architecture, or geology. In writing this text, I have tried to be sensitive to the needs of *all* these students by preparing them for more advanced courses and illustrating through real-life applied problems that knowledge of college trigonometry is fundamental to many disciplines.

Approach

For success in a college trigonometry course, it is essential that students become active rather than passive readers of the text. Therefore, I have written this text using an *interactive approach.* Each key mathematical concept is supported with a step-by-step text example with margin annotations and explanatory notes, and each text example is followed by a practice problem for the student to work out. The practice problem may ask the reader to check the preceding text example, work the preceding text example using an alternative approach, extend the preceding text example by asking for additional information, or try an entirely different problem that has similar mathematical steps. In effect, the practice problems require the student to become involved with the mathematics and constitute a built-in workbook for the student. *A complete detailed solution of each practice problem is given in the back of the text.*

Features

Written in a warm and user-friendly style, this is a traditional college trigonometry text with a contemporary flair in the sense that it starts to address the concerns of writing across the curriculum, group learning, critical thinking, and the use of modern technology in the math classroom. The following features distinguish this text from the existing texts in the market.

Applied Problems: To arouse student interest, each chapter opens with an applied problem and a related photograph. The solution to the applied problem occurs within the chapter after the necessary mathematics has been developed. Other applied problems from the fields of science, engineering, and business are introduced at every reasonable opportunity. They are clearly visible throughout the text and occur in separate subsections at the end of most sections. They are a highlight of this text and tend to show how college trigonometry relates to real-life situations.

Exercise Sets: The heart of any math textbook is its end-of-section exercise sets. It is here that students are given an opportunity to practice the mathematics that has been developed. The exercise sets in this text are broken into three parts: *Basic Skills*, *Critical Thinking*, and *Calculator Activities.*

Basic Skills: These exercises are routine in nature and tend to mimic the text examples that are worked-out in each section.

Critical Thinking: These exercises require the student to "think critically" and transcend the routine application of the basic skills to the next level of difficulty. Exercises in this group may require the student to draw upon skills developed in earlier chapters.

Calculator Activities: These exercises require the use of a calculator to solve basic-skill- and critical-thinking-type problems. Some of the exercises in this group ask the student to use a *graphing calculator* to solve problems in which standard algebraic methods do not apply. Problems that require the use of a graphing calculator are identified by the logo

Some of the exercise sets also contain problems that are *calculus-related.* Designed for students who are taking a college trigonometry course as a prerequisite to calculus, these exercises illustrate the algebraic support that is needed for simplifying derivatives, solving optimization problems, finding the area bounded by two or more curves, and so on. In calculus, the ratio of the change in the variable y to the change in the variable x is designated by $\Delta y/\Delta x$. In this text, we use the logo $\frac{\Delta y}{\Delta x}$ to identify the problems in the exercise sets that are calculus-related.

Chapter Reviews: To help students prepare for chapter exams, each chapter in this text concludes with an extensive chapter review. The chapter reviews are broken into two parts: *Questions for Group Discussion* and *Review Exercises.*

Questions for Group Discussion: In keeping with the interactive approach, these questions allow students to state in their own words what they have learned in the chapter. Since many of these questions have open-ended answers, they are ideally suited for class or group discussions and are extremely valuable to those who believe in cooperative or collaborative learning.

Review Exercises: These exercises reinforce the ideas that are discussed in the chapter and allow the instructor to indicate to the student the types of problems that may appear on a chapter test.

Cumulative Reviews: To help students pull together ideas from several chapters, cumulative review exercises are strategically placed after Chapters 3 and 7. The problems in these exercises are ungraded as far as difficulty and presented in a random order. Some problems are basic and similar to those already studied, while others are more challenging and require some creative thinking.

Pedagogy: Every effort has been made to make this a text from which students can learn and succeed. The following pedagogical features attest to this fact: *Caution notes*, flagged by the symbol Caution, help eliminate misconceptions and bad mathematical habits by pointing out the most common errors that students make.

Introductory comments at the start of many sections introduce vocabulary and inform the reader of the purpose of the section.

