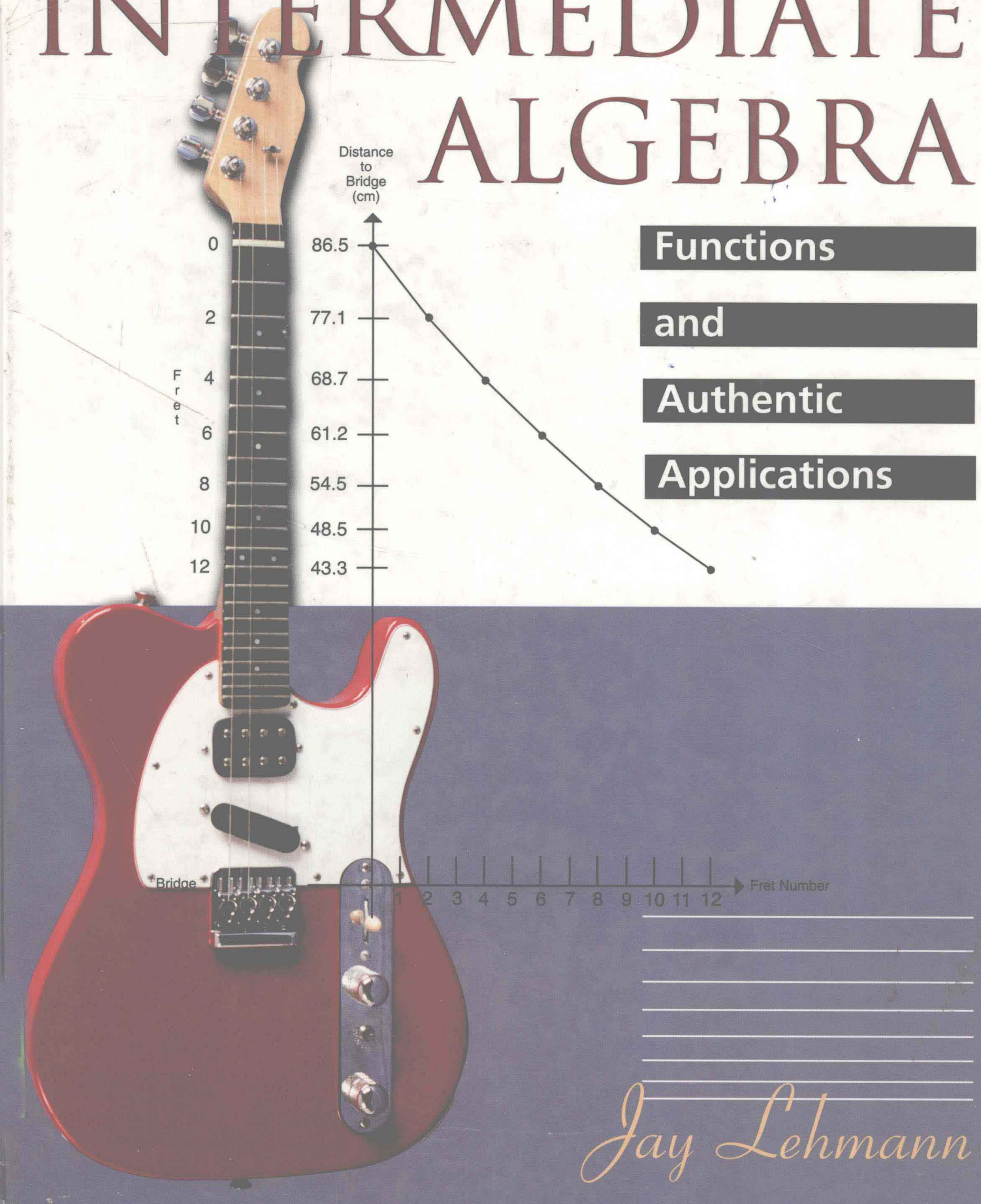
INTERMEDIATE ALGEBRA



Functions and Authentic Applications

Jay Lehmann

INTERMEDIATE ALGEBRA

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Jay Lehmann

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To Keri, who believed in this work
when I doubted and who celebrated
it with me when I remembered.

And to Dylan, who is a continual
reminder that new beginnings
are always possible.

