Progress in Mathematics

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Toshiki Mabuchi
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Alan Weinstein
Editors

Geometry and Analysis on Manifolds

In Memory of Professor Shoshichi Kobayashi

Birkhäuser

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ISSN 0743-1643 ISSN 2296-505X (electronic)
Progress in Mathematics
ISBN 978-3-319-11522-1 ISBN 978-3-319-11523-8 (eBook)
DOI 10.1007/978-3-319-11523-8

Library of Congress Control Number: 2015932660

Mathematics Subject Classification (2010): 53-06, 32Q45, 53C15, 32Q15, 32Q20, 58C10

Springer Cham Heidelberg New York Dordrecht London

Printed on acid-free paper

Springer International Publishing AG Switzerland is part of Springer Science+Business Media (www.springer.com)

Birkhäuser

Progress in Mathematics
Volume 308

More information about this series at http://www.springer.com/series/4848

Preface

Prof. Shoshichi Kobayashi was a recognized international leader in differential geometry and complex geometry. As detailed below, throughout a career lasting over five decades, he contributed crucial new ideas which are still fundamental in these research areas, particularly within complex geometry.

Prof. Kobayashi's early work contributed to a wide range of topics in differential geometry, for example through his well-known work on the study of transformation groups and on the Frankel conjecture, which characterized complex projective spaces. In complex geometry, his work on Hermitian-Einstein holomorphic vector bundles led to the so-called Kobayashi–Hitchin correspondence.

The most well-known mathematical object bearing his name is the Kobayashi pseudometric, which he introduced in 1967. This has become an essential tool for the study of mappings between complex manifolds. He identified a crucial property of "good" spaces, which he called hyperbolic, in this theory, and which in recognition of this fundamental insight are now known as Kobayashi hyperbolic spaces.

Perhaps most importantly, his books, especially the two volume "Foundations of Differential Geometry" with Katsumi Nomizu, have taught differential geometry and complex geometry to generations of students and other researchers. His writing is known for its succinctness and clarity, and the book commonly known as Kobayashi–Nomizu has been in publication for over 50 years.

Over the course of his long career, Prof. Kobayashi produced a most impressive mathematical legacy in the form of 35 Ph.D. students, a long list of contributions to differential geometry, and many influential monographs. As part of his interest in the dissemination of mathematics, he contributed immensely to the communication between US and Japanese mathematicians. He greatly enjoyed mathematical discussions with young researchers, and with great warmth always encouraged them. Prof. Shoshichi Kobayashi lives on in the form of his magnificent mathematical legacy and our memories of a wonderful man.

We are very grateful to Professor Shoshichi Kobayashi for his distinguished contributions to differential geometry and complex geometry, and we are pleased to be able to publish a volume dedicated to him consisting of invited contributions from his PhD students, co-workers and colleagues working on topics close to his research areas. All papers are refereed to the standards of an excellent journal.

Finally, we thank the editors at Birkhäuser for their constant encouragement for this publication.

The editors

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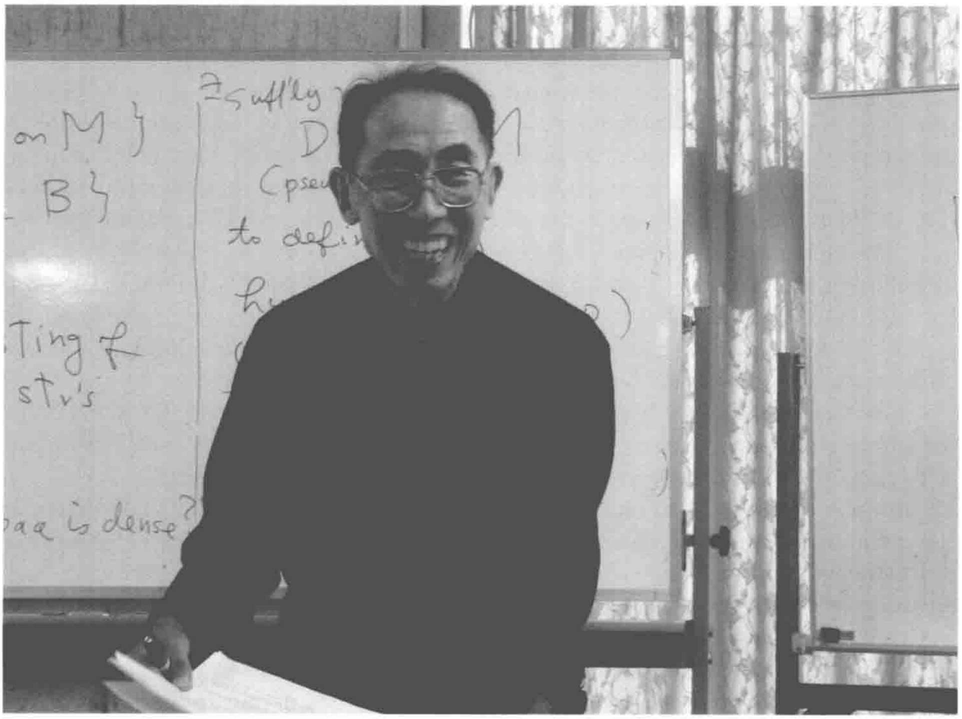