Boxed definitions, formulas, laws, and properties state key mathematical ideas and provide the reader with quick and easy access to this information.

Step-by-step procedural boxes indicate the sequence of steps that a student can follow in order to simplify trigonometric expressions, solve certain types of equations and inequalities, sketch the graph of various functions, find the inverse of a function, and so on.

Development

Chapter 1 provides an extensive review of the basic algebraic topics that are needed in the study of trigonometry such as interval notation, the coordinate plane, graphing techniques, and the functional concept. Section 1.4 lists eight basic functions and their graphs (constant, identity, absolute value, squaring, cubing, reciprocal, square root, and cube root functions) and then applies the vertical and horizontal shift rules, the x- and y-axis reflection rules, and the vertical stretch and shrink rules to sketch the graphs of several other related functions. These eight basic functions and their graphs are then used to discuss composition of functions, inverse functions, applied functions, and variation. The graphing calculator is introduced in this chapter and some of the exercises suggest using this tool to verify results or to explore new ideas.

Chapters 2, 3, and 4 represent the core of a college trigonometry course. *Chapter 2* introduces the trigonometric functions using both angle domains and real number domains. After an introductory section on angles and their measure, Section 2.2 defines the six trigonometric ratios for any angle θ and develops the fundamental trigonometric identities from these definitions. This section also introduces the trigonometric ratios of right triangles and the cofunction relationships. Section 2.3 evaluates the trigonometric functions of angles and uses the unit circle to show that the trigonometric functions of a real number may be found by considering the real number as the radian measure of its corresponding central angle. The graphs of the sine and cosine functions (Section 2.4) are obtained by observing the changes in the x-coordinate and y-coordinate of a point $P(x,y)$ on the unit circle. These graphs are then used to help list the important properties of the sine and cosine functions and to help develop the graphs of the other trigonometric functions. The restricted sine, restricted cosine, and restricted tangent functions are defined in Section 2.6 and the inverse of these functions are then discussed.

Chapter 3 discusses several applications of the trigonometric functions. Section 3.1 shows how the inverse trigonometric functions, in conjunction with a calculator, may be used to help solve a right triangle. This section also includes a variety of applied problems involving angles of elevation, angles of depression, and simple harmonic motion. The method of solving an oblique triangle by using the law of sines and law of cosines is developed in the next two sections of this chapter. Area formulas for oblique triangles, including Hero's formula, are also developed and applied to several problems. Section 3.4 defines a vector, introduces the vector operations of addition, subtraction, and scalar multiplication, and applies vectors to problems involving forces and displacements.

Chapter 4 discusses analytic trigonometry—a branch of mathematics in which algebraic procedures are applied to trigonometry. Section 4.1 suggests a general scheme for verifying trigonometric identities and Section 4.2 states a general procedure for solving trigonometric equations. Some of the exercises in these sections ask the student to use a graphing calculator to verify a trigonometric identity or to solve a trigonometric equation. Several important trigonometric formulas are developed in the other sections of this chapter. They include the sum and difference formulas, multiple-angle formulas, product-to-sum formulas, and sum-to-product formulas. The chapter concludes with a summary of all the important trigonometric identities and formulas discussed in Chapters 2 and 4.

Chapters 5, 6, and 7 include several other topics of interest in a college trigonometry course. *Chapter 5* discusses operations with complex numbers. The rules for adding, subtracting, multiplying, and dividing complex numbers in standard form are discussed in the first section. Section 5.2 defines the trigonometric form of a complex number and develops the rules for multiplying and dividing complex numbers in trigonometric form. DeMoivre's theorem is developed in Section 5.3 and used to find powers and roots of complex numbers. The chapter concludes by finding all the roots of a polynomial equation of degree n and showing that these roots, when plotted in the complex plane, represent the vertices of a regular n-sided polygon whenever $n \geq 3$.

Chapter 6 discusses the conic sections using Cartesian, polar, and parametric equations. The first three sections of this chapter state the geometric properties of each conic section and the distinguishing characteristics of their equations. Section 6.4 uses the rotation formulas and the discriminant to help sketch the graph of general quadratic equations in two unknowns in which $B \neq 0$. The polar coordinate system is discussed in Section 6.5 and the common definition of the conic sections is used to develop the polar equations for the parabola, ellipse, and hyperbola. Parametric equations of the conic sections are developed in Section 6.6 and the parametric mode on a graphing calculator is used to generate the graph of some polar equations.

