

Springer Tracts in Mechanical Engineering

Tianjian Lu
Fengxian Xin

Vibro-Acoustics of Lightweight Sandwich Structures



Science Press
Beijing



Springer

Tianjian Lu • Fengxian Xin

Vibro-Acoustics of Lightweight Sandwich Structures

(轻质夹层板结构的声振耦合理论)



 Science Press
Beijing

 Springer

Tianjian Lu
Fengxian Xin
Xi'an Jiaotong University
Xi'an
China

ISBN 978-7-03-041322-2

Science Press Beijing

ISBN 978-3-642-55357-8

ISBN 978-3-642-55358-5 (eBook)

Springer Heidelberg New York Dordrecht London

© Science Press Beijing and Springer-Verlag Berlin Heidelberg 2014

This work is subject to copyright. All rights are reserved by the Publishers, whether the whole or part of the material is concerned, specifically the rights of translation, reprinting, reuse of illustrations, recitation, broadcasting, reproduction on microfilms or in any other physical way, and transmission or information storage and retrieval, electronic adaptation, computer software, or by similar or dissimilar methodology now known or hereafter developed. Exempted from this legal reservation are brief excerpts in connection with reviews or scholarly analysis or material supplied specifically for the purpose of being entered and executed on a computer system, for exclusive use by the purchaser of the work. Duplication of this publication or parts thereof is permitted only under the provisions of the Copyright Law of the Publishers' locations, in its current version, and permission for use must always be obtained from Springer. Permissions for use may be obtained through RightsLink at the Copyright Clearance Center. Violations are liable to prosecution under the respective Copyright Law.

The use of general descriptive names, registered names, trademarks, service marks, etc. in this publication does not imply, even in the absence of a specific statement, that such names are exempt from the relevant protective laws and regulations and therefore free for general use.

While the advice and information in this book are believed to be true and accurate at the date of publication, neither the authors nor the editors nor the publishers can accept any legal responsibility for any errors or omissions that may be made. The publishers make no warranty, express or implied, with respect to the material contained herein.

Not for sale outside the Mainland of China (Not for sale in Hong Kong SAR, Macau SAR, and Taiwan and all countries, except the Mainland of China)

Vibro-Acoustics of Lightweight Sandwich Structures

(轻质夹层板结构的声振耦合理论)

Preface

The purpose of this book is to present the vibration and acoustical behavior of typical sandwich structures subject to mechanical and/or acoustical loadings, which actually form a class of structural elements of practical importance in huge amounts of engineering applications, such as aircraft fuselage, ship and submarine hulls. The contents of this book has grown out of the research activities of the authors in the field of sound radiation/transmission of/through lightweight sandwich structures.

The book is organized into six chapters: Chapter 1 deals with the vibro-acoustic performance of rectangular multiple-panel partitions with enclosed air cavity theoretically and experimentally, which has accounted for the simply supported and clamp supported boundary conditions. Chapter 2 concerns with the transmission of external jet-noise through a uniform skin plate of aircraft cabin fuselage in the presence of external mean flow. As an extension, Chap. 3 handles with the noise radiation and transmission from/through aeroelastic skin plates of aircraft fuselage stiffened by orthogonally distributed rib-stiffeners in the presence of convected mean flow. Chapter 4 develops a theoretical model for sound transmission through all-metallic, two-dimensional, periodic sandwich structures having corrugated core. Chapter 5 focuses on the sound radiation and transmission characteristics of periodically stiffened structures. Ultimately, Chap. 6 proposes the sound radiation and transmission behaviors of periodical sandwich structures having cavity-filling fibrous sound absorptive materials.

This book is involving multidisciplinary subjects especially including combined knowledge of vibration, aeroelastics and structural acoustics, which pays much attention on showing results and conclusions, in addition to mere theoretical modelling. Therefore this book should be of considerable interest to a wide range of readers in relevant fields. It is hoped that the content of the book will find application not only as a textbook for a wide audience of engineering students, but also a general reference for researchers in the field of vibrations and acoustics.

