



中国科学院研究生教学丛书

算术代数

ALGORITHMIC ALGEBRA

Bhubaneswar Mishra



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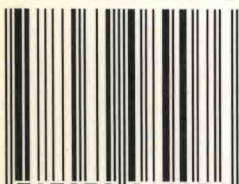
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著者 Bhubaneswar Mishra



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内 容 简 介

本书是中国科学院推荐的研究生原版教材之一,是近年来出版的计算机代数方面的权威著作.书中全面介绍了近20年来该领域的主要成果,包括 Gröbner 基、Wu-Ritt 特征基、系统式法、实代数几何等.这些成果是计算机与代数几何交叉研究所产生的新成果,不仅对代数的发展有很大的影响,也对代数学算法在机器人、计算机视觉等方面的应用提供了基础.本书可作为数学系及计算机系相关专业研究生的教材.

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《中国科学院研究生教学丛书》序

在 21 世纪曙光初露,中国科技、教育面临重大改革和蓬勃发展之际,《中国科学院研究生教学丛书》——这套凝聚了中国科学院新老科学家、研究生导师们多年心血的研究教材面世了。相信这套丛书的出版,会在一定程度上缓解研究生教材不足的困难,对提高研究生教育质量起着积极的推动作用。

21 世纪将是科学技术日新月异,迅猛发展的新世纪,科学技术将成为经济发展的最重要的资源和不竭的动力,成为经济和社会发展的首要推动力量。世界各国之间综合国力的竞争,实质上是科技实力的竞争。而一个国家科技实力的决定因素是它所拥有的科技人才的数量和质量。我国要想在 21 世纪顺利地实施“科教兴国”和“可持续发展”战略,实现邓小平同志规划的第三步战略目标——把我国建设成中等发达国家,关键在于培养造就一支数量宏大、素质优良、结构合理、有能力参与国际竞争与合作的科技大军。这是摆在我国高等教育面前的一项十分繁重而光荣的战略任务。

中国科学院作为我国自然科学与高新技术的综合研究与发展中心,在建院之初就明确了出成果出人才并举的办院宗旨,长期坚持走科研与教育相结合的道路,发挥了高级科技专家多、科研条件好、科研水平高的优势,结合科研工作,积极培养研究生;在出成果的同时,为国家培养了数以万计的研究生。当前,中国科学院正在按照江泽民同志关于中国科学院要努力建设好“三个基地”的指示,在建设具有国际先进水平的科学研究基地和促进高新技术产

业发展基地的同时,加强研究生教育,努力建设好高级人才培养基地,在肩负起发展我国科学技术及促进高新技术产业发展重任的同时,为国家源源不断地培养输送大批高级科技人才。

质量是研究生教育的生命,全面提高研究生培养质量是当前我国研究生教育的首要任务。研究生教材建设是提高研究生培养质量的一项重要基础性工作。由于各种原因,目前我国研究生教材的建设滞后于研究生教育的发展。为了改变这种情况,中国科学院组织了一批在科学前沿工作,同时又具有相当教学经验的科学家撰写研究生教材,并以专项资金资助优秀的研究生教材的出版。希望通过数年努力,出版一套面向 21 世纪科技发展、体现中国科学院特色的高水平的研究生教学丛书。本丛书内容力求具有科学性、系统性和基础性,同时也兼顾前沿性,使阅读者不仅能获得相关学科的比较系统的科学基础知识,也能被引导进入当代科学研究的前沿。这套研究生教学丛书,不仅适合于在校研究生学习使用,也可以作为高校教师和专业研究人员工作和学习的参考书。

“桃李不言,下自成蹊。”我相信,通过中国科学院一批科学家的辛勤耕耘,《中国科学院研究生教学丛书》将成为我国研究生教育园地的一丛鲜花,也将似润物春雨,滋养莘莘学子的心田,把他们引向科学的殿堂,不仅为科学院,也为全国研究生教育的发展作出重要贡献。

钱伯祥

To my parents

Purna Chandra & Baidehi Mishra

Preface

In the fall of 1987, I taught a graduate computer science course entitled “Symbolic Computational Algebra” at New York University. A rough set of class-notes grew out of this class and evolved into the following final form at an excruciatingly slow pace over the last five years. This book also benefited from the comments and experience of several people, some of whom used the notes in various computer science and mathematics courses at Carnegie-Mellon, Cornell, Princeton and UC Berkeley.

