TUP-Springer **Project**

Quan-Lin Li

随机模型构造性计算理论及其应用

RG-分解方法

Constructive Computation in Stochastic Models with Applications

The *RG*-Factorizations

清华大学出版社

Springer

Quan-Lin Li

随机模型构造性计算理论及其应用

RG-分解方法

Constructive Computation in Stochastic Models with Applications

The *RG*-Factorizations

清华大学出版社 Springer

内 容 简 介

本书介绍了随机模型中计算技术的主要基础理论，总结了近十年来国内外所取得的新成果与进展。它构造性地建立了一般马尔可夫过程的 *RG*-分解，其中 *RG*-分解是马尔可夫过程与高斯消元法的完美结合，为求解无限维（或大型）线性方程组提供了有效途径。全书共分为三个部分。第一部分描述了如何把排队系统、可靠性工程、制造系统、计算机网络、交通系统、服务系统等应用随机模型转化为块型结构的马尔可夫过程，这为研究许多实际系统的性能评价、优化与决策提供了统一的数学理论框架。第二部分提供了研究随机模型的计算理论基础，包括 Censoring 不变性、*RG*-分解、*RG*-对偶性、谱分析、稳态计算、瞬态计算、渐近性分析、敏感性分析等。第三部分研究了随机模型中的一些热点问题，例如拟平稳分布、连续状态马尔可夫过程、马尔可夫报酬过程、马尔可夫决策过程、演化博弈论等。

本书的读者对象为代数、应用概率、运筹学、管理科学、制造系统、计算机网络、交通系统、服务系统、生物工程等领域中高年级大学生、研究生、科技人员与工程技术人员。

图书在版编目(CIP)数据

随机模型构造性计算理论及其应用：*RG*-分解方法：英文/李泉林著. --北京：清华大学出版社，2010. 1

ISBN 978-7-302-21501-1

Ⅰ.①随…　Ⅱ.①李…　Ⅲ.①电子计算机－数值计算－英文　Ⅳ.①TP301.6

中国版本图书馆 CIP 数据核字(2009)第 214331 号

责任编辑：韩燕丽　张　彤
责任校对：赵丽敏
责任印制：孟凡玉
出版发行：清华大学出版社　　**地　　址**：北京清华大学学研大厦 A 座
http://www.tup.com.cn　　**邮　　编**：100084
社 总 机：010-62770175　　**邮　　购**：010-62786544
投稿与读者服务：010-62776969, c-service@tup.tsinghua.edu.cn
质 量 反 馈：010-62772015, zhiliang@tup.tsinghua.edu.cn
印 装 者：北京雅昌彩色印刷有限公司
经　　销：全国新华书店
开　　本：153×235　**印　张**：43.25　**字　数**：974 千字
版　　次：2010 年 1 月第 1 版　　**印　　次**：2010 年 1 月第1 次印刷
印　　数：1～1000
定　　价：148.00 元

To my friend Marcel F. Neuts for his pioneering
contributions to stochastic models

Preface

Stochastic systems are involved in many practical areas, such as applied probability, queueing theory, reliability, risk management, insurance and finance, computer networks, manufacturing systems, transportation systems, supply chain management, service operations management, genomic engineering and biological sciences. When analyzing a stochastic system, block-structured stochastic models are found to be a useful and effective mathematical tool. In the study of the block-structured stochastic models, this book provides a unified, constructive and algorithmic framework on two important directions: performance analysis and system decision. Different from those books given in the literature, the framework of this book is systematically organized by means of the UL- and LU-types of *RG*-factorizations, which are completely developed in this book and have been extensively discussed by the author. The *RG*-factorizations can be applied to provide effective solutions for the block-structured Markov chains, and are shown to be also useful for optimal design and dynamical decision making of many practical systems, such as computer networks, transportation systems and manufacturing systems. Besides, this book uses the *RG*-factorizations to deal with some recent interesting topics, for example, tailed analysis, continuous-state Markov chains, quasi-stationary distributions, Markov reward processes, Markov decision processes, sensitivity analysis, evolutionary game and stochastic game. Note that all these different problems can be dealt with by a unified computational method through the *RG*-factorizations. Specifically, this book pays attention to optimization, control, decision making and game of the block-structured stochastic models, although available results on these directions are still few up to now.