Xi'an, China

T.J. Lu
F.X. Xin

Acknowledgements

Although the contents of this book has grown out of the research activities of the authors, we would also like to deeply appreciate the reproduction permission for figures, tables etc. from Elsevier, AIAA, ASME, Acoustical Society of America and Taylor & Francis Ltd. These research activities are supported by the National Basic Research Program of China (Grant No. 2011CB610300), the National Natural Science Foundation of China (Grant Nos. 11102148, and 11321062) and the Fundamental Research Funds for Central Universities (xjj2011005).

Contents

1	Transmission of Sound Through Finite Multiple-Panel Partition	1
1.1	Simply Supported Finite Double-Panel Partitions	2
1.1.1	Introduction	2
1.1.2	Vibroacoustic Theoretical Modeling	4
1.1.3	Mathematic Formulation and Solution	5
1.1.4	Convergence Check for Numerical Results	10
1.1.5	Model Validation	11
1.1.6	Effects of Air Cavity Thickness	13
1.1.7	Effects of Panel Dimensions	17
1.1.8	Effects of Incident Elevation Angle and Azimuth Angle	20
1.1.9	Conclusions	23
1.2	Clamped Finite Double-Panel Partitions	24
1.2.1	Introduction	24
1.2.2	Modeling of the Vibroacoustic Coupled System.....	26
1.2.3	Model Validation	32
1.2.4	Finite Versus Infinite Double-Panel Partition	34
1.2.5	Effects of Panel Thickness on STL	35
1.2.6	Effects of Air Cavity Thickness on STL	37
1.2.7	Effects of Incident Angles on STL	37
1.2.8	Conclusions	40
1.2.9	Sound Transmission Measurements	41
1.2.10	Relationships Between Clamped and Simply Supported Boundary Conditions.....	47
1.2.11	Conclusions	51
1.3	Clamped Finite Triple-Panel Partitions	53
1.3.1	Introduction	53
1.3.2	Dynamic Structural Acoustic Formulation	56
1.3.3	The Principle of Virtual Work	60
1.3.4	Determination of Modal Coefficients	60
1.3.5	Sound Transmission Loss	63
1.3.6	Model Validation	63

1.3.7	Physical Interpretation of STL Dips	64
1.3.8	Comparison Among Single-, Double-, and Triple-Panel Partitions with Equivalent Total Mass	68
1.3.9	Asymptotic Variation of STL Versus Frequency Curve from Finite to Infinite System	69
1.3.10	Effects of Panel Thickness	70
1.3.11	Effects of Air Cavity Depth	74
1.3.12	Concluding Remarks	75
Appendices		77
Appendix A		77
Appendix B		80
References		83
2	Vibroacoustics of Uniform Structures in Mean Flow	87
2.1	Finite Single-Leaf Aeroelastic Plate	88
2.1.1	Introduction	88
2.1.2	Modeling of Aeroelastic Coupled System	90
2.1.3	Effects of Mean Flow in Incident Field	99
2.1.4	Effects of Mean Flow in Transmitted Field	103
2.1.5	Effects of Incident Elevation Angle in the Presence of Mean Flow on Both Incident Side and Transmitted Side	106
2.1.6	Conclusions	108
2.2	Infinite Double-Leaf Aeroelastic Plates	109
2.2.1	Introduction	109
2.2.2	Statement of the Problem	111
2.2.3	Formulation of Plate Dynamics	112
2.2.4	Consideration of Fluid-Structure Coupling	114
2.2.5	Definition of Sound Transmission Loss	117
2.2.6	Characteristic Impedance of an Infinite Plate	117
2.2.7	Physical Interpretation for the Appearance of STL Peaks and Dips	119
2.2.8	Effects of Mach Number	122
2.2.9	Effects of Elevation Angle	127
2.2.10	Effects of Azimuth Angle	128
2.2.11	Effects of Panel Curvature and Cabin Internal Pressurization	129
2.2.12	Conclusions	130
2.3	Double-Leaf Panel Filled with Porous Materials	131
2.3.1	Introduction	131
2.3.2	Problem Description	133
2.3.3	Theoretical Model	134
2.3.4	Validation of Theoretical Model	140
2.3.5	Influence of Porous Material and the Faceplates	141
2.3.6	Influence of Porous Material Layer Thickness	143

