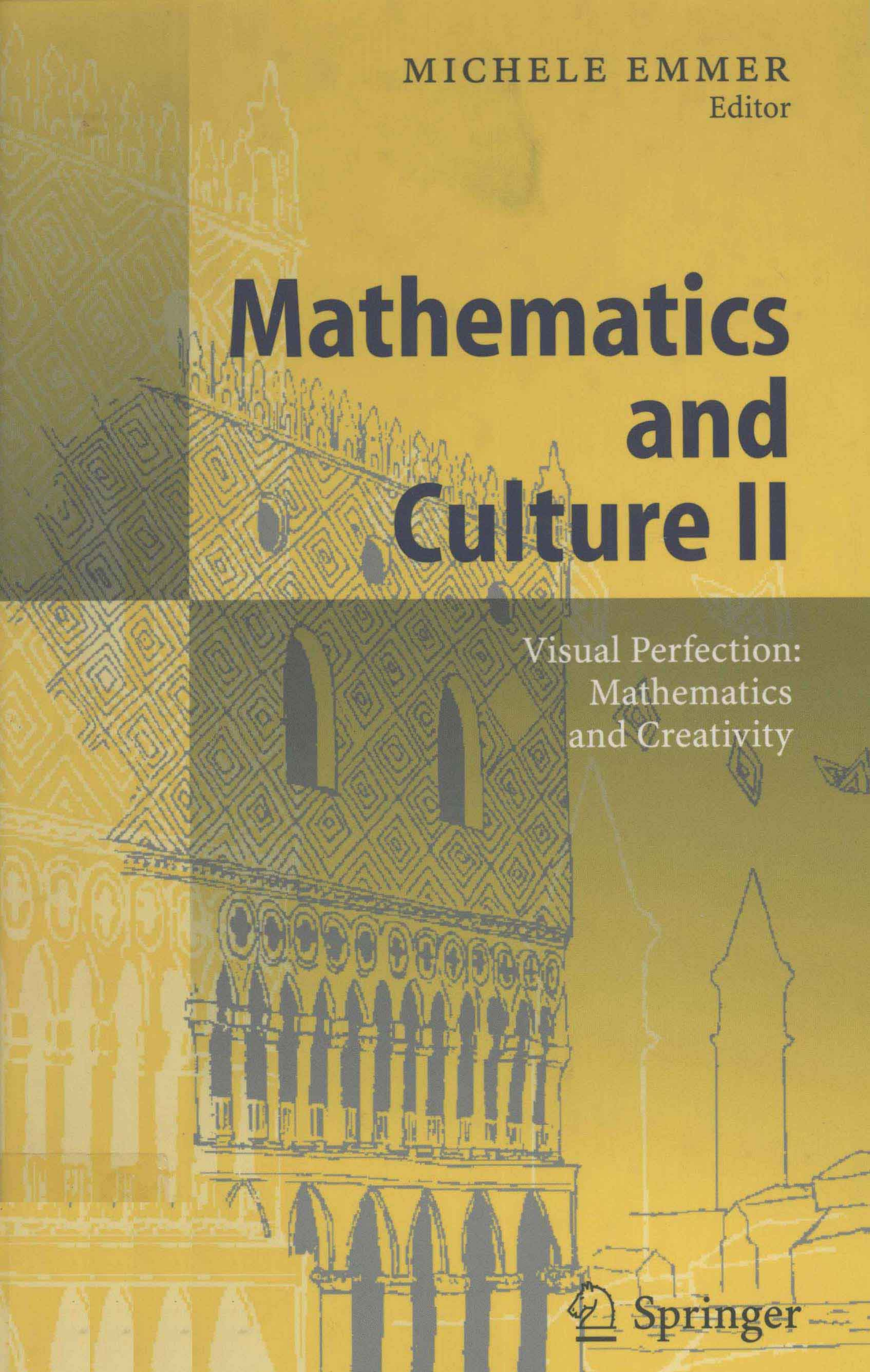
MICHELE EMMER
Editor

Mathematics and Culture II

Visual Perfection: Mathematics and Creativity



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Visual Perfection: Mathematics and Creativity