Preface

The question of common sense is always 'What is it good for?'—a question which would abolish the rose and be answered triumphantly by the cabbage. —James Russell Lowell, 1819–1891, American poet, editor.

The above quote suggests James Russell Lowell must have taught intermediate algebra. How many times have your students asked, "What is it good for?" After years of responding, "You'll find out in the next course," I began a five-year quest to develop a more satisfying and substantial response to my students' query.

Curve Fitting Approach

Although there are many ways to center an intermediate algebra course around authentic applications, I chose a curve fitting approach for several reasons. A curve fitting approach

- allows great flexibility in choosing interesting, authentic, current situations to model.
- emphasizes concepts related to functions in a natural, substantial way.
- encourages students to view functions graphically, numerically, and symbolically as well as to verbally describe concepts related to functions.
- serves as a glue to hold together the many diverse topics of a typical intermediate algebra course.

To curve fit, students learn the following four-step modeling process:

1. Examine the data set to determine which type of model, if any, to use.
2. Find an equation for the model.
3. Verify that the model fits the data.
4. Use the model to make estimates and predictions.

This four-step process weaves together the skills of noticing numerical patterns from data displayed in tables, noticing graphical patterns in scattergrams, finding formulas of functions, graphing functions, evaluating functions, and solving equations.

Curve fitting not only fosters cohesiveness within a chapter, it also creates a parallel theme for each chapter that introduces and discusses another function.

With many intermediate algebra texts, the first third of the course reviews topics typically taught in elementary algebra. Some students find it hard to stay interested in the course because they have "seen it all before." In Section 1.1 students become acquainted with the idea of a model as they learn to describe situations by sketching graphs without scaling on the axes. Although this section could be placed at the beginning of Chapter 2, I have started the text with it to let students see that they will be learning new and interesting things in this course. Then, in Chapter 2, they become acquainted with the curve fitting theme that serves to maintain or increase student interest and involvement throughout the rest of the course.

Technology

To make effective use of students' time in and out of class, the text assumes that students have access to technology such as the TI-83 graphing calculator in the classroom and at home. Technology like this allows students to create scattergrams and verify the fit of a model quickly and accurately.

The text also supports instructors in holding students accountable for all aspects of the course without the aid of technology. For example, although students are encouraged to use technology to verify a model's fit to some data, students must also be able to sketch graphs by hand. Likewise, although a few exercises suggest that a student use technology to find a regression equation, most modeling exercises ask that students use pencil and paper to derive a formula for a model by selecting two or three points (depending on the type of model).

After students solve symbolic problems by hand, technology allows them to reinforce their numerical and graphical intuition because they use graphing calculator tables or graphs to verify their work. Technology also empowers students to explore mathematical concepts and skills that would be too time consuming or difficult to do by hand.

Explorations

Almost every section of the text contains an exploration that supports student investigation of a concept or skill. These explorations have been selected from hundreds that other instructors and I have class tested during the past five years.

An exploration can be used as a collaborative activity during class time or as part of a homework assignment. Some explorations lead students to reflect on concepts and skills of the current section. Other explorations are directed-discovery activities that introduce key concepts and skills to be discussed in the next section. All explorations are optional, so an instructor is free to assign as many explorations as the course schedule allows.

For students who enjoy learning by hands-on activities, explorations open the door for them to see the wonder and beauty of mathematics. It empowers all students to become active explorers of mathematics.

Conceptual Development

As a complement to the explorations, many examples show how to investigate concepts. They also illustrate how to perform skills of the course. This way, students learn *why* they perform skills to solve problems as well as how to solve the problems.

Homework sets have been designed to give an instructor maximum flexibility. Instructors can choose from a wide variety of exercises to balance conceptual-based, skill-based, and application-based exercises.

Key Points

Each section contains a "Key Points" feature that summarizes the definitions, properties, concepts, and skills of that section. This gives students section-by-section support in completing homework assignments. Students can also refer to this feature when preparing for quizzes and tests. A section's key points can serve as a prompt for students to reread portions of the section that address concepts that the students would like to consider further. Each chapter also has a "Key Points of This Chapter" feature.

Tips on Succeeding in This Course

Many sections close with tips that are intended to help students succeed in the course. By sprinkling these tips throughout the text, students have time to experiment with and assimilate new behaviors over time.

