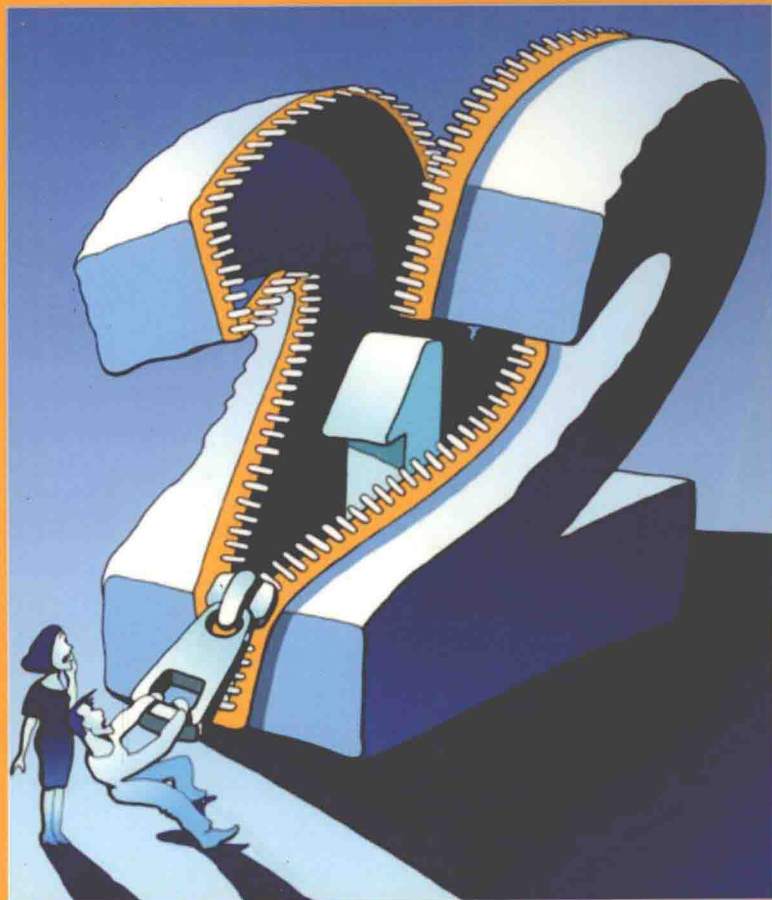


Do I Count?

*Stories from
Mathematics*



Günter M. Ziegler



CRC Press
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AN A K PETERS BOOK

Do I Count?

Stories from Mathematics



Günter M. Ziegler

Translated by Thomas von Foerster



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Preface

What does it mean, “to do mathematics”?

“Let me put up a few numbers to make the discussion more concrete” is a wonderful sentence that all regular talk-show economists should have in their arsenal. I even wanted to use it as the title of this book, but my publisher countered with the argument (though it seemed specious to me) that it was too long for a book title. We compromised on the shorter version you see on this book.

Put up some numbers—and then what? Then they are up. Just so.

I could also have called the book *What Is Mathematics?* but that is already the title of a book by Richard Courant and Herbert Robbins, a book about which one could say a few things. For example, that Courant selected the rather catchy title on the recommendation of Thomas Mann. Aside from the fact that Mann is no longer available to help with my title selection, I also did not want to write a classic mathematics book like the one by Courant and Robbins, in which mathematics is displayed along with mathematical concepts, ideas, considerations, research, and results.

My book is to be about the *doing* of mathematics, the *making* of mathematics. That is something entirely different. Try saying “love” instead of “mathematics.” So what is love? What is making love? Of the latter we have some pretty concrete

ideas, even if, in German, it comes across as a bad translation. My book is to be about the people behind the numbers and the places where mathematics is made. It should be about the battles for precision; about perseverance, errors, and the love of detail; about great emotions—and also about the problems that make the fight worthwhile, about recognition and prizes. This book is a trip into the world of mathematicians. This is no separate secret world. The world of mathematics is *our* world. Mathematics is not at all something distant, strange, and abstract that one can only learn about, and learn to hate, in school. Instead, it designates the corner pennants and goal lines of a playing field across which we move quite unencumbered, a field that we don't first encounter when we learn how to count, one, two, three. Are you aware that mathematics accompanies us from the very first knot in a shoelace until the artificial knee? There is mathematics in housekeeping, in communications, in traffic, and in weather reports (especially when they are accurate).