Chapter 7 discusses the properties of real exponents, defines the exponential function, and develops the logarithmic function as the inverse of the exponential function. By letting the number of compounding periods in the compound interest formula increase without bound, the reader is shown how the number e develops in a real-life situation. The properties of logarithms are used to help graph functions containing logarithmic expressions (Section 7.4) and also to help solve exponential and logarithmic equations (Section 7.5). In Section 7.5, the graphing calculator is used to help solve some exponential equations that are not solvable by ordinary algebraic methods.

Supplements

The following supplements are available for users of this text.

1. *Instructor's Solution Manual* by Eleanor Canter—includes complete worked-out solutions to all the even-numbered exercises.
2. *Student's Solution Manual* by Eleanor Canter—provides worked-out solutions for the odd-numbered exercises from the text.
3. *Instructor's Manual with Test Bank* by Cheryl Roberts—includes sample syllabi, suggested course schedule, chapter outlines with references to videos, homework assignments, chapter tests, and a test bank of multiple choice questions and open-ended problems.
4. *Graphing Calculator Lab Manual*

5. *WESTEST* 3.0—computer generated testing programs include algorithmically-generated questions and are available to qualified adopters. Macintosh and IBM-compatible versions are available.
6. *Video series*—"In simplest Terms" produced by Anneberg/CPB Collection. The videos are referenced in the Instructor's Manual accompanying *College Trigonometry.*
7. *Mathens Tutorial Software*—generates problems of varying degrees of difficulty and guides students through step-by-step solutions.

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D.E. Stevens
Boston, Massachusetts, 1993

CONTENTS

CHAPTER 1

An Introduction To Functions and Graphs

The real estate tax T on a property varies directly as its assessed value V.

(a) Express T as a function of V if $T = \$2800$ when $V = \$112{,}000$.

(b) State the domain of the function defined in part (a), then sketch its graph.

(For the solution, see Example 3 in Section 1.6.)

1.1 Real Numbers and Interval Notation

◆ Introductory Comments

A chef requires 3 eggs and $\frac{2}{3}$ cup of sugar for a cake recipe. A meteorologist reports that the temperature is -4 °C and the barometric pressure is 29.35 inches. A student determines that the side of a right triangle is $\sqrt{5}$ units and the area of a circle is π square units. Each of the numbers

$$3, \quad \frac{2}{3}, \quad -4, \quad 29.35, \quad \sqrt{5}, \quad \text{and} \quad \pi$$

is an element of the **set of real numbers.*** These are the type of numbers that we work with every day. The set of real numbers has five important subsets.

1. **Natural numbers** or **Positive integers:** $\{1, 2, 3, 4, \ldots\}$
2. **Whole numbers:** $\{0, 1, 2, 3, \ldots\}$
3. **Integers:** $\{\ldots, -3, -2, -1, 0, 1, 2, 3, \ldots\}$
4. **Rational numbers:** {All real numbers of the form $\frac{p}{q}$, where p and q are integers and $q \neq 0$.}
   or
   {All decimal numbers that either terminate or repeat the same block of digits.}
   Examples: $\frac{3}{5}$, $\frac{-8}{3}$, $\frac{16}{1}$, $\frac{0}{4}$, 0.75, $-5.343434\ldots$
5. **Irrational numbers:** {All real numbers that are not rational.}
   or
   {All decimal numbers that neither terminate nor repeat the same block of digits.}
   Examples: $\sqrt{2}$, $-\sqrt{3}$, π, $\sqrt[3]{6}$, $3.050050005\ldots$

A geometric interpretation of the real numbers can be shown on the *real number line,* as illustrated in Figure 1.1. For each point on this line there corresponds exactly one real number, and for each real number there corresponds exactly one point on this line. This type of relationship is called a **one-to-one correspondence.**

The real number associated with a point on the real number line is called the

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*The concept of a set is often used in mathematics. A *set* is a collection of objects, and these objects are called the *elements* of the set. A *subset* of a given set is formed by selecting particular elements of the set. Braces { } are used to enclose the elements of sets and subsets.