Taking It to the Lab Sections

Lab assignments have been included at the end of most chapters to increase students' understanding of both concepts and the scientific method. These labs reinforce the idea

that mathematics is useful. They are also an excellent avenue for more in-depth writing assignments.

Many of the labs involve curve fitting data obtained by physical experiments. Students can use data provided in the labs or collect data of their own. The "Topic of Your Choice" labs require students to choose a topic, research the topic, collect data, and analyze this data.

Two or more of these labs can be assigned at the same time to form a project assignment. Project papers include opening and summary paragraphs, typed responses in paragraph form, carefully drawn graphs and tables, and an attractive cover page. During the last three weeks of the course, each of my students uses a linear function, an exponential function, and a quadratic function to model and analyze three situations that they have chosen. This project assignment serves as an excellent review for an important portion of the course.

Margin Comments

"Review" margin comments remind students of concepts learned in preceding courses. Margin "Notes" support ideas in adjacent paragraphs. "Modeling Process" margin comments review the steps of modeling and "Graphing Calculator" margin comments give support to students who are using graphing calculators.

Additional Topics Chapter

Not all topics typically taught in an intermediate algebra course can be connected with a curve fitting approach at the appropriate level. Through extensive polling of instructors across the country, I have assembled in Chapter 11 a collection of such topics that were "musts." Each section contains a "Section Quiz" feature. The union of the section quizzes can be used as a set of review exercises for Chapter 11. For instructors who wish to "cut and paste" sections from Chapter 11 into earlier chapters, these quizzes can be appended to the appropriate chapter review exercises.

Appendix A: Reviewing Prerequisite Material

Appendix A has been included to remind students of important topics typically addressed in an elementary algebra course. Examples and exercises are included in each section.

On the Cover

For a description of the cover, see the "Stringed Instrument Lab" in Chapter 4.

Instructor's Resource Manual

This manual gives section-by-section suggestions for lectures and for the explorations. It also contains many additional explorations to give instructors more options. The manual also includes chapter tests. Because these tests contain hard-to-find data sets, they can be a useful resource even for instructors who prefer to create their own tests.

Student Solutions Manual

This manual contains solutions for the odd-numbered exercises in the Homework sections in the text.

Homepage of Data

As this text focuses on curve fitting for authentic situations, both student and instructor may need data sets. To meet this need, the text has a Homepage that includes current data sets and Web links to more data sets that students, instructors, and I have found. The Homepage also includes ideas for additional labs and other forms of support.

Homepage address
www.prenhall.com/lehmann.

Getting in Touch

I would greatly appreciate receiving your comments regarding this text. I would also like to hear about any data sets you would like to share. If you have questions regarding the text, please ask and I will respond. Finally, let me know if you or your department is interested in attending a workshop to discuss the text in greater detail.

Thank you for your interest in preserving the rose.

Jay Lehmann
MathnerdJay@aol.com

To the Student

You are about to embark on an exciting journey. In this course you will not only learn more about algebra, you will also learn how to apply algebra to describe and make predictions about authentic situations. This text contains data from hundreds of different kinds of situations. Most of the data have been collected from recent newspapers and Internet postings, so the information is current and usually of interest to the general public.

Working with true data will make mathematics more meaningful.

While working with data from true-to-life situations, you will learn the meaning of mathematical concepts. As a result, the concepts will be easier to learn, since they will be connected to familiar contexts. Second, you will see that almost any situation can be seen from a mathematical view. This will help you to understand the situation and make estimates and/or predictions.

Many of the problems you will explore in this course involve data collected during a scientific experiment or a census. The practical way to deal with such data sets is to use technology. So, a graphing calculator or computer system is required.

Applying mathematics to authentic situations is a lifelong skill.

In addition to working with data sets in this text, your instructor may assign some of the labs. Here you will collect data through experiment or research. This will give you a more complete picture of how you can use the approaches presented in this text in everyday life and possibly in your lifelong careers.

Hands-on explorations are rewarding and fun.

Learning is similar to exercising. The more often you work out, the stronger you will be. The more opportunities you take to make an intuitive leap or discover a pattern, the stronger your abilities as a critical thinker will be.