The block-structured stochastic models began with studying the matrix-geometric stationary probability of a Quasi-Birth-And-Death (QBD) process which was first proposed to analyze two-dimensional queues or computer systems, e.g., see Evans (1967) and Wallace (1969). The initial attention was directed toward performance computation. Neuts (1978) extended the results of the QBD processes to Markov chains of $GI/M/1$ type for the first time. Based on the phase type (PH) distribution given in Neuts (1975), Neuts (1981) opened an interesting and crucial door in numerical analysis of stochastic models, which has become increasingly important for dealing with large-scale and complex stochastic systems due to advent more powerful computational ability under fast development of computer technology and communication networks. For a complete understanding of stochastic models, it is necessary to review two key advances. Firstly, Neuts (1981) considered Markov chains of $GI/M/1$ type whose stationary probability vectors are the matrix-geometric form, called *matrix-geometric solution*. For the matrix-geometric solution, the matrix R, the minimal nonnegative solution to the nonlinear matrix equation $R=\sum_{k=0}^{\infty}R^k A_k$, plays an important role. He indicated that numerical computation of Markov chains of $GI/M/1$ type can be transformed to that of the matrix R, and then an infinite-dimensional computation for the stationary probability vector is transformed to another finite-dimensional computation for the censored Markov chain to level 0. Readers may refer to Neuts (1981), Latouche and Ramaswami (1999), Bini, Latouche and Meini (2005) and others therein. Secondly, as a companion research for Markov chains of $GI/M/1$ type, Neuts (1989) provided a detailed discussion for Markov chains of $M/G/1$ type whose stationary probability vector has a complicated form, called *matrix-iterative solution*. Although the two types of Markov chains have different block structure, the matrix-iterative solution has many properties similar to those in the matrix-geometric solution, for example, the matrix-iterative solution is determined by the minimal nonnegative solution G to another key nonlinear matrix equation $G=\sum_{k=0}^{\infty}A_k G^k$.

These results given in Neuts' two books (1981, 1989) are simple, perfect and computable. However, Markov chains of $GI/M/1$ type and Markov chains of $M/G/1$ type are two important examples in the study of block-structured stochastic models, while analysis of many practical stochastic systems needs to use more general block-structured Markov chains, e.g., see retrial queues given in Artalejo and Gómez-Corral (2008) and other stochastic models given in Chapters 1 and 7 of this book. Under the situation, these practical examples motivate us in this book to develop a more general algorithmic framework for studying the block-structured stochastic models, including generalization of the matrix-geometric solution and the matrix-iterative solution from the level independence to the level dependence. It is worthwhile to note that such a generalization is not easy and simple, it needs and requires application of new mathematical methods. During the two decades, the censoring technique is indicated to be a key method for dealing with more general block-structured Markov chains. Grassmann and Heyman (1990) first used the censoring technique to find some basic relationships between the matrix-geometric solution and the matrix-iterative solution from a more general model: Markov chains of $GI/G/1$ type. Furthermore, Heyman (1995) applied the censoring technique to provide an LU-decomposition for any ergodic stochastic matrix of infinite size, Li (1997) gave the LU-decomposition for Markov chains of $GI/M/1$

type and also for Markov chains of $M/G/1$ type, and Zhao (2000) obtained the UL-type RG-factorization for Markov chains of $GI/M/1$ type. From these works, it may be clear that finding such matrix decomposition for general Markov chains is a promissing direction for numerical solution of block-structred stochastic models. Along similar lines, we have systematically developed the UL- and LU-types of RG-factorizations for any irreducible Markov chains in the past ten years, e.g., see Li and Cao (2004), Li and Zhao (2002, 2004) and Li and Liu (2004). This book summarizes many important results and basic relations for block-structured Markov chains by means of the RG-factorizations. The RG-factorizations are derived for any irreducible Markov chains in terms of the Wiener-Hopf equations, while some useful iterative relations among the R-, U- and G-measures are organized in the Wiener-Hopf equations. Specifically, the iterative relations are sufficiently helpful for dealing with performance computation and system decision. On the other hand, this book also provides new probabilistic interpretations for those results obtained by Neuts' method. We may say that the RG-factorizations begin a new era in the study of block-structured stochastic models with an algebraic and probabilistic combination.

