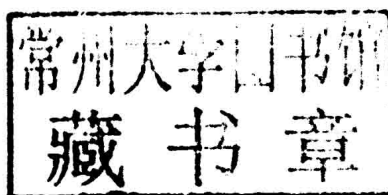
The book cover features a background of a cloudy sky. A dark blue horizontal band spans the middle of the cover, containing the title in white serif font. In the top left corner, there is a small, dark rectangular area with some faint, illegible markings.

Applied Thermodynamics for Meteorologists

SAM MILLER

APPLIED THERMODYNAMICS FOR METEOROLOGISTS

SAM MILLER
Plymouth State University



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32 Avenue of the Americas, New York, NY 10013-2473, USA

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www.cambridge.org

Information on this title: www.cambridge.org/9781107100718

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First published 2015

Printed in the United States of America

A catalog record for this publication is available from the British Library.

Library of Congress Cataloging in Publication Data

Miller, Sam, 1961– author.

Applied thermodynamics for meteorologists / Sam Miller, Plymouth State University.
pages cm

Includes bibliographical references and index.

ISBN 978-1-107-10071-8 (hardback)

1. Atmospheric thermodynamics. I. Title.

QC880.4.T5M55 2015

536'.7—dc23 2015002801

ISBN 978-1-107-10071-8 Hardback

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APPLIED THERMODYNAMICS FOR METEOROLOGISTS

This textbook on atmospheric thermodynamics is for students of meteorology or atmospheric science. It also serves as a reference text for working professionals in meteorology and weather forecasting. It is unique because it provides complete, calculus-based derivations of basic physics from first principles and connects mathematical relationships to real-world, practical weather forecasting applications. Worked examples and practice problems are included throughout.

Sam Miller began his career in meteorology as a weather observer in the U.S. Air Force (USAF) in 1982. In 1984 he attended the weather forecaster school at Chanute Air Force Base, Illinois, and graduated with honors. He worked as a USAF weather observer in Maine, and as a forecaster in northern California, upstate New York, and Adana, Turkey. He eventually earned the rank of Technical Sergeant and left the USAF in 1989 after more than ten years on active duty. Miller then attended the University of New Hampshire and earned a Bachelor of Science in Physics (1996), a Master of Science in Earth Sciences: Oceanography (1999), and a PhD in Earth Sciences (2003), while also working as a weather observer in Portsmouth, New Hampshire, a research scientist at the university, and a teacher in New Hampshire's public university system. From 2003 to 2005, Miller was a weather forecaster with the U.S. National Weather Service in Anchorage, Alaska. Since 2005 he has served as a professor of meteorology at Plymouth State University, New Hampshire, where he teaches courses in basic meteorological analysis, atmospheric thermodynamics, instrumentation, weather forecasting, satellite meteorology, and radar meteorology. Miller is a member of Phi Beta Kappa, the American Meteorological Society, the Royal Meteorological Society, and many other professional organizations. He has published research papers on the sea breeze and has worked extensively as a meteorological consultant in legal matters.

This book is dedicated to weather forecasters everywhere.

Acknowledgments

I want to sincerely thank:

- My wife Virginia and daughter Julia, for patiently indulging me while I invested more than two years of my life in this book, and my parents Catherine Carter-Hancock and Edward J. Miller, who believed in me, even when the determination to do so was justifiably viewed with suspicion.
- The many fine instructors who taught me at the U.S. Air Force's (USAF's) Weather Observer and Weather Forecaster Schools at Chanute Air Force Base, Illinois, in 1982 and 1984, such as Dr. Glenn Van Knowe (he was a technical sergeant back then), *and* the professors and staff at the University of New Hampshire, who helped me thrash through degrees in physics, oceanography, and Earth sciences, between 1989 and 2003.
- Master Sergeant Brian Hammond, who was my boss at two USAF weather stations, and Lieutenant Colonel R. Bruce Telfeyan, who was my boss at a third weather station. Both of these gentlemen provided outstanding examples of careful, conscientious operational meteorologists who took time to mentor youngsters.
- Ms. Özlem Bilgin, Mr. Zeki Çelikbaş, Dr. Ahmet Öztopal, and, *most of all*, Dr. Mikdat Kadioğlu of Istanbul Technical University (ITU), who helped me immensely during my sabbatical leave in Türkiye in the spring of 2012, by providing me with an office and technical support in the ITU Meteorological Engineering Department, where I wrote the first draft of this book.
- ITU's Dr. Aydın Mısırlıoğlu, Dr. Sevinç Sırdaş, Dr. Ali Deniz, Dr. Selahattin İncelik, Dr. Yurdanur Ünal, Dr. Melike Nikbay, Dr. Barış Önel, Dr. Zerefşan Kaymaz, Dr. Levent Şaylan, and Mr. Mehmet Ünal for their kindness and help.
- My colleagues in Plymouth State University's meteorology program, Dr. Jim Koerner (who also served on my PhD committee; retired now), Dr. Eric Hoffman, Dr. Lourdes Aviles, Dr. Eric Kelsey, Dr. Jason Cordeira, Dr. Joe Zabransky (retired), Dr. Lisa Doner, Mr. Anthony (Toby) Fusco, and Mr. Brendon Hoch. The majority of the content of this book began as the notes I use in the thermodynamics course they allow me to teach our undergraduates.
- Ms. Marsi Wisniewski, who helped keep me on the linguistic straight and narrow. Dr. Tom Boucher of Plymouth State University's Mathematics Department checked my math in a few places. Dr. Dennis Machnik checked some of my physics in the first few chapters.

- Four anonymous reviewers, who read six chapters of this text and made several very valuable suggestions, and my editor at Cambridge University Press, who helped me through the publication process.

And most of all, my students at Plymouth State University, who – through their enthusiasm, intelligence, and sense of humor – remind me of why teachers do this kind of work.

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1

Basic Concepts and Terminology

1.1. What Is Thermodynamics?

Thermodynamics is the study of energy and its transformations. Several books on this subject begin with that line, or something very much like it, and I can't think of a better way to start this one. In my experiences as a student, teacher, and user of thermodynamics, I've read several of these books, each one directed to a slightly different audience. The purpose of *this* book is to provide undergraduate meteorology students with a solid theoretical (physical and mathematical) basis for understanding "energy and its transformations" in the Earth's atmosphere, and an appreciation for both the *limitations* and the *practical usefulness* of the thermodynamic models we use to describe the atmosphere. If, at the end of this book, you know where these ideas came from, what their weaknesses are, and how to apply them to your job as a working meteorologist, then you have learned what I hoped you would learn.

It's worth saying a little more about the word *model*. Meteorologists are physical scientists, and as such we like to use equations (models) to describe what's happening in the Earth system. This is pragmatic, because it makes it possible to do quantitative research and make weather forecasts. But some scientists forget about the differences between their models and the real objects they're studying, and this is a mistake. A model (i.e., the physics we use) gives us a glimpse at an underlying reality, but the real atmosphere is much more complex. For example, there are no real "isolated parcels," that is, there are no little packets of air that do not exchange mass or heat with their surrounding environment. This idea is just an approximation that makes it possible for us to solve the equations and make weather forecasts. Over some short period of time, it isn't a *bad* approximation, but it's still only a simplified picture of the real atmosphere. Remember that when you're using these equations.

Thermodynamics evolved into a well-integrated science from several separate threads over the last three centuries. Scholars from many different parts of the world, and highly varied scientific disciplines, have contributed to its evolution. In addition

to this text, I'd like to recommend that you read von Baeyer's fun and interesting *Warmth Disperses and Time Passes*,¹ which describes this history very well. One of the points that von Baeyer mentions early in his book is the mistaken belief, once widely held, that heat is some kind of fluid that "flows" through matter. This fluid – called *caloric* – was thought to come from the spaces between the atoms of a substance, and could be "squeezed out" under the right circumstances. We now know this is wrong, but you'll find that many of the ideas we use to describe heat are the same ideas we use to describe fluid flow. We still use the idea of heat as a fluid, although we know it's only an analogy, because it works very well under *some* circumstances.