This text contains explorations that will allow you to *discover* concepts, rather than hear or read about them. Most explorations contain step-by-step instructions that lead you toward discovering new concepts.

You will also find that discovering a concept greatly improves your chances of remembering it. Since discovering a concept is exciting, it is more likely to leave a lasting impression on you. Over the years, students have remarked to me time and time again that they never dreamed that learning math could be so much fun.

Working in a team will make learning mathematics comfortable.

Discovering concepts with others is fun and rewarding. With this in mind, your instructor may have you work with a team of students to complete the explorations. As a team you will want to share your discoveries with each other. When you do, keep in mind that the student who does the explaining is getting as much, if not more,

out of the exercise as the listener(s). To explain a mathematical concept, you must be able to describe it clearly and concisely, thus sharpening your understanding of the concept.

This text contains special features to support you in succeeding.

Each section contains a "Key Points" feature to help you learn, review, and retain concepts and skills addressed in the section. After reading a section, look through the Key Points and decide whether you understand the concepts and skills described. If you do not understand a Key Point, then it is time to reread the portion of the section that addresses that point. This feature can also help you prepare for quizzes and exams. However, there is no substitute for regular, careful reading of the text and your notes, and doing lots of the exercises.

Each section also contains a "Tips on Succeeding in This Course" feature. These tips are meant to inspire you to try new strategies to help you succeed in this course and future courses. Some tips may remind you of strategies that you have used successfully in the past, but have forgotten. If you browse through all of the tips early in the course you can take advantage of as many of them as you wish. Then, as you progress through the text, you'll be reminded of your favorite strategies once more.

I have also included a review of key concepts and skills from elementary algebra in Appendix A. Before the course begins or shortly after it starts, consider reading this appendix and completing the exercises. If you need more review, refer to an elementary algebra text or ask your instructor, a tutor, or a friend.

Feel free to contact me.

Throughout the text, I tried to choose realistic exercises that will interest you. Some of the data sets in this book were researched by teams of students that chose the topics for their lab assignments. If you would like to share a data set with me for possible inclusion in future editions, please send it to my e-mail address. I will post your data on a Homepage, so that the Homepage can grow as a data resource for all students. I also welcome any comments you would like to share with me about the text.

Jay Lehmann
My e-mail: MathnerdJay@aol.com
Text's Homepage: www.prenhall.com/lehmann

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Jim Ryan, *Madera Community College*

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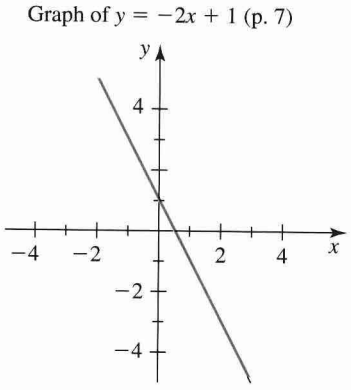

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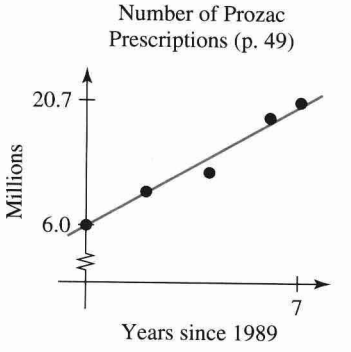

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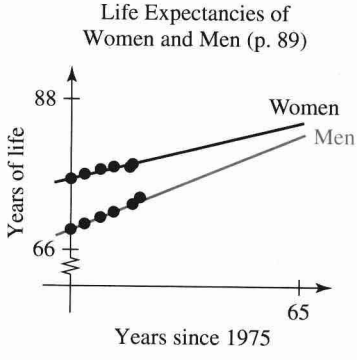

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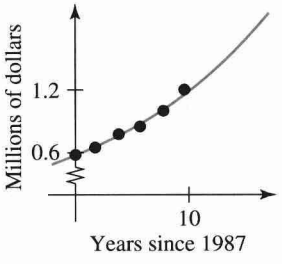

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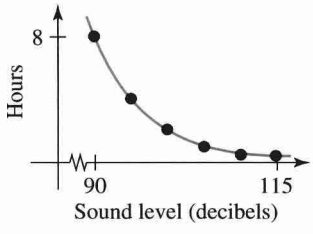