2.3.7	Influence of External Mean Flow	144
2.3.8	Influence of Incident Sound Elevation Angle	147
2.3.9	Influence of Sound Incident Azimuth Angle	148
2.3.10	Conclusion	150
Appendix	151
	Mass-Air-Mass Resonance	151
	Standing-Wave Attenuation	152
	Standing-Wave Resonance.....	152
	Coincidence Resonance.....	153
References	154
3	Vibroacoustics of Stiffened Structures in Mean Flow	159
3.1	Noise Radiation from Orthogonally Rib-Stiffened Plates.....	160
3.1.1	Introduction	160
3.1.2	Theoretical Formulation	162
3.1.3	Effect of Mach Number	170
3.1.4	Effect of Incidence Angle	172
3.1.5	Effect of Periodic Spacings	173
3.1.6	Concluding Remarks	175
3.2	Transmission Loss of Orthogonally Rib-Stiffened Plates	176
3.2.1	Introduction.....	176
3.2.2	Theoretical Formulation	178
3.2.3	Model Validation	187
3.2.4	Effects of Mach Number of Mean Flow.....	189
3.2.5	Effects of Rib-Stiffener Spacings.....	190
3.2.6	Effects of Rib-Stiffener Thickness and Height.....	193
3.2.7	Effects of Elevation and Azimuth Angles of Incident Sound.....	194
3.2.8	Conclusions.....	197
Appendices	198
	Appendix A.....	198
	Appendix B.....	201
References	203
4	Sound Transmission Across Sandwich Structures with Corrugated Cores	207
4.1	Introduction	207
4.2	Development of Theoretical Model	209
4.3	Effects of Core Topology on Sound Transmission Across the Sandwich Structure	214
4.4	Physical Interpretation for the Existence of Peaks and Dips on STL Curves.....	215
4.5	Optimal Design for Combined Sound Insulation and Structural Load Capacity.....	219
4.6	Conclusion	220
References	221

5	Sound Radiation, Transmission of Orthogonally Rib-Stiffened Sandwich Structures	225
5.1	Sound Radiation of Sandwich Structures	226
5.1.1	Introduction	226
5.1.2	Theoretical Modeling of Structural Dynamic Responses	228
5.1.3	Solutions	234
5.1.4	Far-Field Radiated Sound Pressure	238
5.1.5	Validation of Theoretical Modeling	239
5.1.6	Influences of Inertial Effects Arising from Rib-Stiffener Mass	240
5.1.7	Influence of Excitation Position	242
5.1.8	Influence of Rib-Stiffener Spacings	243
5.1.9	Conclusions	244
5.2	Sound Transmission Through Sandwich Structures	245
5.2.1	Introduction	245
5.2.2	Analytic Formulation of Panel Vibration and Sound Transmission	248
5.2.3	The Acoustic Pressure and Continuity Condition	257
5.2.4	Solution of the Formulations with the Virtual Work Principle	258
5.2.5	Virtual Work of Panel Elements	259
5.2.6	Virtual Work of x -Wise Rib-Stiffeners	260
5.2.7	Virtual Work of y -Wise Rib-Stiffeners	261
5.2.8	Combination of Equations	261
5.2.9	Definition of Sound Transmission Loss	264
5.2.10	Convergence Check for Space-Harmonic Series Solution	265
5.2.11	Validation of the Analytic Model	266
5.2.12	Influence of Sound Incident Angles	267
5.2.13	Influence of Inertial Effects Arising from Rib-Stiffener Mass	269
5.2.14	Influence of Rib-Stiffener Spacings	270
5.2.15	Influence of Airborne and Structure-Borne Paths	271
5.2.16	Conclusions	272
	Appendices	273
	Appendix A	273
	Appendix B	278
	References	285
6	Sound Propagation in Rib-Stiffened Sandwich Structures with Cavity Absorption	289
6.1	Sound Radiation of Absorptive Sandwich Structures	290
6.1.1	Introduction	290
6.1.2	Structural Dynamic Responses to Time-Harmonic Point Force	292
6.1.3	The Acoustic Pressure and Fluid-Structure Coupling	295

