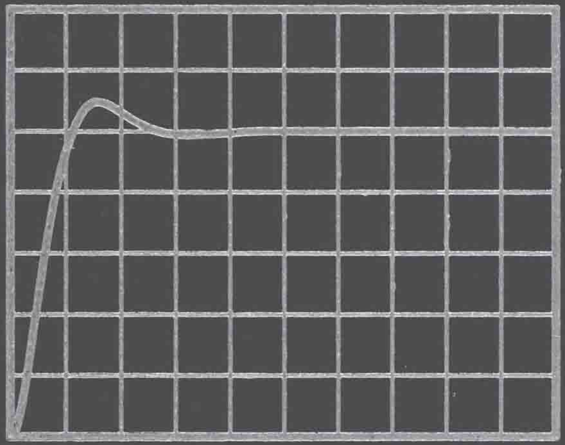
MODERN CONTROL ENGINEERING

Fourth Edition



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Modern Control Engineering

Fourth Edition

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Preface

This book presents a comprehensive treatment of the analysis and design of control systems. It is written at the level of the senior engineering (mechanical, electrical, aerospace, and chemical) student and is intended to be used as a text for the first course in control systems. The prerequisite on the part of the reader is that he or she has had introductory courses on differential equations, vector-matrix analysis, circuit analysis, and mechanics.

The main revision made in the fourth edition of the text is to present two-degrees-of-freedom control systems to design high performance control systems such that steady-state errors in following step, ramp, and acceleration inputs become zero. Also, newly presented is the computational (MATLAB) approach to determine the pole-zero locations of the controller to obtain the desired transient response characteristics such that the maximum overshoot and settling time in the step response be within the specified values. These subjects are discussed in Chapter 10. Also, Chapter 5 (primarily transient response analysis) and Chapter 12 (primarily pole placement and observer design) are expanded using MATLAB. Many new solved problems are added to these chapters so that the reader will have a good understanding of the MATLAB approach to the analysis and design of control systems. Throughout the book computational problems are solved with MATLAB.

This text is organized into 12 chapters. The outline of the book is as follows. Chapter 1 presents an introduction to control systems. Chapter 2 deals with Laplace transforms of commonly encountered time functions and some of the useful theorems on Laplace transforms. (If the students have an adequate background on Laplace transforms, this chapter may be skipped.) Chapter 3 treats mathematical modeling of dynamic systems (mostly mechanical, electrical, and electronic systems) and develops transfer function models and state-space models. This chapter also introduces signal flow graphs. Discussions of a linearization technique for nonlinear mathematical models are included in this chapter.

Chapter 4 presents mathematical modeling of fluid systems (such as liquid-level systems, pneumatic systems, and hydraulic systems) and thermal systems. Chapter 5 treats transient response analyses of dynamic systems to step, ramp, and impulse inputs. MATLAB is extensively used for transient response analysis. Routh's stability criterion is presented in this chapter for the stability analysis of higher order systems. Steady-state error analysis of unity-feedback control systems is also presented in this chapter.

Chapter 6 treats the root-locus analysis of control systems. Plotting root loci with MATLAB is discussed in detail. In this chapter root-locus analyses of positive-feedback systems, conditionally stable systems, and systems with transport lag are included. Chapter 7 presents the design of lead, lag, and lag–lead compensators with the root-locus method. Both series and parallel compensation techniques are discussed.

Chapter 8 presents basic materials on frequency-response analysis. Bode diagrams, polar plots, the Nyquist stability criterion, and closed-loop frequency response are discussed including the MATLAB approach to obtain frequency response plots. Chapter 9 treats the design and compensation techniques using frequency-response methods. Specifically, the Bode diagram approach to the design of lead, lag, and lag–lead compensators is discussed in detail.

Chapter 10 first deals with the basic and modified PID controls and then presents computational (MATLAB) approach to obtain optimal choices of parameter values of controllers to satisfy requirements on step response characteristics. Next, it presents two-degrees-of-freedom control systems. The chapter concludes with the design of high performance control systems that will follow a step, ramp, or acceleration input without steady-state error. The zero-placement method is used to accomplish such performance.

Chapter 11 presents a basic analysis of control systems in state space. Concepts of controllability and observability are given here. This chapter discusses the transformation of system models (from transfer-function model to state-space model, and vice versa) with MATLAB. Chapter 12 begins with the pole placement design technique, followed by the design of state observers. Both full-order and minimum-order state observers are treated. Then, designs of type 1 servo systems are discussed in detail. Included in this chapter are the design of regulator systems with observers and design of control systems with observers. Finally, this chapter concludes with discussions of quadratic optimal regulator systems.