The world of mathematicians is also nothing foreign. You will find in this book all the rubrics familiar from magazines and daily papers, because they are interesting: celebrities, history, travels, politics, science and technology, weather, clever puzzles, a look into the future, and not a lot of esoterica. For which points to emphasize, I have of course oriented myself toward my own interests, preferences, and aversions by painting, in a manner of speaking, a picture of this fascinating and far-reaching world of mathematics and the making of mathematics *as I see it*. Think of it as an adventure trip with a personal tour guide. In that sense: welcome aboard!

Contents

Preface	ix
1 On the Number Line	1
3—Can Bees Count?	2
5—Can Chickens Compute?	4
10—And the Name of the Rose	6
13—Bad Luck?.....	11
42—The Answer to Everything?	15
91—The Numbers on the Bone.....	17
1729—Hardy’s Taxi	22
119/100—High Percentages	24
π —As Beautiful as the Mona Lisa?.....	25
$\sqrt{-1}$ —Victim of a Character Assassination	29
\aleph_0 —The End of the Number Line?.....	31
2 The Never-Ending Story of Prime Numbers	35
Euclid Is Still Right	36
How Many Prime Numbers Are There?	37
Fermat Made a Mistake.....	40
The “Mozart of Mathematics” Makes Use of an Error	42
Another Search for Errors	45
A Well-Insured Million.....	46
3 The Mathematical Perspective	49
Estimates.....	50
Random Numbers	53

	Everything Far Above Average	57
	Integers	60
4	Caution: Equations.....	63
	Equations for Everything?	64
	The Body Mass Index.....	71
	The Huntington Affair	74
	Pythagoras Lives	79
	Equations as Art.....	82
5	The Small Puzzles.....	85
	Sudokus.....	86
	$3x + 1$	90
	The Perfect Monster.....	92
	The Great Puzzles	94
6	Where Mathematics Is Created	103
	At the Desk	104
	At the Coffee Machine	105
	At the Café.....	106
	In the Computer.....	110
	In Bed.....	112
	In Church	114
	In Captivity.....	116
	In an Attic Room in Princeton.....	118
	On a Beach	120
	In a Paradise with a Library	122
	Knowledge in the ArXiv.....	124
	Research in the Internet?	126
7	The Book of Proofs.....	129
	About Proofs	130
	Concerning Errors	135
	About Computer Proofs	137
	Concerning Precision.....	140
	Concerning Surprises.....	143

8	Three Legends	147
	Mathematician vs. Mathematician.....	148
	Was It Kovalevskaya's Fault?	152
	The Disappearance of Alexander Grothendieck	158
9	What Kinds of People Are These?	163
	Paul Erdős: Traveler.....	165
	Gian-Carlo Rota: Provocateur	169
	Persi Diaconis: Magician	173
	Daniel Biss: Politician	176
	Caroline Lasser: Colleague.....	179
10	What Mathematicians Can Do.....	185
	Self-Confidence and Visions	185
	"Unfortunately Difficult" vs. "The Right Stuff"	188
	Record Races	191
	You Know More Math Than You Think	194
	"Mathematics Is ..."	195
	Read More	197
	Index	207

Chapter 1

On the Number Line

“All is number,” the motto of the Pythagoreans, designates the belief that the regular behavior of the world can be grasped and expressed with numbers. We still believe that today, and not without reason.

But if numbers are so fundamental, then we have to allow ourselves to ask, “What are numbers, and what are they good for?” The question may seem stupid or naive, but it is neither. The number theorist Richard Dedekind formulated the question in this way as the title of his famous book *Was sind und was sollen die Zahlen?* And the question has no simple answer. As indeed it cannot; it may well be that the regularities in the world can be phrased and understood in terms of numbers, but if so, then “one, two, three” will certainly not suffice. In fact, the so-called *natural* numbers (one, two, three, and so forth), which seem to us so concrete and obvious, already present problems—philosophical problems, of course, but also very concrete problems.