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Mathematics and Culture II

Mathematics and Culture: Ever New Ideas

MICHELE EMMER

One of the interesting questions in the study of links between mathematics, art and creativity is whether a mathematician's creativity leads him to invent a new world, or rather makes him discover one that already exists in its own right. It could seem like a superfluous question of very little interest. Though it might seem so to non mathematicians, it certainly isn't so for many mathematicians like Roger Penrose, who has dedicated a part of the book *The Emperor's New Mind* [1] to this subject. "In mathematics, should one talk of invention or of discovery?" asks Roger Penrose. There are two possible answers to the question: when the mathematician obtains new results, he creates only some elaborate mental constructions which, although have no relation to physical reality whatsoever, nevertheless possess such power and elegance that they are able to make the researcher believe that these "mere mental constructions" have their very own reality. Alternatively, do mathematicians discover that "these mere mental constructions" are already there, a truth whose existence is completely independent of their workings out? Penrose is inclined toward the second point of view, even though he adds that the problem is not as simple as it seems. His opinion is that in mathematics one determines situations for which the term discovery is certainly more appropriate than the term invention. There are cases in which the results essentially derive from the structure itself, more than from the input of mathematicians. Penrose cites the example of complex numbers: "Later we find many other magical properties that these complex numbers possess, properties that we had no inkling about at first. These properties are just there. They were not put there by Cardano, nor by Bombelli, nor Wallis, nor Coates, nor Euler, nor Wessel, nor Gauss, despite the undoubted farsightedness of these, and other, great mathematicians; such magic was inherent in the very structure that they gradually uncovered."

When mathematicians discover a structure of this kind, it means that they stumbled upon that which Penrose calls "works of God". So, are mathematicians mere explorers? Fortunately not all mathematical structures are so strictly predetermined. There are cases in which "the results are obtained equally by merit of the structure and of the mathematicians' calculations"; in this case, Penrose says, it is more appropriate to use the word invention than the word discovery. Hence there is room for what he calls works of man, though he notes that the discoveries of structures that are works of God are of vastly greater importance than the 'mere' inventions that are the works of man.

One can make analogous distinctions in the arts and in engineering. "Great works of art are indeed 'closer to God' than are lesser ones."

Among artists, Penrose claims, the idea that their most important works reveal eternal truths is not uncommon, that they have "some kind of prior ethereal existence", while the lesser important works have a more personal character, and are arbitrary, mortal constructions. In mathematics, this need to believe in an immaterial and eternal existence, at least of the most profound mathematical concepts (the works of God), is felt even more strongly. Penrose observes that "there is a compelling uniqueness and universality in such mathematical ideas which seems to be of quite a different order from that which one could expect in the arts or or engineering." A work of art can be appreciated or brought into question in different epochs, but no-one can put in doubt a correct proof of a mathematical result.

Penrose explains very explicitly that mathematicians think of their discipline as a highly creative activity that has nothing to envy of the creativity of artists and indeed, because of the uniqueness and universality of mathematical creation, one that should be regarded as superior to the artistic discipline; Penrose doesn't write it explicitly, but many mathematicians think that mathematics is the true Art; a difficult, laborious art, with it's very own language and symbolism, that produces universally accepted results.

Penrose takes up the question of the existence in its own right of the world of mathematical ideas, a question that has dogged the science of mathematics since its inception. It comes from the book *Matière à pensée*, co-authored by a mathematician, Alain Connes, winner of the Fields medal, and a neurobiologist, Jean-Pierre Changeux [2].

In one of the chapters in the book, entitled *Invention ou découverte* the neurobiologist, speaking of the nature of mathematical objects, recalls that there is a realist attitude directly inspired by Plato, an attitude that can be summarised in the phrase: the world is populated by ideas that have a reality separate from physical reality. Supporting his observation, Changeux quotes a claim of Dieudonné, according to whom "mathematicians accept that mathematical objects possess a reality distinct from physical reality", a reality that can be compared to that which Plato accords to his Ideas. From this point of view it is of secondary importance whether or not the mathematical world is a divine creation, as the mathematician Cantor (1874–1918) believed: "The highest perfection of God is the possibility of creating an infinite set, and his immense bounty leads him to do so."

We are in complete divine mathesis, in complete metaphysics. This is surprising to serious scientists, comments Changeux. Connes, the mathematician, not at all bothered by the biologist's arguments, replies very clearly that he identifies strongly with the realist point of view. After emphasising that the sequence of prime numbers has a more stable reality than the material reality surrounding us, he notes that "one can compare the work of a mathematician to that of an explorer discovering the world." If practical experience leads us to discover pure and simple facts such as, for example, that the sequence of prime numbers appear to have no end, the work of a mathematician consists of proving that there exists an infinite number of primes. Once this property has been proved, no-one can ever claim to having found the largest prime of all. It would be easy to show them that they are wrong. Connes concludes "so we clash with a reality just as incontestable as the physical one". Indeed, it is arguably more real than any physical reality.