6.1.4	Far-Field Sound-Radiated Pressure	300
6.1.5	Convergence Check for Numerical Solution	300
6.1.6	Validation of Theoretical Modeling	301
6.1.7	Influence of Air-Structure Coupling Effect	303
6.1.8	Influence of Fibrous Sound Absorptive Filling Material	305
6.1.9	Conclusions	307
6.2	Sound Transmission Through Absorptive Sandwich Structure	308
6.2.1	Introduction	308
6.2.2	Analytic Formulation of Panel Vibration and Sound Transmission	309
6.2.3	Application of the Periodicity of Structures.....	313
6.2.4	Solution by Employing the Virtual Work Principle.....	316
6.2.5	Model Validation	321
6.2.6	Effects of Fluid-Structure Coupling on Sound Transmission.....	322
6.2.7	Sound Transmission Loss Combined with Bending Stiffness and Structure Mass: Optimal Design of Sandwich	324
6.2.8	Conclusions	327
Appendices		328
Appendix A.....		328
Appendix B.....		330
Appendix C.....		333
References		338

Chapter 1

Transmission of Sound Through Finite Multiple-Panel Partition

Abstract This chapter is organized as three parts: in the first part, the vibroacoustic performance of a rectangular double-panel partition with enclosed air cavity and simply mounted on an infinite acoustic rigid baffle is investigated analytically. The sound velocity potential method rather than the commonly used cavity modal function method is employed, which possesses good expandability and has significant implications for further vibroacoustic investigations. The simply supported boundary condition is accounted for by using the method of modal function, and double Fourier series solutions are obtained to characterize the vibroacoustic behaviors of the structure. Results for sound transmission loss (STL), panel vibration level, and sound pressure level are presented to explore the physical mechanisms of sound energy penetration across the finite double-panel partition. Specifically, focus is placed upon the influence of several key system parameters on sound transmission, including the thickness of air cavity, structural dimensions, and the elevation angle and azimuth angle of the incidence sound. Further extensions of the sound velocity potential method to typical framed double-panel structures are also proposed.

In the second part, the air-borne sound insulation performance of a rectangular double-panel partition clamp mounted on an infinite acoustic rigid baffle is investigated both analytically and experimentally, and compared with that of a simply supported one. With the clamped (or simply supported) boundary accounted for by using the method of modal function, a double series solution for the sound transmission loss (STL) of the structure is obtained by employing the weighted residual (Galerkin) method. Experimental measurements with Al double-panel partitions having air cavity are subsequently carried out to validate the theoretical model for both types of the boundary condition, and good overall agreement is achieved. A consistency check of the two different models (based separately on clamped modal function and simply supported modal function) is performed by

extending the panel dimensions to infinite where no boundaries exist. The significant discrepancies between the two different boundary conditions are demonstrated in terms of the STL versus frequency plots as well as the panel deflection mode shapes.

In the third part, an analytical model for sound transmission through a clamped triple-panel partition of finite extent and separated by two impervious air cavities is formulated. The solution derived from the model takes the form of that for a clamp-supported rectangular plate. A set of modal functions (or more strictly speaking, the basic functions) are employed to account for the clamped boundary conditions, and the application of the virtual work principle leads to a set of simultaneous algebraic equations for determining the unknown modal coefficients. The sound transmission loss (STL) of the triple-panel partition as a function of excitation frequency is calculated and compared with that of a double-panel partition. The model predictions are then used to explore the physical mechanisms associated with the various dips on the STL versus frequency curve, including the equivalent “mass-spring” resonance, the standing-wave resonance, and the panel modal resonance. The asymptotic variation of the solution from a finite-sized partition to an infinitely large partition is illustrated in such a way as to demonstrate the influence of the boundary conditions on the soundproofing capability of the partition. In general, a triple-panel partition outperforms a double-panel partition in insulating the incident sound, and the relatively large number of system parameters pertinent to the triple-panel partition in comparison with that of the double-panel partition offers more design space for the former to tailor its noise reduction performance.