Shoshichi Kobayashi

Contents

Part 1

About Shoshichi Kobayashi

Progress in Mathematics, Vol. 308, 3–12

In Memory of Professor Shoshichi Kobayashi

Takushiro Ochiai

As I stop and reflect on the life and academic achievements of Professor Shoshichi Kobayashi, "the beautiful theorem" on how a great mathematician should be comes to mind. His original and sharp theorems read like masterpiece short stories. His writings splendidly harmonize and play like a symphony. Though I am aware of my inability to reach the height of his talent, I dare to write this article to introduce the fine personal character and remarkable academic achievements of Professor Shoshichi Kobayashi.

I. Encounter with good teachers and friends

Shoshichi, the first of five sons of Kyuzo and Yoshie Kobayashi, was born on January 4th in Kofu City, Yamanashi Prefecture, Japan, his parents' hometown. Soon after his birth, his family moved to Tokyo in order for his father to start a new business.

He entered elementary school in the Kouenji school district in Suginami-Ku. According to his own testimony, his teacher told him about the Pythagorean Theorem during a class break when he was in the 5th or 6th grade. "Since I knew neither the necessity for nor the existence of the proof of the theorem at that age, I measured the three edges of several right triangles, verified the fact by myself and was impressed. Since the first step of getting into mathematics is to be deeply moved by a good theorem, this was probably the first step in my case." (Sugaku Seminar, May, 1973)

This is the kind of situation which many renowned great mathematicians have often experienced, another example of how "Genius displays itself even in childhood."

Shoshichi went on to Chitose Junior High Hchool in Setagaya-Ku in 1944. At that time, when the air raids by American B29 bombers hit the Tokyo area, which started on November 14, 1944 during the Pacific War, he often had to run to a nearby air raid shelter whenever an alarm sounded. According to his

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This is an English translation of an extended version of my article "Kobayashi – sensei wo shinonde", which appeared in Sugaku Seminar, 2013.

For more information of Professor Shoshichi Kobayashi, refer to the website www.ShoshichiKobayashi.com.

younger brother Hisashi, Shoshichi brought mathematical books and candles with him every time, which shows the fervent passion for mathematics that he had throughout his whole life.

When the air raids became intense in 1945, which was the last year of the Pacific War, Shoshichi moved to his aunt's house in Kofu City and changed his school to Kofu Junior High School. As was common in those days, he spent every day either by working from the morning to the evening on farms on fine days or by attending lectures at school on rainy days.

Since the medicine factory managed by the Army where his father was employed was evacuated into Nagano Prefecture, he changed in June 1945 from the school in Kofu that he had attended only several days to Nozawa Junior High School (the present Nozawa North Senior High School) located in Nozawa, Saku-City in Nagano Prefecture. As a fourth-year student at Nozawa Junior High School, Shoshichi learned fairly advanced mathematics after the school hours from a newly-appointed teacher, Mr. Muneo Hayashi (who had studied function theory of one complex variable under the supervision of Professor Noshiro at Nagoya University) who had moved to Nozawa on his medical doctor's advice.