In this book, the basic concepts involved are emphasized and highly mathematical arguments are carefully avoided in the presentation of the materials. Mathematical proofs are provided when they contribute to the understanding of the subjects presented. All the material has been organized toward a gradual development of control theory.

Throughout the book, carefully chosen examples are presented at strategic points so that the reader will have a clear understanding of the subject matter discussed. In addition, a number of solved problems (A-problems) are provided at the end of each chapter, except Chapter 1. These solved problems constitute an integral part of the text. Therefore, it is suggested that the reader study all these problems carefully to obtain a deeper understanding of the topics discussed. In addition, many problems (without solutions) of various degrees of difficulty are provided (B-problems). These problems may be used as homework or quiz purposes. An instructor using this text can obtain a complete solutions manual (for B-problems) from the publisher.

Most of the materials including solved and unsolved problems presented in this book have been class tested in senior level courses on control systems at the University of Minnesota.

If this book is used as a text for a quarter course (with 40 lecture hours), most of the materials in the first 10 chapters (except perhaps Chapter 4) may be covered. [The first nine chapters cover all basic materials of control systems normally required in a first course on control systems. Many students enjoy studying computational (MATLAB) approach to the design of control systems presented in Chapter 10. It is recommended that Chapter 10 be included in any control courses.] If this book is used as a text for a semester course (with 56 lecture hours), all or a good part of the book may be covered with flexibility in skipping certain subjects. Because of the abundance of solved problems (A-problems) that might answer many possible questions that the reader might have, this book can also serve as a self-study book for practicing engineers who wish to study basic control theory.

I would like to express my sincere appreciation to Professors Athimoottil V. Mathew (Rochester Institute of Technology), Richard Gordon (University of Mississippi), Guy Beale (George Mason University), and Donald T. Ward (Texas A & M University), who made valuable suggestions at the early stage of the revision process, and anonymous reviewers who made many constructive comments. Appreciation is also due to my former students, who solved many of the A-problems and B-problems included in this book.

Katsuhiko Ogata

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1

Introduction to Control Systems

1–1 INTRODUCTION

Automatic control has played a vital role in the advance of engineering and science. In addition to its extreme importance in space-vehicle systems, missile-guidance systems, robotic systems, and the like, automatic control has become an important and integral part of modern manufacturing and industrial processes. For example, automatic control is essential in the numerical control of machine tools in the manufacturing industries, in the design of autopilot systems in the aerospace industries, and in the design of cars and trucks in the automobile industries. It is also essential in such industrial operations as controlling pressure, temperature, humidity, viscosity, and flow in the process industries.

Since advances in the theory and practice of automatic control provide the means for attaining optimal performance of dynamic systems, improving productivity, relieving the drudgery of many routine repetitive manual operations, and more, most engineers and scientists must now have a good understanding of this field.

Historical Review. The first significant work in automatic control was James Watt's centrifugal governor for the speed control of a steam engine in the eighteenth century. Other significant works in the early stages of development of control theory were due to Minorsky, Hazen, and Nyquist, among many others. In 1922, Minorsky worked on automatic controllers for steering ships and showed how stability could be determined from the differential equations describing the system. In 1932, Nyquist developed a relatively simple procedure for determining the stability of closed-loop systems on the basis of open-loop response to steady-state sinusoidal inputs. In 1934, Hazen, who introduced the term servomechanisms for position control systems, discussed the design of relay servomechanisms capable of closely following a changing input.

During the decade of the 1940s, frequency-response methods (especially the Bode diagram methods due to Bode) made it possible for engineers to design linear closed-loop control systems that satisfied performance requirements. From the end of the 1940s to the early 1950s, the root-locus method due to Evans was fully developed.

The frequency-response and root-locus methods, which are the core of classical control theory, lead to systems that are stable and satisfy a set of more or less arbitrary performance requirements. Such systems are, in general, acceptable but not optimal in any meaningful sense. Since the late 1950s, the emphasis in control design problems has been shifted from the design of one of many systems that work to the design of one optimal system in some meaningful sense.

As modern plants with many inputs and outputs become more and more complex, the description of a modern control system requires a large number of equations. Classical control theory, which deals only with single-input–single-output systems, becomes powerless for multiple-input–multiple-output systems. Since about 1960, because the availability of digital computers made possible time-domain analysis of complex systems, modern control theory, based on time-domain analysis and synthesis using state variables, has been developed to cope with the increased complexity of modern plants and the stringent requirements on accuracy, weight, and cost in military, space, and industrial applications.