1.1 Simply Supported Finite Double-Panel Partitions

1.1.1 Introduction

Double-leaf partition structures have found increasingly wide applications in noise control engineering due to their superior sound insulation capability over single-leaf configurations. Typical examples include transportation vehicles, grazing windows and partition walls in buildings, aircraft fuselage shells, and so on [1–12].

Considerable efforts have been devoted to understanding and predicting the transmission of sound across single-leaf [13–15] and double-leaf [16–29] partitions. In fact, research about the former is often a prerequisite for studying the latter. For instance, Lomas [14] developed Green function solution for the steady-state vibration of an elastically supported rectangular plate coupled to a semi-infinite acoustic medium. An important feature of the investigation is the treatment of the elastic support boundary condition which was taken into account by assuming the rotational motion along the boundary controlled by distributions of massless

rotary springs and by introducing the corresponding moments into the governing equations. The problem of sound radiation by a simply supported unbaffled panel was investigated by Laulagnet [13]. Both pressure jump and plate displacement in series of the simply supported plate models were developed.

Early sound transmission studies [16, 28–30] of double-panel structures with air cavity in between generally simplified the structure as infinite and hence did not account for the elastic boundary conditions on the periphery. For typical examples, Antonio et al. [17] gave an analytical evaluation of the acoustic insulation provided by double infinite walls and also did not take elastic boundary condition into account. Kropp et al. [19] addressed the optimization of sound insulation of double-panel constructions by dividing the frequency range into three cases, i.e., where the double wall resonance frequency is much higher (or closer or much lower) than the critical frequency of the total construction. Recently, Tadeu et al. [20] adopted an analytical method to assess the airborne sound and impact insulation properties of single- and double-leaf panels by neglecting the elastic boundary conditions. Bao and Pan [31] presented an experimental study on active control of sound transmission through double walls with different approaches, including cavity control, panel control, and room control.

For simply supported, finite rectangular double-panel structures, existing studies [3, 22–27, 32–37] concerned mainly with the loss of sound transmission across the structure, without detailed analysis about the energy transmission, the vibroacoustic coupling effects, and the physical mechanisms of sound transmission process across the structure. In particular, previous studies on double-panel partitions focus on either infinite extent or finite extent, without exploring the natural relationship between the two. The present study squarely addresses these deficiencies from the new perspectives of the integration analysis of STL, panel vibration level, and sound pressure level, with more details and the physical nature of sound penetration through double-panel partitions revealed. Since the rigid baffle bounds the cavity as well as the panel so that the cavity boundaries restrict the field to sinusoidal distributions parallel to the panel plane, analytical solutions in double Fourier series are proposed by applying the sinusoidal distributed sound velocity potential method. This method can be easily expanded to the vibroacoustic analysis of rib-stiffened double-panel structures, accounting for both the structure-borne route (i.e., structural connections between the two panels) and the airborne route (i.e., air cavity between the two panels), and hence can be regarded as an alternative of the cavity mode method in certain engineering applications. The model predictions are validated by comparing the analytical results with existing experimental data. The influences of key system parameters such as air cavity thickness, panel dimensions, and elevation angle and azimuth angle of incident sound on the sound insulation capability of the finite double-panel partition are systematically investigated. The results and conclusions of the present study should be referentially significant to others due to the similar physical nature of the vibroacoustic problem.