During this time, Shoshichi was introduced to the well-known book on the theory of functions of one complex variable by Tanzou Takeuchi and got to know about the existence of this branch of mathematics. Mr. Hayashi and Shoshichi maintained their teacher-student relationship throughout their whole lives. Shoshichi recollected, "He taught me the joy of being impressed by beautiful mathematical theorems." Great researchers are often blessed by encountering an excellent teacher at just the opportune time.

The junior high school system of those days was a five-year program. But if exceptionally excellent students in junior high schools passed an extremely difficult entrance examination taken in the fourth year, they were allowed to enter, by skipping the fifth year, one of the elite high schools. In April of 1948, having passed the above-mentioned examination, Shoshichi entered 'Number One Higher School' (Daiichi Koto Gakko or Ichiko, for short) in Tokyo, the most difficult Higher School to get into. In 1949, Japan's school system was overhauled and changed to a new system, which has been in effect up to now. Ichiko was terminated and became the Junior College of "Todai" (the University of Tokyo), at which time all the freshmen of Ichiko lost their status. Therefore, Shoshichi had to take another entrance examination, which he successfully passed, to enter the junior college of "Todai." He then continued on to the Mathematics Department as a junior in the School of Science.

This period was an important phase of changes in mathematics, as classical differential geometry was transforming itself into modern differential geometry. That is, while the genealogy of present-day differential geometry has been the legacy of Gauss (1777–1855), Riemann (1826–1866), Cartan (1869–1951), and Chern (1911–2004), those days were the time when their geometry was about to be developed into the strict and rigorous form of today. However, in those days, lectures were not provided according to a well-organized curriculum as they are today;

the students had to learn on their own all the necessary mathematical materials through discussions with colleagues or at seminars with instructors. Exceptionally, Kobayashi was able to study the then latest results in differential geometry through Professor Kentaro Yano's lectures (which were carried out based on the manuscript of the book "Curvature and Betti Numbers", a collaboration with Bochner, before its publication). They were lectures into which he infused the breath of a new era of differential geometry in a novel theory drawing the topological property of a Riemannian space from conditions on curvature, using the maximum principle in analysis. In the beginning, Kobayashi intended to study complex function theory of several variables, but he changed his plan and joined Professor Yano's seminar on differential geometry and chose harmonic integral theory as his study topic. There were students who chose Rauch's very original paper and Morse's theory. All these "Seminars" dealt with topics running ahead of the times and were filled with joy and enthusiasm. Thus, Kobayashi was led through Yano's lectures and seminar, to the forefront of research in the differential geometry of those days.

Kobayashi graduated from the mathematics department in 1953. Professor Yano had received a scholarship for graduate studies from the French Government and had spent two years (1936–1938) in France under E. Cartan at the same time as S.S. Chern by coincidence. At Yano's suggestion, Shoshichi took the examination for a scholarship of the French Government for graduate studies when he was a senior in the mathematics department. He won the scholarship and from 1953 to 1954 he studied at the Poincaré Institute of Mathematics in Paris for a half-year and at the University of Strasbourg for another half-year. At the University of Strasbourg, he studied under Charles Ehresmann (1905–1975). Ehresmann completed the theory of fiber bundles with his student Jacques Feldbau in the 1940s (a few years before and after the end of World War II) and developed the geometry of Cartan's connection (which was rather difficult to understand at the time) in a rigorous form that is in use today. Thus, Kobayashi chose Cartan's connections as the subject for his thesis. An encounter with Katsumi Nomizu, eight years his senior, who was in France for research in differential geometry, changed Kobayashi's fate significantly during this period. Kobayashi recollected that not only had he learned the theory of holonomy and transformation groups from Nomizu and André Lichinerowicz, but also he was egged on by Nomizu to pursue "studying in the United States instead of returning directly to Japan." (Sugaku Seminar, July, 1982)