Still in the aforementioned book, we read that mathematics is a universal language, but that is not all, for the generative nature of mathematics has a crucial role. Otherwise one could not explain the incredible and unforeseeable usefulness of mathematics, from meteorology to the structure of DNA, to the simulation of boats for the America's Cup.

Imagination is not enough to do mathematics; you need knowledge of the language and techniques; imagination, creativity, the ability to make problems explicit, to formalise and solve them, whenever possible.

In recent years, thanks to the advent of computer graphics, the role of visualisation in some sectors of mathematics has increased significantly. It was inevitable that this sort of new *Visual Mathematics* would open new ways for links between creativity in the arts and in mathematics. We wish to deal with some of these aspects in this new book of the series called significantly "Mathematics and Culture", which was born of an idea of Valeria Marchiafava and Michele Emmer in September 1997, in Turin.

References

[1] Penrose, R.: *The Emperor's New Mind.* Oxford University Press, New York (1989).
[2] Changeux, Je.-P., Connes, A.: *Matière à Penseée.* ed. Odile Jacob, Paris (1989).

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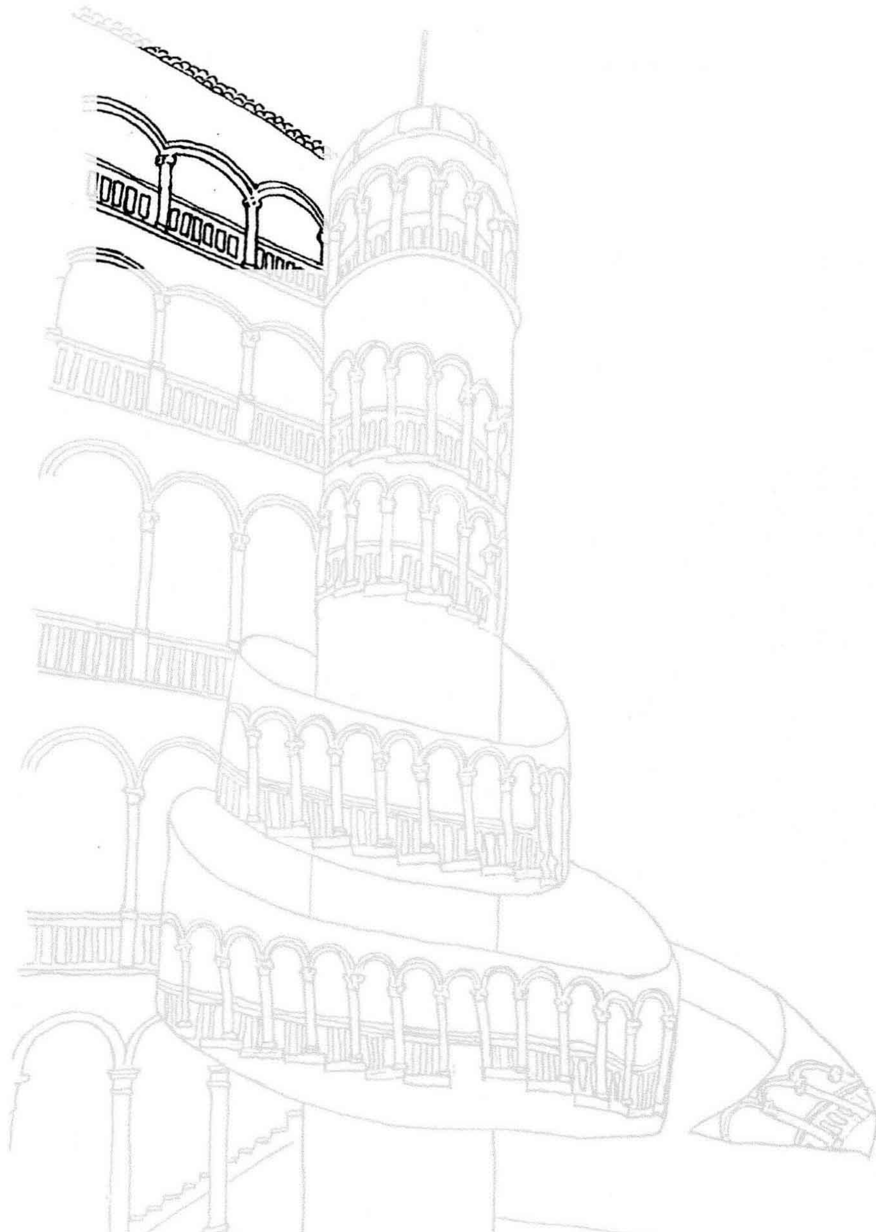

