

FOURIER OPTICS: An Introduction

E. G. STEWARD, D.Sc., F.Inst. P.

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E. G. STEWARD, D.Sc., F.Inst. P.
Professor of Physics and Molecular Medicine
The City University, London



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To encourage and help the reader to consult the whole span of the literature the text is referenced throughout, in addition to the inclusion of a bibliography. I have assumed that the reader already has a knowledge of mathematics up to the usual university entrance level, an additional mathematics being developed in the context in which it is used. In the main, the treatment is restricted to one dimension so as to keep the mathematical treatment as generally accessible as possible.

Author's Preface

It is a great pleasure to record my sincere thanks to Dr R. S. Longman for stimulating discussions and suggestions at many stages in the preparation of the first manuscript, to be able to tap his experience as author of a major undergraduate textbook on optics has been invaluable. Similarly, I am grateful to Professor Henry Lipson who kindly read and commented on Chapters 1, 2 and 3.

This book is intended for students who seek a simple introduction to the Fourier principles of modern optics and an insight into the similar role they play in other branches of science and engineering. Fourier transforms, associated with the operations of convolution and correlation, form the basis not only of image formation and processing with lens systems, but also of studies ranging from the atomic structure of matter to the galactic structure of the universe, and some modern methods of spectroscopy. They also apply to the communication and information sciences of electrical engineering, including the processing of information that is not optical in origin. As a supplement to the undergraduate mainstream textbooks on optics the book bridges the gap to the more advanced treatises in the various specialized fields.

Incoherent imaging is described from the point of view of convolution and transfer functions, with emphasis on the features of linearity and invariance shared with many types of electrical network (non-linear systems being outside the scope of this book). The double Fourier transformation process of coherent image formation is illustrated with particular reference to its application in X-ray crystallography.

As an introduction to the wide range of applications of optical filtering and image processing the basic ideas of amplitude-, phase- and holographic filtering are described, with illustrations drawn from optical and electron microscopy, and from the developing field of pattern recognition. Energy-spectrum correlation and geometrical optics-based processing are also briefly described.

Historical aspects are mentioned wherever they particularly help to convey the spirit of science and show how developments take place: to understand fully the present state of a subject, and where it is going, it is essential to know where it has come from. For example, in Chapter 6 it is explained how the modern uses of interferometry in radio and optical astronomy, and spectroscopy, have their origins in Michelson's stellar and spectral interferometers: the methods that Michelson devised, and his brilliant awareness of their potential, form the natural starting point for understanding their present-day counterparts.

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E. G. Steward

Preliminaries

1.1 INTRODUCTION

This book is primarily concerned with the Fourier aspects of two inter-related topics in physical optics: (i) the formation and processing of images and (ii) the study of the spatial distribution and spectral composition of radiation sources. They are brought together within the same covers because they are inter-related in a number of ways, and when such relationships exist there is, usually, as here, an increased benefit to be obtained from studying the topics together. Their wider implications, referred to in the Preface, will be introduced as they arise.

As we work through these pages we can think of both topics in the context of visible light, but we also deal with some important applications in other parts of the electromagnetic spectrum. In image formation this takes us from the optical region to the way in which X-rays are used to deduce the atomic structure of matter, and at the other extreme, to astronomy and the structure of the universe. In spectroscopy, Fourier-based methods are now used over a wide spectral range.

The mathematical methods named after **J. B. J. Fourier** are extremely powerful in these topics. They are introduced in Chapters 3 and 4, and are used extensively in the closing chapters. For the mathematical representation of light the '**scalar-wave approximation**' as described in general physics textbooks is entirely adequate and is employed throughout. Appendix A gives a resumé of the notation and basic equations used, together with reminders of the meaning of such terms as path difference and phase difference, and the use of phasor diagrams for summing waves having different amplitudes and phases.

In this chapter we identify, in general terms, some of the physical processes involved in the two main topics. These can be introduced very conveniently in the context of Young's experiment and the phenomena of the Newton's rings type.

The reader will already know that the historical experiment performed by **Thomas Young** in 1801 provided crucial evidence in support of the wave theory of light. We must start by reminding ourselves of some of the details of this experiment and of our present-day interpretation of it.