From 1954 to 1956, Kobayashi became a research assistant at the invitation of Carl B. Allendoerfer of the University of Washington in Seattle, who had proven the Gauss–Bonnet theorem for higher dimensions. From Allendoerfer, Kobayashi received lectures that traced the historic development of the Gauss–Bonnet theorem. He also received clear lectures on complex manifolds from H.C. Wang, who was newly appointed when Kobayashi was in his second year, and became seriously interested in complex manifolds. But he was hesitant about cutting ties with differential geometry and started studying the Bergman metric, when he heard that Bergman would be in Seattle for a few days and he went to meet with him. However, Kobayashi's most valuable experience in mathematics during his two-year stay in Seattle was the Summer Institute of differential geometry, which was held in the summer of 1956 and lasted more than a month. Kobayashi, the youngest among the participants, recollected: "Allendoerfer and Boothman were most senior, followed by the younger group of Yano, Chern and Samuelson, as well as the even younger group of Ambrose, Singer, Bott, Calabi, Eells and others. By the discovery of the theorem of Gauss–Bonnet, Chern classes and so on, it was the days when the field of differential geometry, which had become a rather localized and isolated mathematical discipline by then, was changing into a global differential geometry, which would have something to do with such subjects as topology, differential topology, algebraic geometry and analysis, and was becoming a very attractive field of research. Every delivered talk was rich in its content, tearing down the then-existing walls around differential geometry and showing that differential geometry ushered in a new era." Many young mathematicians must have participated. As he had become acquainted with quite a few differential geometers, he recalled that the summer institute was a most valuable event for mathematics. Kobayashi received his Ph.D. degree in 1955.

A most important encounter during that time for Kobayashi was the one with his wife Grace (Yukiko), to whom he was introduced by the fiancee of Kobayashi's close friend. They became engaged shortly afterwards and were married a year later. According to the testimony of Hisashi, his younger brother, Kobayashi's character changed greatly by this marriage, a change which was even reflected in photographs, where he was always smiling or laughing. The author thinks that his innate character of being tolerant and broad-minded with other people began to reveal itself after his marriage to Yukiko.

Shoshichi spent two years from the fall of 1956 as a post doctoral fellow at the Institute for Advanced Study in Princeton, and then a year and a half (from the fall of 1958 until the end of 1959) at MIT without any teaching duty, thanks to the support from a grant of Ambrose and Singer. In the fall of 1959, Kobayashi received suddenly a letter from Professor Kelley, the then chair of Mathematics Department of UCB (the University of California, Berkeley), offering him a position at Berkeley. Kobayashi, having heard that Chern was about to move from Chicago to Berkeley, accepted this offer most willingly. Later, Kobayashi came to know that this offer came through the recommendation of Chern. When he moved from Chicago to Berkeley in 1960, Chern was asked to form a new group of geometry. As Kobayashi was required to change his visa status from J-1 to an immigrant visa, Chairman Kelley arranged for him to teach at the University of British Columbia, Canada for two and a half years, after which he settled in at Berkeley in the summer of 1962.

Kobayashi was promoted to associate professor in 1963, to full professor in 1966 and retired, in 1994 under the Voluntary Early Retirement Incentive Program (VERIP) of the University of California and became Professor Emeritus retaining his office. Together with Chern, he contributed over 50 years as a leader in Berkeley, a world center of differential geometry.

Kobayashi was a mathematician globally respected not only in Japan, but also in the whole world. Professor Tadashi Nagano, who was his elder classmate in the mathematics department at the University of Tokyo and a research collaborator, stated in an article introducing Kobayashi's mathematical achievements in the occasion when Kobayashi was awarded the first Geometry Prize of the Mathematical Society of Japan in 1987: "Kobayashi has many friends, and values them. People are glad to react to the stimulus which he emits and achieve successful results. It shows the height of the quality of his mathematics." On another occasion, Professor Ogus, the chairman of the mathematics department of UCB, who had been a longtime colleague of Kobayashi, delivered a memorial address at the funeral service: "He was also a very kind man with a quiet strength and a disarming smile, whose company was simultaneously comforting and awe-inspiring." No doubt, the above feelings are shared by all who had the pleasure of getting to know Shoshichi Kobayashi.