1.2. Systems

The fundamental model underlying practical atmospheric thermodynamics breaks the atmosphere up into components. The first component, called a *system*, is an object, such as a quantity of matter consisting of many particles, which take up a volume of space (see Figure 1.1). It is separated from its *surroundings* by a *boundary*, which is an imaginary barrier that may or may not allow mixing between the system and the surroundings. In meteorology, we use the word *parcel* as a substitute for "system," and *environment* or *ambient* (or *background*) *air* as a substitute for "surroundings." This model assumes that changes can occur to a system completely independent of its surroundings, or as a result of some forcing *by* the surroundings, but that these internal changes have no important effect *on* the surroundings.

The parcel may be *open*, *closed*, or *isolated*.

- In an open parcel, the boundary permits the exchange of both energy and matter between the parcel and its environment.²
- In a closed parcel, the boundary permits the exchange of energy but prevents the exchange of matter.
- In an isolated parcel, the boundary prevents the exchange of both matter and energy with the environment.

Open parcels (exchanging both matter and energy with the environment) can often be treated as closed parcels (exchanging energy only) by considering the surface-to-volume ratios of small and large spheres. Recall that the surface area (A) of a sphere is given by:

$$A = 4\pi r^2 \tag{1.1}$$

where r is the radius of the sphere. The volume (V) of a sphere is given by:

$$V = \frac{4}{3}\pi r^3 \tag{1.2}$$

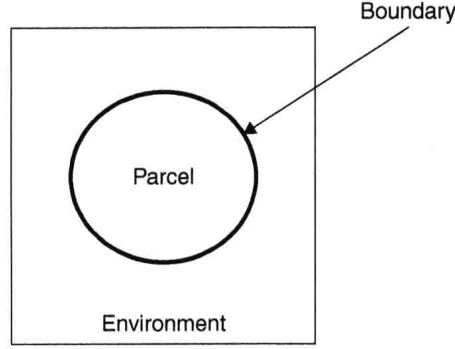


Figure 1.1. Parcel, environment, and boundary.

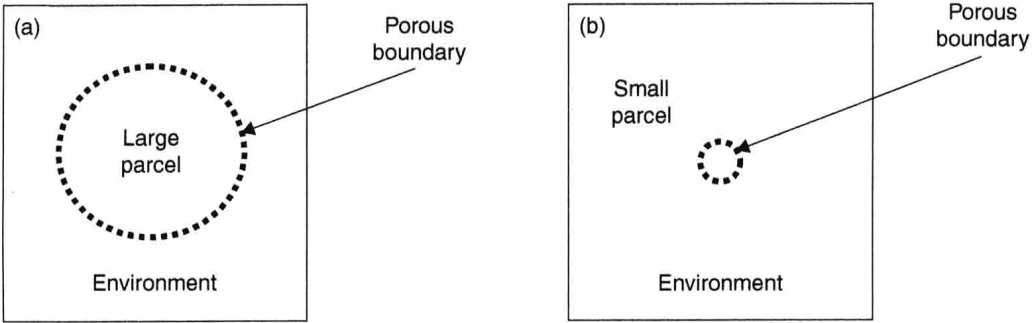


Figure 1.2. (a) Large and (b) small parcels with porous boundaries. Changes to the internal volume of the large parcel occur slowly, because of the relatively small surface area enclosing it. Changes to the internal volume of the small parcel are assumed to have already occurred.

The surface-to-volume ratio is therefore:

$$\frac{A}{V} = \frac{4\pi r^2}{\frac{4}{3}\pi r^3} = \frac{3}{r} \quad (1.3)$$

which gets smaller as the radius of the sphere increases. As the volume increases (scaled to r^3), the surface area (scaled to r^2) enclosing it doesn't increase as quickly. Because exchanges between a parcel and its environment occur through the boundary, (1.3) implies that there is a diminishing effect on the internal volume of a parcel as its radius increases.