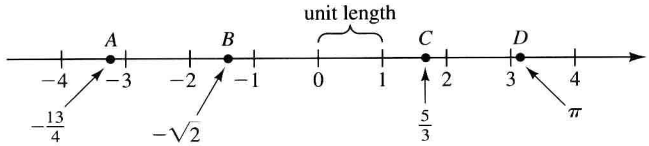

FIGURE 1.1
On the real number line there exists a *one-to-one correspondence* between the set of real numbers and the set of points on the line.

coordinate of the point. Referring to Figure 1.1, $-\frac{13}{4}$ is the coordinate of point A, $-\sqrt{2}$ is the coordinate of point B, $\frac{5}{3}$ is the coordinate of point C, and π is the coordinate of point D.

In this section, we discuss various sets of real numbers and the notation that is used to describe these sets. We begin by reviewing the inequality symbols that we use to compare two distinct real numbers.

Inequality Symbols

The real number line gives us a convenient way to compare two distinct real numbers a and b. If a is to the *right* of b on the real number line, then ***a* is greater than *b*,** and we write

$$a > b$$

Referring to Figure 1.1, since π is to the right of $\frac{5}{3}$, we have $\pi > \frac{5}{3}$.

If a is to the left of b on the real number line, then ***a* is less than *b*,** and we write

$$a < b$$

Referring to Figure 1.1, since $\frac{5}{3}$ is to the left of π, we have $\frac{5}{3} < \pi$. In general, for real numbers a and b,

$$a < b \quad \text{if and only if} \quad b > a$$

Note: The phrase *if and only if* occurs frequently in mathematics. In the preceding statement, it implies two statements:

1. If $a < b$, then $b > a$ and, conversely,
2. If $b > a$, then $a < b$.

The symbols $>$ and $<$ are called **inequality symbols,** and expressions such as $a > b$ and $a < b$ are called **inequalities.** Two other inequality symbols used frequently are

$\leq$ read "less than or equal to" and $\geq$ read "greater than or equal to."

Inequalities can be used to indicate if a number is *positive, negative, nonnegative,* or *nonpositive,* as shown in Table 1.1

Table 1.1
Some inequalities and their meanings

Inequality			Meaning
$a > 0$	or	$0 < a$	a is positive
$a < 0$	or	$0 > a$	a is negative
$a \geq 0$	or	$0 \leq a$	a is nonnegative
$a \leq 0$	or	$0 \geq a$	a is nonpositive

Figure 1.2 shows three distinct real numbers a, b, and c on a real number line. To indicate that b is between a and c on this line, we can write either

FIGURE 1.2
Three distinct real numbers on the real number line with $a < b < c$.

$$a < b < c \quad \text{or} \quad c > b > a$$

Each of these expressions is a **double inequality.** When using expressions like $a < b < c$ and $c > b > a$, be sure all the inequality symbols point in the same direction. For example, the expression $a < c > b$ is completely meaningless.

EXAMPLE 1 Rewrite each statement using inequality symbols.

(a) a is at most 6. **(b)** b is at least -2.

(c) c is nonnegative and less than 10.

SOLUTION

(a) a is at most 6 is written as $a \leq 6$ **(b)** b is at least -2 is written as $b \geq -2$

(c) c is nonnegative *and* less than 10 is written as

$0 \leq c$ *and* $c < 10$ or, more compactly, $0 \leq c < 10$. ◆

PROBLEM 1 Repeat Example 1 for each statement.

(a) a is not more than 8. **(b)** b is negative and at least -4. ◆

◆ Distance between Two Points on the Real Number Line

The distance between zero and a number a on the real number line, without regard to direction, is called the *absolute value* of a and is denoted $|a|$. Because distance is independent of direction and is always nonnegative, the absolute value of any real number is also nonnegative. That is, $|a| \geq 0$. A more formal definition of **absolute value** follows.