Mathematics, Art and Architecture

MICHELE EMMER

The care that mathematicians have for the aesthetic quality of their discipline is noteworthy: this gives rise to many mathematicians' idea that mathematical activity and artistic activity are, in some sense, very similar and comparable.

Morris Kline, a mathematician, (and he is but one of many examples that we could cite) has dedicated several pages of his book *Mathematics In Western Culture* [1] to this subject. After recalling that mathematicians have for hundreds of years accepted what the Greeks maintained, that is, the fact that mathematics is an art and that mathematical work must satisfy aesthetic requirements, Kline asks the fundamental question of why many people maintain that the inclusion of mathematics in the arts is unjustified.

One of the most common objections is that mathematics does not evoke any emotion. Kline, on the other hand, observes that mathematics provokes undeniable feelings of aversion and reaction, and moreover generates great joy in the researchers when they manage to make a precise formulation of their ideas and on obtaining clever and inspired proofs. The problem lies in the fact that it is only researchers who can experience these emotions and no-one else. "Just like in the arts, each particular of the final work is not discovered, but composed. Of course the creative process must produce a work that has design, harmony and beauty. These qualities too are present in mathematical creations."

Commenting on these words of Kline, I have observed that [2] "if it is not interesting, in this realm, to discuss mathematicians' ideas on art, it is however worthwhile highlighting how this artistic aspiration is widespread in the mathematical community. Complementary to this requirement is the need for recognition of the artistic creativity of the mathematician by the mathematical laity; a recognition that is not generally given, in particular by those who take a professional interest in art. Especially since this would entail having to understand something of contemporary mathematics. Everyone can look at a work of art, listen to a symphony, but one cannot look at or listen to mathematics. Kline clearly recognises this when he says that the definitive test of a work of art is it's contribution to aesthetic pleasure or to beauty.

Fortunately, or unfortunately, the test in question is subjective, one that depends on the level of culture in a specific sector. The question as to whether or not mathematics possesses its own kind of beauty can thus be given a reply only by those who have a culture in this discipline ... Unfortunately, to master mathematical ideas requires study and there exists no direct route that substantially accomplishes this ".

References

[1] Kline, M.: *Mathematics in Western Culture,* Oxford University Press, New York, 1953
[2] Emmer, M.: *La perfezione visibile: arte matematica.* Theoria, Roma (1991)

From Tiling the Plane to Paving Town Square

Judith Flagg Moran, Kim Williams

Introduction

Every culture which has left us its artifacts has in so doing left evidence for the universal human fascination with patterns, and humans' propensity, if not compulsion, to ornament their environment, objects, and persons. [1] At the beginning of the current century, neuroscientists and cognitive neuropsychologists such as Stanislas Dehaene and Brian Butterworth are using formidable imaging techniques to actually watch the brain process numbers. But even a PET scan of a brain perceiving the pattern of a Peruvian blanket does not address the question of why this activity should be pleasurable or why the original weaver felt impelled to create the pattern in the first place. Writers on the psychology of art such as E. H. Gombrich and Rudolph Arnheim have addressed these questions by linking the perception of pattern to cognitive development through the perception of structure and so to human evolutionary success. In *New Essays on the Psychology of Art,* Arnheim states,

> *Perception must look for structure. In fact, perception is the discovery of structure. Structure tells us what the components of things are and by what sort of order they interact.* [2]

In A Sense of Order, Gombrich states his belief that the human sense of order is "rooted in man's biological heritage" and links pattern perception more directly to survival:

> *I believe that in the struggle for existence organisms developed a sense of order not because their environment was generally orderly but rather because perception requires a framework against which to plot deviations from regularity.* [3]

More recently, John Barrow in *The Artful Universe* echoes Gombrich in linking recognition of order in the environment with survival skills: recognition of order is beneficial for survival; survival requires being able to pick out the shape of the tiger against the pattern of the leaves in the jungle. At some point, however, the ability to recognize order in the environment, an adaptive skill, became an end in itself, that is, the recognition of order became pleasureable: aesthetics was born.