We can treat open parcels as closed parcels by making the following simplifying assumptions (see Figure 1.2):

- For *small* parcels, the surface-to-volume ratio is *large*, and we can assume that any changes to the internal volume of the parcel that are going to occur *have already done so*.

- For *large* parcels, the surface-to-volume ratio is *small*, and we can assume that the rate of change in the internal volume of the parcel is so small that it can be ignored for short periods of time.

Two questions ought to immediately pop into your mind. The first is “what about middle-sized parcels?” The answer to this one is not very satisfying, but we’ll use it anyway: there are no “middle-size” parcels in our model atmosphere. The second question is “who cares?” *Why is this important?* To answer that question, we have to talk about energy and equilibrium.

1.3. Energy and Equilibrium

Energy is an abstract idea for something that appears in nature in many different forms. In practical terms, it is often described as the ability to do *work*, and so energy and work share the same physical units (*Joules*, in the *Système International d’Unités* or SI system). Energy can be divided into categories such as *external* and *internal*. In thermodynamic terms, “external energy” is the energy some object has in relation to its surroundings, such as the energy of position (known as *potential energy*), or the energy of motion (*kinetic energy*). “Internal energy” is determined using information about the internal *state* of the system. The “state” is the collection of physical properties describing the system, such as its temperature.

The internal state of a system may undergo changes, such as phase changes between solids, liquids, or gases, but, if left undisturbed, eventually all of these changes will cease, and the system reaches *internal equilibrium*. A system in internal equilibrium has a fixed set of properties that can be measured with a high degree of precision. Any small perturbations to a system in internal equilibrium won’t result in any large changes to its state; once the perturbation ends, the system will return to equilibrium. An open or closed system can also be in *external equilibrium*, which means that any exchanges between the system and the environment are exactly balanced by exchanges in the opposite direction.

One example of external equilibrium is the classic bowling ball at the bottom of a valley (Figure 1.3). A small nudge to the bowling ball to either the left or right may get it moving back and forth a little, but eventually it will settle back at the lowest point in the valley. This is equivalent to stating that the bowling ball is in a minimum potential energy state. As long as the kinetic energy of the nudge is less than the energy necessary to push the bowling ball all the way up the hill, it will never escape,³ and over time, will return to its equilibrium position at the bottom of the valley. Another example of external equilibrium involves temperature: When you bring two objects together that begin at different temperatures, over time the temperature difference between them will disappear, and reach an equilibrium point that’s somewhere between the two starting temperatures. We’ll return to this example again in Chapter 3.

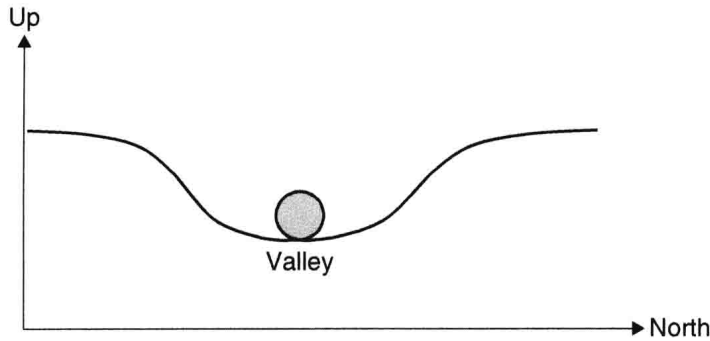


Figure 1.3. A state of external equilibrium: Bowling ball at the bottom of a valley. In classical physics, the only way to get the bowling ball out of this potential energy well is to impart enough kinetic energy to push it up the hill on either side.