Therefore, once again, what are numbers? Are they something that designates a quantity? In that case, is $\frac{1}{2}$ a number? Or -1 ? Or $\sqrt{2}$? Something with which one can count? In that case, is “infinity” a number? Something with which one can

compute? A domain in which one can solve equations? In that case, the “imaginary unit” $i = \sqrt{-1}$ is a number. And, as if the question were not already unclear enough, it seems as if mathematicians can’t get enough, they are always creating new numbers—or will they at some point finally be satisfied?

3—Can Bees Count?

In January 2009, many newspapers around the world reported the news that “Bees can count to three.” Researchers at the Bee Group of the University of Würzburg were said to have determined that bees could indeed count to three.

However, I also remembered seeing a headline not too long before that bees can count to four. What was going on? In this case it was not my bad memory, since the much more complete and thorough memory of Google confirmed that “Bees can count to four” was reported in October 2008 by *Netzeitung* (www.netzeitung.de), for example.

Let us leave aside for the moment that we find it extremely surprising that bees can count at all, never mind whether it is up to three or up to four. And let us leave aside any mysterious auras that surround these numbers. Three, like each of the smaller integers, is freighted with all sorts of symbolism, including, for example, the Christian Trinity, which involves the belief that the Father, Son, and Holy Ghost are together *one* God, or in simple terms, “one equals three.” This opens up all sorts of avenues for further discussions, but that will not be our theme. Three is for us nothing but a number—but what does that mean?

The two news reports were referring to different experiments by different teams of bee researchers. In the first experiment, bees were trained to fly toward panels that showed objects, where they were rewarded with sugar water. The bees learned that it was only panels with three objects that led to food, not panels with four or six objects. They were able to make this

distinction whether the panels had apples, flowers, or red or black spots. The researchers concluded that the bees had been able to form an “abstract” notion of the number 3 and could differentiate it from 4. They could, after training, fly toward panels with three objects instead of panels with four, five, or six objects. The bees could, however, not be trained to prefer panels with four objects to those with five objects, which led the researchers to conclude that bees cannot distinguish 4 from 5 nor 5 from 6. Thus the report that bees can count to three. Quite an achievement for a little animal with a brain the size of a sesame seed. Chimpanzees and humans can recognize four objects at a glance, but no more. With five or six objects it has been shown not to be possible to see “at a glance” how many there are; for that many objects, one has to start counting.

Do we now have to imagine the clever bee Maya and her somewhat duller friend Willy who mutter “one, two, three...” to themselves and point their fingers in the air? But we know bees don’t have fingers. And they probably mumble only in animated television series.

As for the second report, to make bees count, one could try the following experiment: let bees fly through a tube with marks on the side and try to train them to look for food after the third mark. The marks could be put at different distances each time, so that the bees would be prevented from looking for the food at some particular distance. And indeed, the bees can learn to count to three, that is, to fly to the third marker. They can also be trained to count to four, that is, to fly to the fourth marker. But farther than that they cannot count, not even after very patient training.

Thus it came to be that in German newspapers it was reported that “bees can count to four”—and perhaps raising the question of whether journalists can count at all in a reader’s mind a few weeks later when one saw the headline “Bees Can Count to Three.” In any case, Professor Srinivasan, one of the investigators, reports that there are slower learners and

faster learners among the bees, that is, cleverer and duller bees. But we knew that already, and we like Willy anyway.

So, do the bees then know what the number 3 *really* is? That is a fruitful philosophical question that we cannot leave entirely to the bees. However, among the reliable and firm foundations of mathematics is, naturally, an expectation that mathematicians will have neat and clear answers and concepts for such questions. They have them, too, but not for as long as one might think: this is not one of the questions answered in the depths of time or even in some Greek dialog from classic times. It was only toward the end of the nineteenth century that Georg Cantor clarified the difference between cardinal and ordinal numbers in the course of his investigations of set theory. The former describes the size of a set (a set can have one, two, three, or more elements; one is concerned with the quantity). The latter arises in the sorting of elements and then in counting them off (in which position is an element in a sequence?). There is an enormous difference, even when bees are counting. Only the journalists (and the headline writers) missed the distinction. It is, after all, not simple.

