Elementary *and* Intermediate Algebra

Third Edition

Mark Dugopolski

For DeAnza College

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Elementary and Intermediate Algebra

Third Edition

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DEFINITIONS, RULES, AND FORMULAS

Subsets of the Real Numbers

Natural Numbers = $\{1, 2, 3, \ldots\}$

Whole Numbers = $\{0, 1, 2, 3, \ldots\}$

Integers = $\{\ldots -3, -2, -1, 0, 1, 2, 3, \ldots\}$

Rational = $\left\{\frac{a}{b} \middle| a \text{ and } b \text{ are integers with } b \neq 0\right\}$

Irrational = $\{x \mid x \text{ is not rational}\}$

Properties of the Real Numbers

For all real numbers a, b, and c

$a + b = b + a$; $a \cdot b = b \cdot a$ Commutative

$(a + b) + c = a + (b + c)$; $(ab)c = a(bc)$ Associative

$a(b + c) = ab + ac$; $a(b - c) = ab - ac$ Distributive

$a + 0 = a$; $1 \cdot a = a$ Identity

$a + (-a) = 0$; $a \cdot \frac{1}{a} = 1 \ (a \neq 0)$ Inverse

$a \cdot 0 = 0$ Multiplication property of 0

Absolute Value

$$|a| = \begin{cases} a & \text{for } a \geq 0 \\ -a & \text{for } a < 0 \end{cases}$$

$\sqrt{x^2} = |x|$ for any real number x.

$|x| = k \leftrightarrow x = k \text{ or } x = -k \quad (k > 0)$

$|x| < k \leftrightarrow -k < x < k \quad (k > 0)$

$|x| > k \leftrightarrow x < -k \text{ or } x > k \quad (k > 0)$

(The symbol $\leftrightarrow$ means "if and only if.")

Interval Notation

$(a, b) = \{x | a < x < b\}$ $\qquad [a, b] = \{x | a \leq x \leq b\}$

$(a, b] = \{x | a < x \leq b\}$ $\qquad [a, b) = \{x | a \leq x < b\}$

$(-\infty, a) = \{x | x < a\}$ $\qquad (a, \infty) = \{x | x > a\}$

$(-\infty, a] = \{x | x \leq a\}$ $\qquad [a, \infty) = \{x | x \geq a\}$

Exponents

$a^0 = 1$ $\qquad a^{-1} = \frac{1}{a}$

$a^{-r} = \frac{1}{a^r} = \left(\frac{1}{a}\right)^r$ $\qquad \frac{1}{a^{-r}} = a^r$

$a^r a^s = a^{r+s}$ $\qquad \frac{a^r}{a^s} = a^{r-s}$

$(a^r)^s = a^{rs}$ $\qquad (ab)^r = a^r b^r$

$\left(\frac{a}{b}\right)^r = \frac{a^r}{b^r}$ $\qquad \left(\frac{a}{b}\right)^{-r} = \left(\frac{b}{a}\right)^r$

Roots and Radicals

$a^{1/n} = \sqrt[n]{a}$ $\qquad a^{m/n} = \left(\sqrt[n]{a}\right)^m = \sqrt[n]{a^m}$

$\sqrt[n]{ab} = \sqrt[n]{a} \cdot \sqrt[n]{b}$ $\qquad \sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}$

Factoring

$a^2 + 2ab + b^2 = (a + b)^2$

$a^2 - 2ab + b^2 = (a - b)^2$

$a^2 - b^2 = (a + b)(a - b)$

$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$

$a^3 + b^3 = (a + b)(a^2 - ab + b^2)$

Rational Expressions

$\frac{a}{b} + \frac{c}{b} = \frac{a + c}{b}$ $\qquad \frac{a}{b} - \frac{c}{b} = \frac{a - c}{b}$

$\frac{ac}{bc} = \frac{a}{b}$ $\qquad \frac{a}{b} + \frac{c}{d} = \frac{ad + bc}{bd}$

$\frac{a}{b} \cdot \frac{c}{d} = \frac{ac}{bd}$ $\qquad \frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \cdot \frac{d}{c}$

If $\frac{a}{b} = \frac{c}{d}$, then $ad = bc$.

Quadratic Formula

The solutions to $ax^2 + bx + c = 0$ with $a \neq 0$ are

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

Distance Formula

The distance from (x_1, y_1) to (x_2, y_2), is

$$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}.$$

Midpoint Formula

The midpoint of the line segment with endpoints (x_1, y_1) and (x_2, y_2) is

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right).$$

Slope Formula

The slope of the line through (x_1, y_1) and (x_2, y_2) is

$$\frac{y_2 - y_1}{x_2 - x_1} \quad (\text{for } x_1 \neq x_2).$$

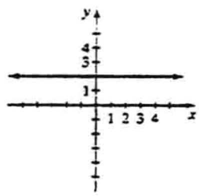
Linear Function

$f(x) = mx + b$ with $m \neq 0$

Graph is a line with slope m.

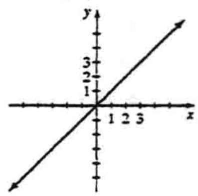
A constant function

$f(x) = 2$



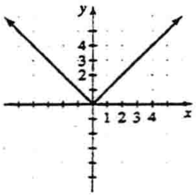
The identity function

$f(x) = x$



Absolute Value Function

$f(x) = |x|$

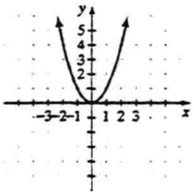


Quadratic Function

$f(x) = ax^2 + bx + c$ with $a \neq 0$

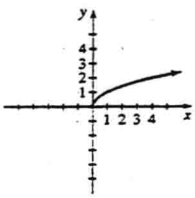
Graph is a parabola.