During the years from 1960 to 1980, optimal control of both deterministic and stochastic systems, as well as adaptive and learning control of complex systems, were fully investigated. From 1980 to the present, developments in modern control theory centered around robust control, H_∞ control, and associated topics.

Now that digital computers have become cheaper and more compact, they are used as integral parts of control systems. Recent applications of modern control theory include such nonengineering systems as biological, biomedical, economic, and socioeconomic systems.

Definitions. Before we can discuss control systems, some basic terminologies must be defined.

Controlled Variable and Manipulated Variable. The *controlled* variable is the quantity or condition that is measured and controlled. The *manipulated* variable is the quantity or condition that is varied by the controller so as to affect the value of the controlled variable. Normally, the controlled variable is the output of the system. *Control* means measuring the value of the controlled variable of the system and applying the manipulated variable to the system to correct or limit deviation of the measured value from a desired value.

In studying control engineering, we need to define additional terms that are necessary to describe control systems.

Plants. A plant may be a piece of equipment, perhaps just a set of machine parts functioning together, the purpose of which is to perform a particular operation. In this book, we shall call any physical object to be controlled (such as a mechanical device, a heating furnace, a chemical reactor, or a spacecraft) a plant.

Processes. The *Merriam–Webster Dictionary* defines a process to be a natural, progressively continuing operation or development marked by a series of gradual changes that succeed one another in a relatively fixed way and lead toward a particular result or end; or an artificial or voluntary, progressively continuing operation that consists of a series of controlled actions or movements systematically directed toward a particular result or end. In this book we shall call any operation to be controlled a *process*. Examples are chemical, economic, and biological processes.

Systems. A system is a combination of components that act together and perform a certain objective. A system is not limited to physical ones. The concept of the system can be applied to abstract, dynamic phenomena such as those encountered in economics. The word system should, therefore, be interpreted to imply physical, biological, economic, and the like, systems.

Disturbances. A disturbance is a signal that tends to adversely affect the value of the output of a system. If a disturbance is generated within the system, it is called *internal*, while an *external* disturbance is generated outside the system and is an input.

Feedback Control. Feedback control refers to an operation that, in the presence of disturbances, tends to reduce the difference between the output of a system and some reference input and does so on the basis of this difference. Here only unpredictable disturbances are so specified, since predictable or known disturbances can always be compensated for within the system.

1–2 EXAMPLES OF CONTROL SYSTEMS

In this section we shall present several examples of control systems.

Speed Control System. The basic principle of a Watt's speed governor for an engine is illustrated in the schematic diagram of Figure 1–1. The amount of fuel admitted

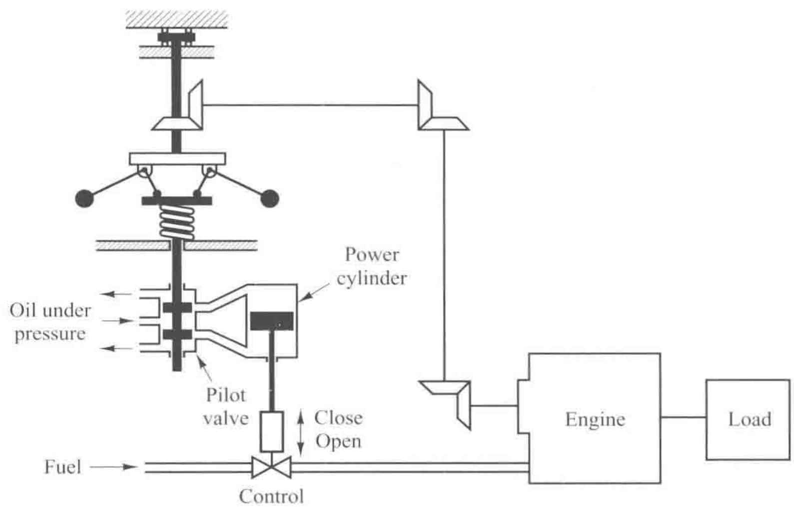

Figure 1–1
Speed control system.