While accepting Barrow's basic idea that "Perhaps the most basic of all (human responses honed by natural selection) is an ability to sense and classify pattern" [4], we find two aspects of his argument particularly intriguing. First, that cognition as a biological trait is influenced over time, that is, it is "subject to evolution".[5] Second, that as our ability to perceive order has evolved, so has our sense of what constitutes order and pattern. This has been especially true in the latter part of the twentieth century, when we have come to accept new configurations as orderly because we have learned and accepted new criteria, in other words, we have extended and developed our "sense of order". Two of the most well-known of these new families of patterns are the fractals and the non-periodic patterns such as Penrose tilings and their descendants and relations. In part, generation and recognition of these patterns has been made possible through the revolution in information technology. Whereas previously, traditional patterns were generated by humans using the mechanism of repeated translation, the technique of rapid iteration, made feasible by the development of the computer, produces the resultant striking images of fractals and nonperiodic tilings in seconds. The instantaneous nature of modern communication technology disseminates the new images rapidly. And in the case of these two families of patterns, their use in modeling has ensured they were brought to the attention of a significant part of the general public. (Fractals can serve as the basis for the creation of natural features in computer graphics; Penrose tilings, discovered at the end of the 1970's, were used in the 1980's as a visual model for quasicrystals [6], the "new form of matter", the discovery of which was splashed across the pages of the *New York Times* and other media.) Once such patterns enter the domain of the general public, they become available to artists, who in turn bring them into the realm of aesthetics and make them part of their culture's pattern vocabulary. The role of artists in the integration of a sense of order into our environment is fundamental. One of the most pattern-obsessed of contemporary artists, M.C. Escher said in 1965:

> *Although I am even now still a layman in the area of mathematics, and although I lack theoretical knowledge, the mathematicians, and in particular the crystallographers, have had considerable influence on my work ... The laws of the phenomena around us – order, regularity, cyclical repetitions, and renewals – have assumed greater and greater importance for me. The awareness of their presence gives me peace and provides me with support. I try in my prints to testify that we live in a beautiful and orderly world, and not in a formless chaos, as it sometimes seems.*[7]

It is intriquing to imagine the use Escher (who died in 1972) might have made of the scaffolding for his work afforded by fractal and non-periodic patterns.

In this paper we have chosen to look at order through the vehicle of decorated pavements, that is, the application of geometric patterns to floors in architectural monuments and urban spaces. Decoration of buildings is common to all cultures and all time periods; the choice of paving designs often reflects cultural values about order. For instance, during the Renaissance, Leon Battista Alberti wrote, *And I would have the Composition of the Lines of the Pavement full of Musical and geometrical Proportions; to the Intent that which-soever Way we turn our Eyes, we may be sure to find Employment for our Minds.* [8]

Alberti makes clear the Renaissance preference for the so-called harmonic proportions that were a hallmark of the architecture of that age. But it is significant that Alberti singles out the pavement for this prescription. The pavement often serves as a sort of canvas for the representation of ideas inherent in the architecture. After all, the floor is often the largest unbroken surface in a building. Paving patterns are not merely orderly in themselves, but contribute significantly to a larger sense of order in the built environment. They do this by indicating the organization that governs the building or space being decorated. Paving designs can indicate a hierarchy of spaces inside a building, they can indicate direction of movement through these spaces, and they can indicate a rhythm and speed of this movement. The present paper of course in no way serves as a survey of pavement design. Our intent is to contrast some pavement designs executed near the beginning of the second millennium with the pavements of two designers working at its end to illustrate how our sense of order and pattern has and continues to evolve.