The main contribution of this book is to construct a unified computational framework to study stochastic models both from stationary solution and from transient solution. When a practical system is described as a block-structured Markov chain, performance computation and system decision can always be organized as a system of linear equations: $xA = 0$ or $xA = b$ where $b \neq 0$. This book provides two different computational methods to deal with the system of linear equations. At the same time, it is seen from the computational process that the middle diagonal matrix of the RG-factorizations plays an important role based on the state classification of Markov chains.

Method I In this method the matrix A can be shown to have a UL-type RG-factorization

$$A = (I - R_U) \cdot \mathrm{diag}\,(\Theta_0, \Theta_1, \Theta_2, \ldots) \cdot (I - G_L),$$

where the size of the matrix Θ_0 is always small and finite in level 0. This book summarizes two important conclusions:

(1) If the block-structured Markov chain is positive recurrent, then the matrix Θ_0 is singular and all the other matrices Θ_k for $k \geqslant 1$ are invertible. In this case, the UL-type RG-factorization can be used to solve the system of linear equations: $xA = 0$ given in Section 2.4, and then such a solution can be used to obtain stationary performance analysis.

(2) If the block-structured Markov chain is transient, then the matrix Θ_k is invertible for $k \geqslant 0$. In this case, the UL-type RG-factorization is used to solve the system of linear equations: $xA = b$ with

$$x = b(I - G_L)^{-1} \cdot \mathrm{diag}\,(\Theta_0^{-1}, \Theta_1^{-1}, \Theta_2^{-1}, \ldots) \cdot (I - R_U)^{-1},$$

which leads to transient performance analysis.

Method Ⅱ In this method the matrix A can be shown to have an LU-type RG-factorization

$$A = (I - \overline{R}_L) \cdot \text{diag}(\varLambda_0, \varLambda_1, \varLambda_2, \ldots) \cdot (I - \overline{G}_U),$$

where the matrix $\varLambda_k$ is invertible for $k \geqslant 0$. Therefore, the LU-type RG-factorization can be used to deal with the system of linear equations: $xA = b$ with

$$x = b(I - \overline{G}_U)^{-1} \cdot \text{diag}(\varLambda_0^{-1}, \varLambda_1^{-1}, \varLambda_2^{-1}, \ldots) \cdot (I - \overline{R}_L)^{-1},$$

which further leads to transient performance analysis of a stochastic model.

This book has grown out of my research and lecture notes on the matrix-analytic methods since 1997. Although I have made an effort to introduce explanations and definitions for mathematical tools, crucial concepts and basic conclusions in this book, it is still necessary for readers to have a better mathematical background, including probability, statistics, Markov chains, Markov renewal processes, Markov decision processes, queueing theory, game theory, matrix analysis and numerical computation. Readers are assumed to be familiar with the basic materials or parts of them.

The organization of this book is strictly logical and more complete from performance computation to system decision. This book contains eleven chapters whose structured relationship is shown in Fig. 0.1. Chapters 1 and 7 introduce motivating examples from different research areas, such as queueing theory, computer networks and manufacturing systems. The examples are first described as the block-structured Markov chains, then they will help readers to understand the basic structure of practical stochastic models. Chapters 2, 3, 5, 6 and 9 systematically develop the construction of the RG-factorizations for Markov chains, Markov renewal processes and β-discounted transition matrices. Chapters 4, 8, 10 and 11 apply the RG-factorizations to deal with some current interesting topics including tailed analysis, Markov chains on a continuous state space, transient solution, Markov reward processes, sensitivity analysis and game theory, respectively. Finally, we also provide two useful appendices which may be basically helpful for readers to understand the contents of this book. Every chapter consists of a brief summary, a main body and a discussion with "Notes in the Literature". At the same time, every chapter also contains a number of problems whose purpose is to help readers understand the corresponding concepts, results and conclusions.