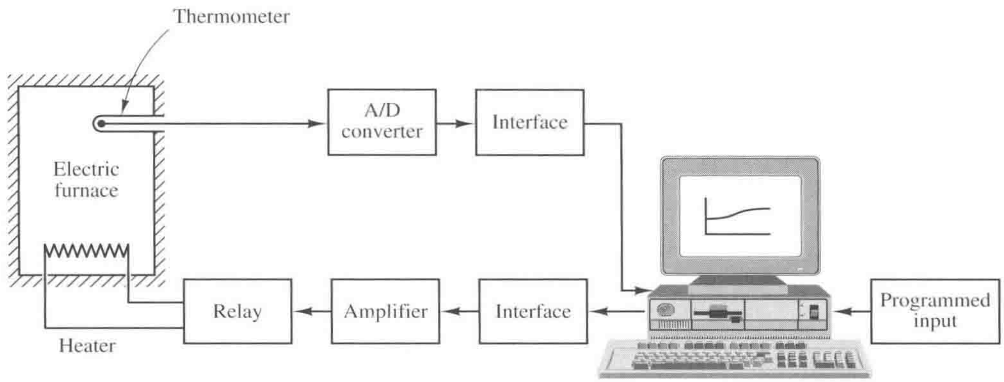

Figure 1–2
Temperature control system.

to the engine is adjusted according to the difference between the desired and the actual engine speeds.

The sequence of actions may be stated as follows: The speed governor is adjusted such that, at the desired speed, no pressured oil will flow into either side of the power cylinder. If the actual speed drops below the desired value due to disturbance, then the decrease in the centrifugal force of the speed governor causes the control valve to move downward, supplying more fuel, and the speed of the engine increases until the desired value is reached. On the other hand, if the speed of the engine increases above the desired value, then the increase in the centrifugal force of the governor causes the control valve to move upward. This decreases the supply of fuel, and the speed of the engine decreases until the desired value is reached.

In this speed control system, the plant (controlled system) is the engine and the controlled variable is the speed of the engine. The difference between the desired speed and the actual speed is the error signal. The control signal (the amount of fuel) to be applied to the plant (engine) is the actuating signal. The external input to disturb the controlled variable is the disturbance. An unexpected change in the load is a disturbance.

Temperature Control System. Figure 1–2 shows a schematic diagram of temperature control of an electric furnace. The temperature in the electric furnace is measured by a thermometer, which is an analog device. The analog temperature is converted to a digital temperature by an A/D converter. The digital temperature is fed to a controller through an interface. This digital temperature is compared with the programmed input temperature, and if there is any discrepancy (error), the controller sends out a signal to the heater, through an interface, amplifier, and relay, to bring the furnace temperature to a desired value.

EXAMPLE 1–1 Consider the temperature control of the passenger compartment of a car. The desired temperature (converted to a voltage) is the input to the controller. The actual temperature of the passenger compartment must be converted to a voltage through a sensor and fed back to the controller for comparison with the input.

Figure 1–3 is a functional block diagram of temperature control of the passenger compartment of a car. Note that the ambient temperature and radiation heat transfer from the sun, which are not constant while the car is driven, act as disturbances.

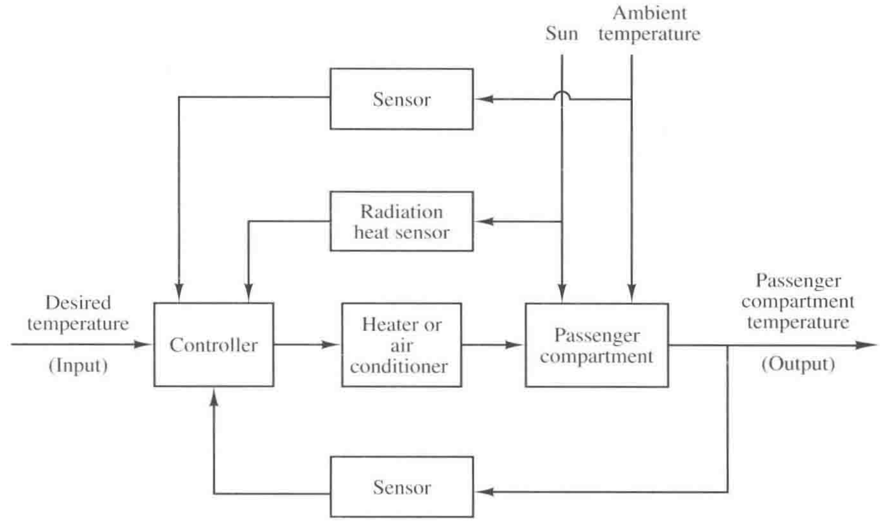

Figure 1–3 Temperature control of passenger compartment of a car.