Even so, should we be impressed that bees can count to four? Actually, no. Much more impressive is the “waggle dance” in which bees dance in a geometric pattern to tell their hive mates the location of food sources. It’s a powerful dance: the angle from the vertical of the line along which the bee waggles indicates the angle from the direction to the sun in which the bees must fly to find the buffet. Clearly, bees are inclined more to geometry than to arithmetic. As you can see, mathematics is rich and varied enough that all can use their talents.

5—Can Chickens Compute?

Another one of these headlines that seems to undermine our presumed supremacy in the realm of mathematics: “Chicks Can Compute—At Least Up to Five.” An Italian researcher named

Rosa Rugani and her colleagues found this out. The news was circulated around the world on April 1, 2009, occasionally with an explicit comment that this was *not* an April Fool's joke. The BBC website published the result with the provocative headline "Baby Chicks Do Basic Arithmetic" and added appropriately adorable pictures of baby chicks. The scientific publication supporting the claim, according to the report, appeared in the renowned *Proceedings of the Royal Society B: Biological Sciences*.

In fact, I was able to find, with help of Google, the appropriate page, also published online on April 1. There it is claimed that freshly hatched chicks can compute, for example, " $2 + 3 = 5$ " in their heads (where else?), which Rosa Rugani was able to confirm through a tricky set of experiments.

I was skeptical—and disturbed. The little chicks can do math? Is this an April Fool's joke after all? Suppose we take it seriously, which would mean that at least primitive arithmetic operations such as addition are not only child's play but even something freshly hatched chicks can do. But what would the little fuzzy beasts gain thereby? Some evolutionary advantage? I am definitely of the belief that intelligence has brought us (!) some (?) evolutionary advantages (although this is far from proven)—but chicks?

So I wrote on my blog "Mathematik im Alltag" (www.wissenlogs.de) with a headline "Chicks Can Compute? Help!" and asked readers for clarification and explanation. The first response was from a reader who was quite certain that Ms. Rugani conducted only serious research that was contributing further proof that chickens are smarter than we think. The reader's name was Martin Huhn (but if my blog were in English, he probably would have signed himself Martin Chicken).

And then I remembered that already in the 1980s a certain Luigi Malerba from Italy had reported:

A learned hen wanted to teach her colleagues to count and to add. So she wrote the numbers 1 through 9 onto

one of the walls of the chicken coop and explained that one can get even larger numbers by combining these. To teach the others addition, she wrote on the next wall: $1 + 1 = 11$, $2 + 2 = 22$, $3 + 3 = 33$, and so forth, until $9 + 9 = 99$. The hens learned to add and found it quite useful.

This clarifies everything.

10—And the Name of the Rose

A friend, and proud father, recently told me that his two-year-old son could already count to five. Only, he does not like two, so he counts, “one, (short pause), three, four, five.” Now we have to clarify just what he means by “three,” and we better do it soon, before he *really* learns to count.

The same problem occurs elsewhere: most airplanes have no row to which the label “13” is attached; of course, the passengers in the row labeled “14” are nonetheless sitting in the thirteenth row (and we hope they feel safer because the row is mislabeled). In many theaters and opera houses, the pleasure of having cadged a seat in the first row evaporates when one finds that one is sitting seven rows from the stage, with heads from rows A, B, C, etc., or even AA, BB, ... blocking the view. (Sometimes this is a good thing—for example, if the stage is so high that the people sitting in what is really the first row need chiropractic assistance after the performance.) Or think of Douglas Adams’s science fiction trilogy *Hitchhiker’s Guide to the Galaxy*. The cover of the fifth (!) volume included a note that this is a book that “gives an entirely new meaning to the concept of trilogy.” That it does.

The redefinition of numbers is an everyday phenomenon that we trip over everywhere. Nonetheless, the father’s concern about his son’s “wrong counting” is noteworthy. Of course, one could