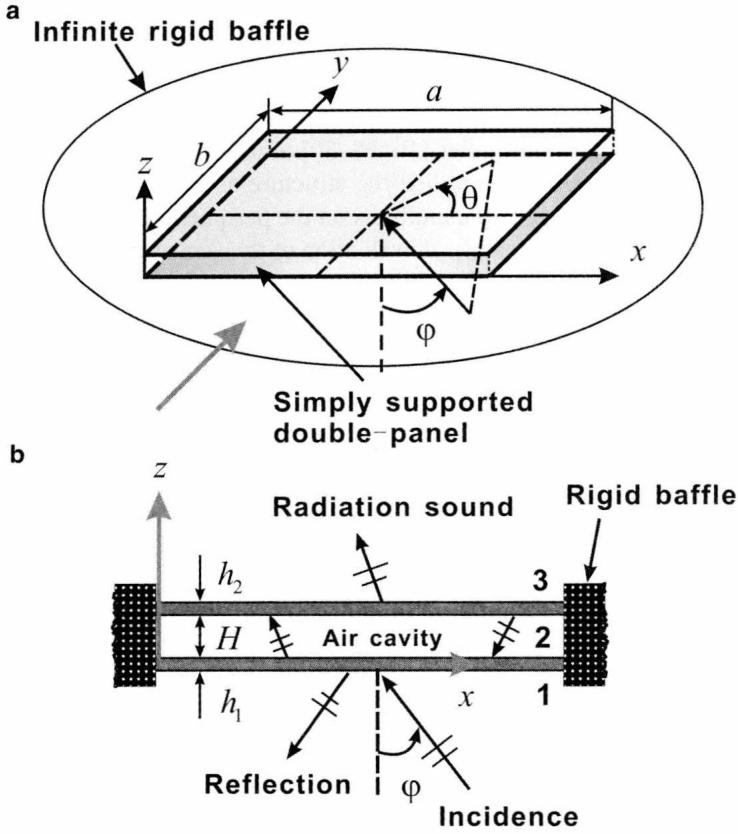


Fig. 1.1 Schematic of sound transmission through a baffled, rectangular, simply supported double-panel partition: (a) global view; (b) side view in the *arrow direction* in (a) (With permission from ASME)

1.1.2 Vibroacoustic Theoretical Modeling

The finite double-panel partition with enclosed air cavity is assumed to be rectangular, baffled, and simply supported along its boundaries, as shown in Fig. 1.1. The two panels are homogenous and isotropic and modeled as classical thin plate. The following geometrical dimensions are considered: the incident (bottom) panel and the radiating (top) panel have identical length a and width b , but may have different thicknesses h_1 and h_2 (Fig. 1.1b); the thickness of the air cavity is H (Fig. 1.1b). The whole configuration is mounted on an infinite acoustic rigid baffle which separates the space into two fields, i.e., sound incidence field ($z < 0$) and sound radiating field ($z > H$). A uniform plane sound wave varying harmonically in time is obliquely incident on the bottom panel, with incident elevation angle φ and azimuth angle θ (Fig. 1.1b). The vibration of the incident panel induced by the incident sound is transmitted through the enclosed air cavity to the radiating panel, which radiates

sound into the acoustic medium. The vibroacoustic behaviors of the double-panel structure coupling with air cavity as well as sound transmission loss across the structure are to be solved analytically with the sound velocity potential method.

1.1.3 Mathematic Formulation and Solution

For an obliquely incident uniform plane sound wave varying harmonically in time, its acoustic velocity potential can be expressed as

$$\phi = Ie^{-j(k_x x + k_y y + k_z z - \omega t)} \quad (1.1)$$

where I is the amplitude; $j = \sqrt{-1}$; ω is the angular frequency; and k_x , k_y , and k_z are the wavenumber components in the x -, y -, and z -directions, respectively:

$$k_x = k_0 \sin \varphi \cos \theta, \quad k_y = k_0 \sin \varphi \sin \theta, \quad k_z = k_0 \cos \varphi \quad (1.2)$$

Here, $k_0 = \omega/c_0$ is the acoustic wavenumber in air, with c_0 denoting the sound speed in air.