The temperature of the passenger compartment differs considerably depending on the place where it is measured. Instead of using multiple sensors for temperature measurement and averaging the measured values, it is economical to install a small suction blower at the place where passengers normally sense the temperature. The temperature of the air from the suction blower is an indication of the passenger compartment temperature and is considered the output of the system.

The controller receives the input signal, output signal, and signals from sensors from disturbance sources. The controller sends out an optimal control signal to the air conditioner or heater to control the amount of cooling air or warm air so that the passenger compartment temperature is about the desired temperature.

Business Systems. A business system may consist of many groups. Each task assigned to a group will represent a dynamic element of the system. Feedback methods of reporting the accomplishments of each group must be established in such a system for proper operation. The cross-coupling between functional groups must be made a minimum in order to reduce undesirable delay times in the system. The smaller this cross-coupling, the smoother the flow of work signals and materials will be.

A business system is a closed-loop system. A good design will reduce the managerial control required. Note that disturbances in this system are the lack of personnel or materials, interruption of communication, human errors, and the like.

The establishment of a well-founded estimating system based on statistics is mandatory to proper management. Note that it is a well-known fact that the performance of such a system can be improved by the use of lead time, or *anticipation*.

To apply control theory to improve the performance of such a system, we must represent the dynamic characteristic of the component groups of the system by a relatively simple set of equations.

Although it is certainly a difficult problem to derive mathematical representations of the component groups, the application of optimization techniques to business systems significantly improves the performance of the business system.

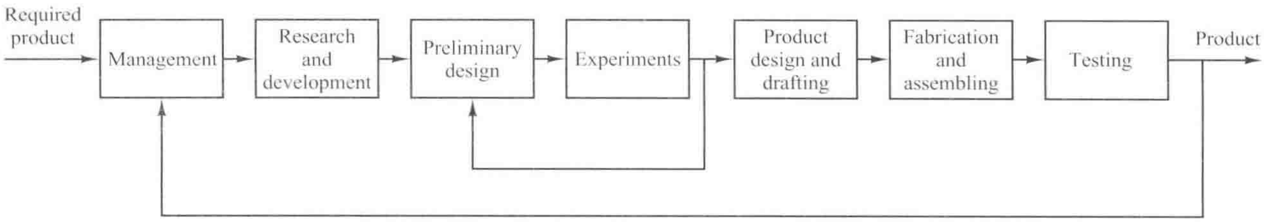

Figure 1–4
Block diagram of an engineering organizational system.

EXAMPLE 1–2 An engineering organizational system is composed of major groups such as management, research and development, preliminary design, experiments, product design and drafting, fabrication and assembling, and testing. These groups are interconnected to make up the whole operation.

Such a system may be analyzed by reducing it to the most elementary set of components necessary that can provide the analytical detail required and by representing the dynamic characteristics of each component by a set of simple equations. (The dynamic performance of such a system may be determined from the relation between progressive accomplishment and time.) Draw a functional block diagram showing an engineering organizational system.

A functional block diagram can be drawn by using blocks to represent the functional activities and interconnecting signal lines to represent the information or product output of the system operation. A possible block diagram is shown in Figure 1–4.

1–3 CLOSED-LOOP CONTROL VERSUS OPEN-LOOP CONTROL

Feedback Control Systems. A system that maintains a prescribed relationship between the output and the reference input by comparing them and using the difference as a means of control is called a *feedback control system*. An example would be a room-temperature control system. By measuring the actual room temperature and comparing it with the reference temperature (desired temperature), the thermostat turns the heating or cooling equipment on or off in such a way as to ensure that the room temperature remains at a comfortable level regardless of outside conditions.

Feedback control systems are not limited to engineering but can be found in various nonengineering fields as well. The human body, for instance, is a highly advanced feedback control system. Both body temperature and blood pressure are kept constant by means of physiological feedback. In fact, feedback performs a vital function: It makes the human body relatively insensitive to external disturbances, thus enabling it to function properly in a changing environment.

Closed-Loop Control Systems. Feedback control systems are often referred to as *closed-loop control* systems. In practice, the terms feedback control and closed-loop control are used interchangeably. In a closed-loop control system the actuating error signal, which is the difference between the input signal and the feedback signal (which may be the output signal itself or a function of the output signal and its derivatives and/or integrals), is fed to the controller so as to reduce the error and bring the output of the system to a desired value. The term closed-loop control always implies the use of feedback control action in order to reduce system error.

Open-Loop Control Systems. Those systems in which the output has no effect on the control action are called *open-loop control systems*. In other words, in an open-