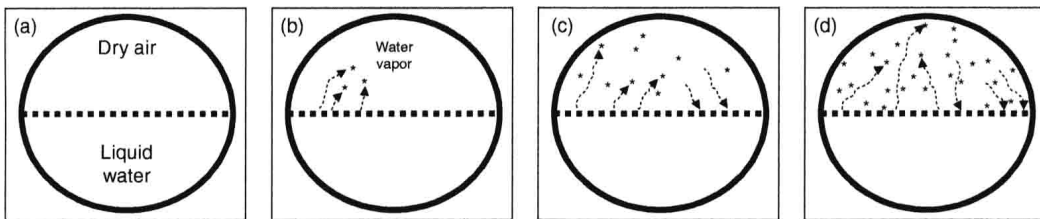


Figure 1.4. Saturation – a compound system in equilibrium. Initially (a) the system consists of dry air above and liquid water below. In (b) evaporation begins, adding water vapor to the air. The vapor molecules float around in the air above the liquid water. In (c), condensation begins, and some of the vapor molecules stick back onto the surface of the liquid water. At first the evaporation rate is much greater than the condensation rate, but as the amount of water vapor in the air increases, the condensation rate increases too. Eventually (d) the evaporation and condensation rates are the same – that is, reach an equilibrium state – and the air is saturated.

An example of internal equilibrium is a compound system consisting of a subsystem of water vapor plus dry air, and another subsystem of liquid water (Figure 1.4). The two subsystems are open to each other (but isolated from the external environment), meaning that both mass and energy can move back and forth between them (but nothing can leave or enter from outside). Water vapor molecules will escape from the surface of the liquid water (evaporate), causing it to lose mass. At the same time, vapor molecules will also stick back to the surface of the liquid water (condense), causing the liquid to gain mass. If the two phase changes are occurring at the same rate, the system is in equilibrium. Evaporation and condensation are still occurring, but the two exactly cancel each other. Once equilibrium is reached, there will be no noticeable change in the masses of the liquid water or the water vapor over time. This kind of equilibrium is called *saturation*.

1.4. Extensive and Intensive Properties; State Variables

Extensive properties are dependent on the mass of the parcel. One example of an extensive property is the *total heat capacity* (C), which can be used to describe the transfer of heat when two objects with different temperatures are brought into direct contact. In Table 1.4, volume (V) is an extensive property. *Intensive properties* are independent of mass, that is, the mass of the parcel is divided out of the property. These are also referred to *specific properties*. In Table 1.4, specific volume (α) is an intensive property, and has units of m^3/kg . As we begin to discuss work and heat and the *Ideal Gas Law*, specific volume will be used quite a bit.

A parcel is called *homogeneous* if its intensive properties are the same at every point. If a parcel is composed of an *ideal gas*,⁴ and is homogeneous and in equilibrium, the Ideal Gas Law can be used to describe the relationship between its state variables. For this reason, the Ideal Gas Law belongs to a class of mathematical models known as *equations of state*. There are additional equations of state that can be invoked if a gas is not “ideal.”

For a parcel composed of an ideal gas, the Ideal Gas Law describes the relationship between its *state variables* – that is, its pressure (p), volume (V), mass (m), and temperature (T). If mass is held constant, or the behavior of the parcel is described in terms of intensive (specific) properties, mass can be eliminated, leaving a total of only three state variables in the relationship. Knowledge of any two state variables provides information about the third, through an equation of state. There are additional state variables, such as chemical composition, internal energy (u), and enthalpy (h). The latter two will be discussed later.

Now, to answer the question posed in the preceding text: we want a “large” or “small” parcel (but not a “medium-size” parcel), so that we can, in turn, assume it reaches equilibrium, making an equation of state (such as the Ideal Gas Law) a valid way to describe the relationship between its state variables. Without this, we can’t write an equation relating pressure, volume, and temperature to each other, which, you’ll see in later chapters, would leave us unable to model most of the atmosphere.

1.5. Fundamental Quantities and Units

In classical physics (i.e., the physics before relativity and quantum mechanics intruded in the twentieth century), there were seven *fundamental* quantities. By fundamental, we mean quantities that are not dependent on one another and cannot be reduced to functions of other quantities. These are:

- Length,
- Mass,
- Time,
- Temperature,