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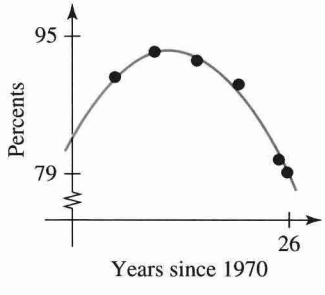

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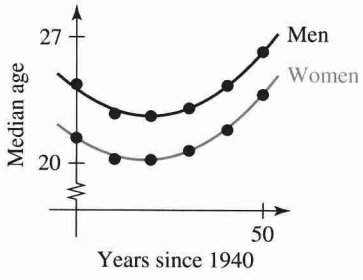

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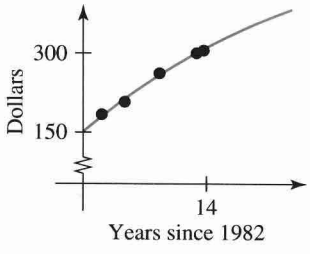

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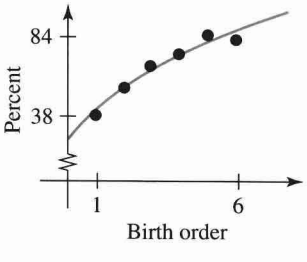

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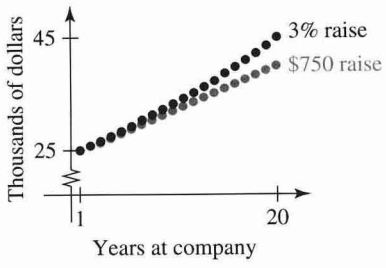

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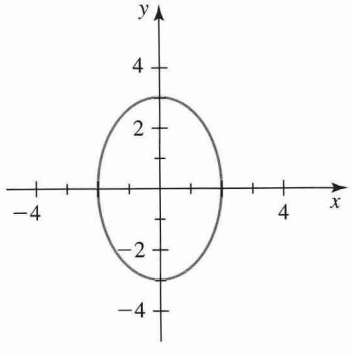

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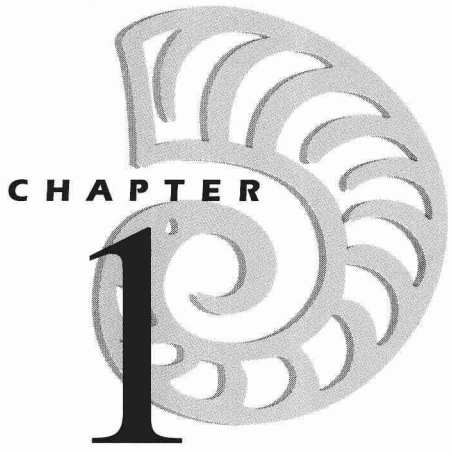


Linear Functions

I see a certain order in the universe and math is one way of making it visible. —Mary Sarton, *As We Are Now*, 1973.

1.1 Using Qualitative Graphs to Describe Situations

OBJECTIVES

- Describe situations using qualitative graphs.
- Identify independent variables and dependent variables.
- Know the meaning of an *intercept* of a curve.
- Identify increasing curves and decreasing curves.

How often have you seen or heard predictions? Perhaps you have read a prediction about world population or the AIDS epidemic. Or maybe you have seen a news report about the estimated age of an artifact discovered at an archeological site. It is virtually impossible to find a business magazine that does not make predictions about the economy. Many of these estimates and predictions are made using mathematics.

A major objective of this text is to help you view the world in a mathematical manner. This viewpoint will allow you to recognize important patterns—patterns that will enable you to make estimates and predictions like the ones mentioned above.

To begin, in this section you will learn how to describe situations using a special type of graph called a **qualitative graph**. This is a graph that has no scaling on the axes.

Note

"Scaling" consists of the tick marks and their numbers on the axes.

Example 1

Since 1986, Michael Jordan has endorsed a successful line of shoes, called Air Jordan® . Let p represent the retail price of Air Jordan shoes and t represent the number of years since 1986. (For example, $t = 1$ represents the year 1987.) The prices of the shoes are described by the qualitative graph displayed in Fig. 1. What does the graph tell us?