II. Great Creation

According to his list of writings, Kobayashi published papers in academic journals every year without fail, starting from his virgin paper in 1954 until his last days. Among a total of 134 papers, there are 85 single-author papers and 49 collaborative works. The relatively many collaborative works attest to his broad mathematical mind-set. A unique feature of his single-author publications is their brevity; e.g., there are 34 papers of less than 5 pages, 28 papers of less than 10 pages and 15 papers of less than 28 pages. Every one of them is deep and rich in content with transparent and easily comprehensible explanations. They seem to me oracles of God's revelations, and they will keep their shining brightness forever, as is evidenced by his invention of the concepts of what is called "Kobayashi distance," hyperbolic complex manifolds and the Hermitian-Einstein vector bundles. Inspired by Chern's result, which further improved on the generalization of the classical Schwarz' lemma by L. Ahlfors ("The holomorphic maps between Hermitian manifolds" by S.S. Chern), Kobayashi became interested in Schwarz' lemma and read all the related papers, one by one. He especially admired Carathéodory's point of view which brought him to devote himself to the holomorphic maps, which, as he admitted, led him to the discovery of the Kobayashi distance.

We can classify his papers into the following fields:

(1) Geometry of connections
(2) Geometric transformation groups
(3) Complex manifolds and holomorphic maps
(4) Geometry of complex vector bundles

Utilizing his many works, expanding them with attention to their historic background and development, Kobayashi subsequently published thirteen long self-contained books, every one of which is easily comprehensible by graduate students. They can be called great works which contain the result of his own research and the history of mathematical development of the subjects. Kobayashi had once said to me: "When I write books, I prepare one or two years for the first manuscript and make certain to give lectures based on it for one or two years in order to deepen the contents before finally completing the final manuscript."

III. Transparent mathematical philosophy

To convey to the public his philosophy of mathematics, or rather, of differential geometry, I include here passages from his essays in Japanese which were published in well-known magazines on mathematical sciences, and deeply reflect his mathematical achievements. The transparency, beauty, and open-mindedness of his viewpoints, which he sharpened well to the limit, are my personal and eternal goal.

"Differential geometry is one viewpoint over mathematics and in itself, a method in mathematics. When various fields in mathematics reach the point of being well understood, the phenomenon called 'algebrization' occurs. I think, however, that it is possible to see different fields of mathematics from a differential geometric viewpoint as well as from an algebraic viewpoint. The raison d'etre of differential geometry is to offer a new viewpoint and powerful methods rather than itself being a theory of numbers. Moreover, the concepts and methods understood from a geometric point of view are so natural (different from artificially created nonsense) that development beyond anticipation later occurs in many cases. Differential geometry can produce limitless developments by carrying its methods into all the fields of mathematics. In particular, fields such as the theory of functions (one or several complex variables), algebraic geometry, topology and differential equation theory will be relatively easy examples. It is especially important, then, to know how to make use of the differential geometric method to make connections in those fields." (Suurikagaku, August, 1965)

"When a theorem in differential geometry is proved by the concepts and techniques of differential geometry, that is, when its argument remains in the closed world of differential geometry, I cannot get so impressed to the extent of excitement, even if it is a good theorem to be admired. I feel a true joy when differential geometry and other fields are connected well enough for problems to be solved by the techniques of differential geometry. When I came to know Chern's work, which clarified thoroughly the essence of Nevanlinna's theorem by differential geometric methods, I wanted to borrow the word 'Eureka!' which Archimedes exclaimed when he found, while taking a bath, a method to determine the amount of pure gold in a crown. Anyone doing mathematics will experience a few cases where he can say, 'Ah, I got it not in form but in reality!' but he is truly ecstatic when he unexpectedly finds an interaction between two fields." (Sugaku Seminar, April, 2003)

"Wonderful theorems in mathematics are proven with breakthroughs of originality which cause everyone to understand them. A mathematician gets the greatest feeling of happiness when he finds such a new idea. A problem, which is solved merely by understanding and following a routine process, is a petty problem, and a theorem whose proof does not use any idea which comes involuntarily, is really tedious. It seems that any problem which does not open a new field, or any result which has no application in any other field of mathematics, disappears after a while. An eternal life is given only to a beautiful result." (Sugaku Seminar December, 1965 issue)