Due to the excitation of the incident sound wave, the double-panel configuration with enclosed air cavity vibrates and radiates sound. The vibroacoustic behaviors of the structure are governed by

$$D_1 \nabla^4 w_1 + m_1 \frac{\partial^2 w_1}{\partial t^2} - j\omega \rho_0 (\Phi_1 - \Phi_2) = 0 \quad (1.3)$$

$$D_2 \nabla^4 w_2 + m_2 \frac{\partial^2 w_2}{\partial t^2} - j\omega \rho_0 (\Phi_2 - \Phi_3) = 0 \quad (1.4)$$

where ρ_0 is the air density and (w_1, w_2) , (m_1, m_2) and (D_1, D_2) are the transverse displacements, surface densities, and flexural rigidities of the incident and radiating panels, located at $z=0$ and $z=H$, respectively (Fig. 1.1). By introducing the loss factor of the panel material, the flexural rigidity of the panel D_i ($i=1, 2$) can be written in terms of the complex Young's modulus $E_i(1+j\eta_i)$ as

$$D_i = \frac{E_i h_i^3 (1 + j\eta_i)}{12 (1 - \nu_i^2)} \quad (1.5)$$

The hard-walled cavity modal function $\phi_{mnl}^c = \cos(m\pi x/a)\cos(n\pi y/b)\cos(l\pi z/c)$ can only accurately model the sound field in a rigidly bounded cavity volume. It will therefore deviate somewhat from the precise results when the hard-walled cavity modal function is employed here to model the cavity bounded by two large flexural panels. In order to avoid this drawback, the sound velocity potential method is adopted, which is completely different from previous investigations based on cavity

modal function. Let Φ_i ($i = 1, 2, 3$) denote the velocity potentials of the three acoustic fields, i.e., sound incidence field, air cavity field, and structure radiating field (Fig. 1.1b), respectively. The velocity potential for the incident field can be defined as

$$\Phi_1(x, y, z; t) = Ie^{-j(k_x x + k_y y + k_z z - \omega t)} + \beta e^{-j(k_x x + k_y y - k_z z - \omega t)} \quad (1.6)$$

where the first and second terms represent separately the velocity potential of the incident and the reflected plus radiating sound waves and I and β are the amplitudes of the incident (i.e., positive-going) and the reflected plus radiating (i.e., negative-going) waves, respectively. Similarly, the velocity potential in the air cavity can be written as

$$\Phi_2(x, y, z; t) = \varepsilon e^{-j(k_x x + k_y y + k_z z - \omega t)} + \zeta e^{-j(k_x x + k_y y - k_z z - \omega t)} \quad (1.7)$$

where ε is the amplitude of positive-going wave and ζ is the amplitude of negative-going wave. In the radiating field, there exist no reflected waves; thus, the velocity potential is only for radiating waves:

$$\Phi_3(x, y, z; t) = \xi e^{-j(k_x x + k_y y + k_z z - \omega t)} \quad (1.8)$$

where ξ is the amplitude of radiating (i.e., positive-going) wave. The local acoustic velocities and sound pressure are related to the velocity potentials by

$$\hat{\mathbf{u}}_i = -\nabla \Phi_i, \quad p_i = \rho_0 \frac{\partial \Phi_i}{\partial t} = j\omega \rho_0 \Phi_i \quad (i = 1, 2, 3) \quad (1.9)$$

For simply supported boundary condition, the transverse displacement and the transverse force are constrained to be zero at the periphery of the panel. Given that the double-panel structure is rectangular, the boundary conditions can be expressed as

$$x = 0, a : \quad w_1 = w_2 = 0, \quad \frac{\partial^2 w_1}{\partial x^2} = \frac{\partial^2 w_2}{\partial x^2} = 0 \quad (1.10)$$

$$y = 0, b : \quad w_1 = w_2 = 0, \quad \frac{\partial^2 w_1}{\partial y^2} = \frac{\partial^2 w_2}{\partial y^2} = 0 \quad (1.11)$$

At the air-panel interface, the normal velocity should be continuous, yielding the following velocity compatibility equations:

$$z = 0 : \quad -\frac{\partial \Phi_1}{\partial z} = j\omega w_1, \quad -\frac{\partial \Phi_2}{\partial z} = j\omega w_1 \quad (1.12)$$

$$z = H : \quad -\frac{\partial \Phi_2}{\partial z} = j\omega w_2, \quad -\frac{\partial \Phi_3}{\partial z} = j\omega w_2 \quad (1.13)$$