It is hoped that this book will be useful for the first-year graduate students or advanced undergraduates, as well as researchers and engineers who are interested in, for example, applied probability, queueing theory, reliability, risk management, insurance and finance, communication networks, manufacturing systems, transportation systems, supply chain management, service operations management, performance evaluation, system decision, and game theory with applications. We suggest a full semester course with two or three hours per week. Shorter courses can be also based on part of the chapters, for instance, engineering students or researchers may only study Chapters 1, 2, 6, 8, 10 and 11.

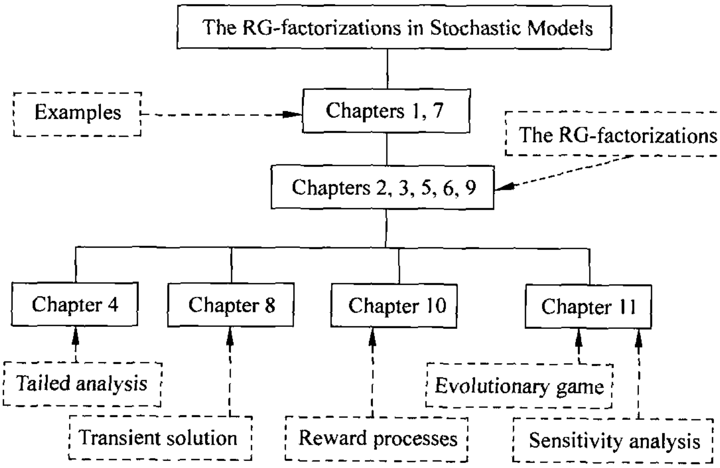

Figure 0.1 Organization of this book

It is a pleasure to acknowledge Marcel F. Neuts for his pioneering work which developed an important area: Numerical computation of stochastic models, his comments and suggestions are valuable and useful for improving the presentation of this book. I also thank Yiqiang Zhao for his cooperation on the *RG*-factorizations and block-structured stochastic models from 1999 to 2003. In fact, some sections of this book directly come from his or our collaboration works. Special thanks go to Jinhua Cao, Ming Tan, Naishuo Tian and Dequan Yue who encouraged me in the study of stochastic models during my master and Ph.D. programs. I am grateful to my friends J.R. Artalejo, N.G. Bean, Xiuli Chao, A. Dudin, A. Gómez-Corral, Fuzhou Gong, Xianping Guo, Qiming He, Zhengting Hou, Guanghui Hsu, Ninjian Huang, Haijun Li, Wei Li, Zhaotong Lian, Chuang Lin, Ke Liu, Zhiming Ma, Zhisheng Niu, T. Takine, Peter G. Taylor, Jeffery D. Tew, Jingting Wang, Dinghua Shi, Deju Xu, Susan H. Xu, David D. Yao and Hanqin Zhang for their great help and valuable suggestions on the matrix-analytic methods. I am indebted to Xiren Cao, Liming Liu and Shaohui Zheng for the financial support for my visits to Hong Kong University of Science and Technology in the recent years. Their valuable comments and suggestions helped me to develop new and interesting fields, such as perturbation analysis and Markov decision processes. I thank my master and Ph.D. students, such as Dejing Chen, Shi Chen, Yajuan Li, Jinqi Wang, Yang Wang, Yitong Wang, Xiaole Wu, Jia Yan and Qinqin Zhang. This book has benefited from the financial support provided by National Natural Science Foundation of China (Grant No. 10671107, Grant No. 10871114, Grant No. 60736028) and the National Grand Fundamental Research (973) Program of China (Grant No. 2006CB805901). I thank all of my colleagues in the Department of Industrial Engineering, Tsinghua

University, each of who always provides me an ideal research environment. Finally, a special thanks go to my wife Shuang Chen and my son Mingyi Li for with constant understanding and support in my family for many years.

Dr. Quan-Lin Li

Department of Industrial Engineering

Tsinghua University, Beijing, P. R. China

May 30, 2009

Contents