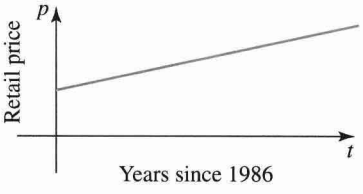
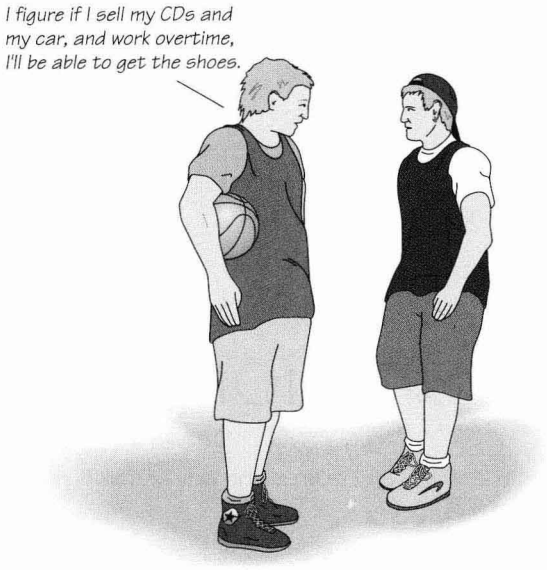


Figure 1 Retail price of Air Jordan shoes

Solution

The graph (or curve) tells us that the retail price of Air Jordans has steadily increased. The curve is said to be **linear** because it forms a straight line. —

Since the retail price of Air Jordans changes from year to year, we can say that the price p *depends* on the year. Due to inflation and to increasing popularity of the



shoes, the price increases over time. Note that the year does *not* depend on the price. Raising or lowering the price of the shoes has no effect on the passage of time. Time is *independent* of the price. Because p depends on t, we call p the **dependent variable**, and since time is independent of price, we call t the **independent variable**.

Note that in Fig. 1 we let the horizontal axis be the t-axis and the vertical axis be the p-axis. We usually match the horizontal axis with the independent variable and the vertical axis with the dependent variable (see Fig. 2).

Note

For a review of constants and variables, see Section A.4.

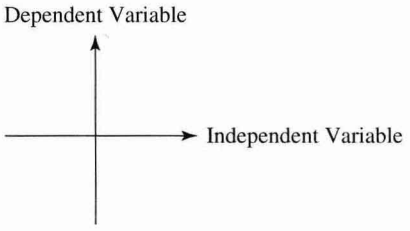


Figure 2 Match the vertical axis with the dependent variable and the horizontal axis with the independent variable

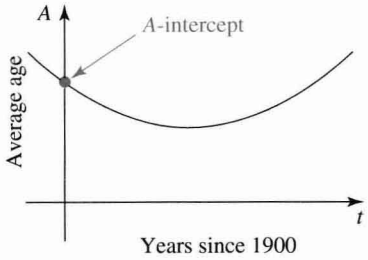


Figure 3 The average age when men first married

Example 2

Let A represent the average age when men first married and let t represent the number of years since 1900. In Fig. 3, the graph describes the relationship between the variables t and A for the years between 1900 and the present. What does the graph tell us?

Solution

The graph tells us that the average age when men first married decreased each year over the first half of the 20th century and increased over the second half. We say that the curve sketched in Fig. 3 is **quadratic** because it has the shape of a **parabola**. —

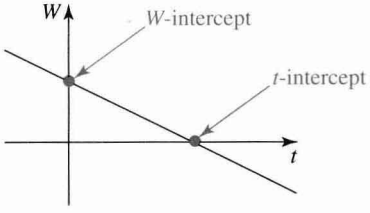


Figure 4 Intercepts of a line

In Fig. 3, note that the curve and the A-axis intersect. The point of intersection is an ***A*-intercept**. Two more examples of intercepts are shown in Fig. 4.

In Examples 3–5, we sketch qualitative graphs that describe given situations.

Example 3

Let C represent the cost of a 30-second ad slot during the Super Bowl at t years since 1987. Each year, the cost has increased by more than the previous increase. Sketch a qualitative graph that describes the relationship between C and t.