The Pavements of the Cosmati

In spite of the wear and tear of almost a thousand years, Cosmatesque pavements still overwhelm the senses with their vibrancy, richness and variety – colorful carpets of marble that contrast with the austere simplicity of the Romanesque architecture that they adorn (Figure 1).[9]

The term 'Cosmatesque' refers to a particular style of polychrome decoration created through the use of tesserae or small tiles of marble, granite or ceramic to form geometric patterns. This kind of decoration takes its name from that of members of several families of artisans who created this type of ornament in the twelfth and thirteenth centuries in Italy. Cosmatesque pavements function as space organizers and direction indicators within their spaces. Romanesque churches are based on the basilican plan, that is, a longitudinal nave flanked by side aisles connects the entrance at one end of the church through the choir space to the altar placed before a round apse at the other end. Cosmatesque pavements are always composed of a linear element that marches up the nave and through the choir to arrive at the altar. This linear element may be composed of one or a combination of the two leitmotives of the Cosmati: the guilloche (Figure 2), a series of disks or roundels connecting by interweaving borders, and the quincunx, an arrangements of four roundels around a fifth, also connected by interweaving borders (Figure 3). Much of the rest of the floor area is subdivided into a grid of rectangles, each of which is filled with a geometric pattern that has a repeat in two directions, like wallpaper (Figure 4). (In fact, such patterns are commonly called wallpaper patterns, even by mathematicians.)

The pavement arrangement functions at two different levels. The linear pattern defines the space of the nave both as an architectural element, a corridor, and

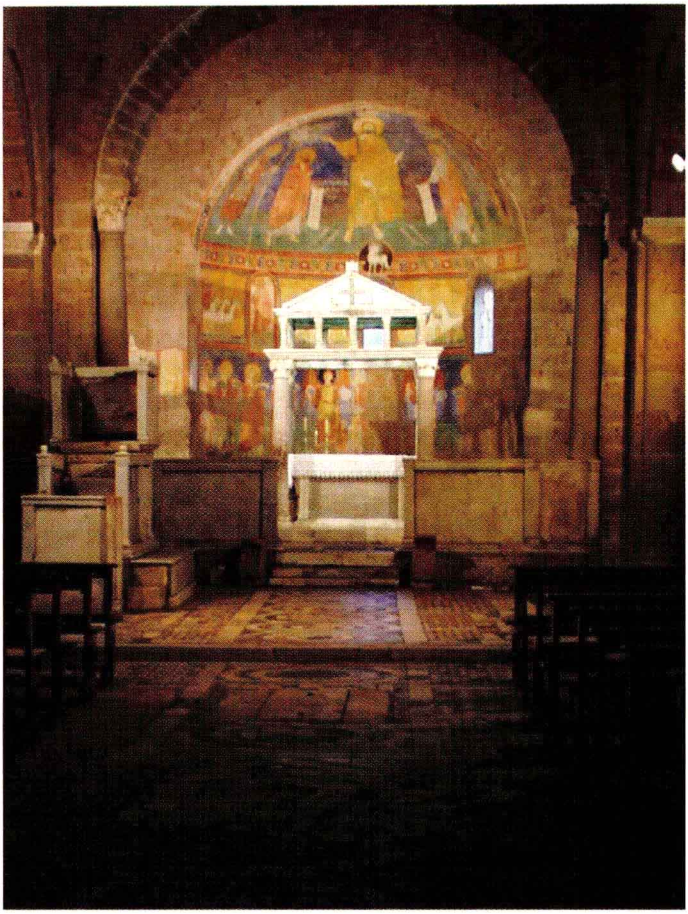

Fig. 1.
The Pavements of the Cosmati

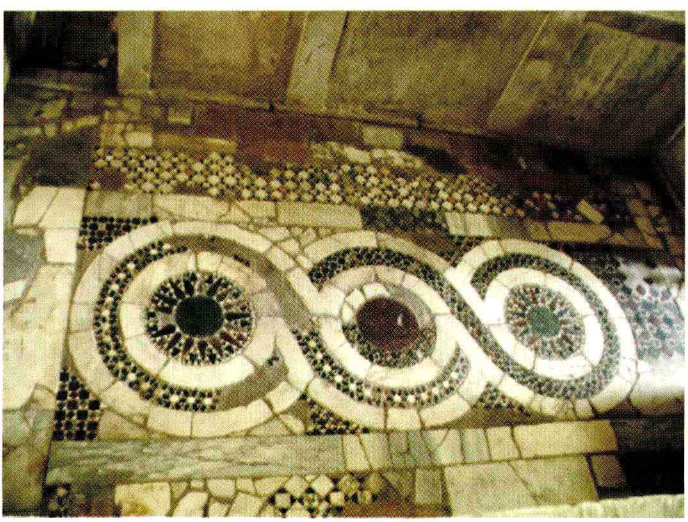

Fig. 2.
Cosmati:
The guilloche