The book is meant for graduate students with a training in theoretical computer science, who would like to either do research in computational algebra or understand the algorithmic underpinnings of various commercial symbolic computational systems: *Mathematica*, *Maple* or *Axiom*, for instance. Also, it is hoped that other researchers in the robotics, solid modeling, computational geometry and automated theorem proving communities will find it useful as symbolic algebraic techniques have begun to play an important role in these areas.

The main four topics—Gröbner bases, characteristic sets, resultants and semialgebraic sets—were picked to reflect my original motivation. The choice of the topics was partly influenced by the syllabii proposed by the Research Institute for Symbolic Computation in Linz, Austria, and the discussions in Hearn’s Report (“Future Directions for Research in Symbolic Computation”).

The book is meant to be covered in a one-semester graduate course comprising about fifteen lectures. The book assumes very little background other than what most beginning computer science graduate students have. For these reasons, I have attempted to keep the book self-contained and largely focussed on the very basic materials.

Since 1987, there has been an explosion of new ideas and techniques in all the areas covered here (e.g., better complexity analysis of Gröbner basis algorithms, many new applications, effective Nullstellensatz, multivariate resultants, generalized characteristic polynomial, new stratification algorithms for semialgebraic sets, faster quantifier elimination algorithm for Tarski sentences, etc.). However, none of these new topics could be included here without distracting from my original intention. It is hoped that this book will prepare readers to be able to study these topics on their own.

Also, there have been several new textbooks in the area (by Akritas, Davenport, Siret and Tournier, and Mignotte) and there are a few more on the way (by Eisenbaud, Robbiano, Weispfenning and Becker, Yap, and Zippel). All these books and the current book emphasize different materials, involve different degrees of depth and address different readerships. An instructor, if he or she so desires, may choose to supplement the current book by some of these other books in order to bring in such topics as factorization, number-theoretic or group-theoretic algorithms, integration or differential algebra.

The author is grateful to many of his colleagues at NYU and elsewhere for their support, encouragement, help and advice. Namely, J. Canny, E.M. Clarke, B. Chazelle, M. Davis, H.M. Edwards, A. Frieze, J. Gutierrez, D. Kozen, R. Pollack, D. Scott, J. Spencer and C-K. Yap. I have also benefited from close research collaboration with my colleague C-K. Yap and my graduate students G. Gallo and P. Pedersen. Several students in my class have helped me in transcribing the original notes and in preparing some of the solutions to the exercises: P. Agarwal, G. Gallo, T. Johnson, N. Oliver, P. Pedersen, R. Sundar, M. Teichman and P. Tetali.

I also thank my editors at Springer for their patience and support. During the preparation of this book I had been supported by NSF and ONR and I am gratified by the interest of my program officers: Kamal Abdali and Ralph Wachter.

I would like to express my gratitude to Prof. Bill Wulf for his efforts to perform miracles on my behalf during many of my personal and professional crises. I would also like to thank my colleague Thomas Anantharaman for reminding me of the power of intuition and for his friendship. Thanks are due to Robin Mahapatra for his constant interest.

In the first draft of this manuscript, I had thanked my imaginary wife for keeping my hypothetical sons out of my nonexistent hair. In the interim five years, I have gained a wife Jane and two sons Sam and Tom, necessarily in that order—but, alas, no hair. To them, I owe my deepest gratitude for their understanding.

Last but not least, I thank Dick Aynes without whose unkind help this book would have gone to press some four years ago.

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