$f(x) = x^2$ (the squaring function)



Square-Root Function

$f(x) = \sqrt{x}$

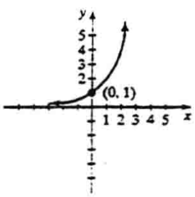


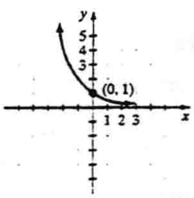
Exponential Function

$f(x) = a^x$ for $a > 0$ and $a \neq 1$

One-to-one property: $a^m = a^n \leftrightarrow m = n$

$f(x) = a^x$ for $a > 1$ $\qquad$ $f(x) = a^x$ for $0 < a < 1$





Logarithmic Function

$f(x) = \log_a(x)$ for $a > 0$ and $a \neq 1$

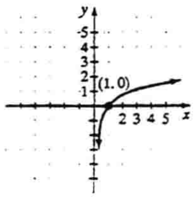
One-to-one property: $\log_a(m) = \log_a(n) \leftrightarrow m = n$

Base-a logarithm: $y = \log_a(x) \leftrightarrow a^y = x$

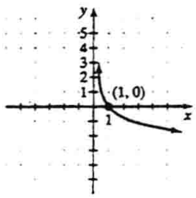
Natural logarithm: $y = \ln(x) \leftrightarrow e^y = x$

Common logarithm: $y = \log(x) \leftrightarrow 10^y = x$

$f(x) = \log_a(x)$
for $a > 1$



$f(x) = \log_a(x)$
for $0 < a < 1$



Properties of Logarithms

$\log_a(a) = 1$ $\qquad$ $\log_a(1) = 0$

$\log_a(a^M) = M$ $\qquad$ $a^{\log_a(M)} = M$

$\log_a(MN) = \log_a(M) + \log_a(N)$

$\log_a\left(\frac{M}{N}\right) = \log_a(M) - \log_a(N)$

$\log_a(M^N) = N \cdot \log_a(M)$

$\log_a\left(\frac{1}{N}\right) = -\log_a(N)$

$\log_a(M) = \frac{\log_b(M)}{\log_b(a)} = \frac{\ln(M)}{\ln(a)} = \frac{\log(M)}{\log(a)}$

Interest Formulas

A = amount and P = principal

Compound interest: $A = P(1 + i)^n$, where n = number of periods and i = interest rate per period

Continuous compounding: $A = Pe^{rt}$, where r = annual interest rate and t = time in years

Variation

Direct: $y = kx \quad (k \neq 0)$

Inverse: $y = \frac{k}{x} \quad (k \neq 0)$

Joint: $y = kxz \quad (k \neq 0)$

Straight Line

Slope-intercept form: $y = mx + b$

Slope: m $\qquad$ y-intercept: $(0, b)$

Point-slope form: $y - y_1 = m(x - x_1)$

Slope: m $\qquad$ Point: (x_1, y_1)

Standard form: $Ax + By = C$

Horizontal: $y = k$ $\qquad$ Vertical: $x = k$

Parabola

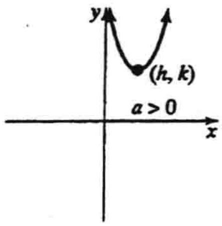
$y = a(x-h)^2 + k \quad (a \neq 0)$

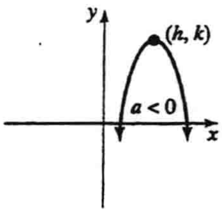
Vertex: (h, k)

Axis of symmetry: $x = h$

Focus: $(h, k+p)$, where $a = \dfrac{1}{4p}$

Directrix: $y = k + p$





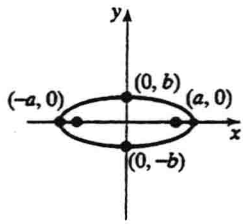
Ellipse

$\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1 \quad (a > b > 0)$

Center: $(0, 0)$

Foci: $(\pm c, 0)$,

where $c^2 = a^2 - b^2$

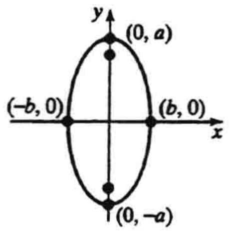


$\dfrac{x^2}{b^2} + \dfrac{y^2}{a^2} = 1 \quad (a > b > 0)$

Center: $(0, 0)$

Foci: $(0, \pm c)$,

where $c^2 = a^2 - b^2$



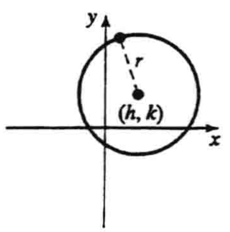
Circle

$(x-h)^2 + (y-k)^2 = r^2 \quad (r > 0)$

Center: (h, k) Radius: r

$x^2 + y^2 = r^2$

Center: $(0, 0)$ Radius: r



Hyperbola

$\dfrac{x^2}{a^2} - \dfrac{y^2}{b^2} = 1$

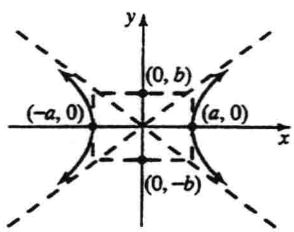
Center: $(0, 0)$

x-intercepts: $(\pm a, 0)$

Foci: $(\pm c, 0)$,

where $c^2 = a^2 + b^2$

Asymptotes: $y = \pm\dfrac{b}{a}x$



$\dfrac{y^2}{a^2} - \dfrac{x^2}{b^2} = 1$

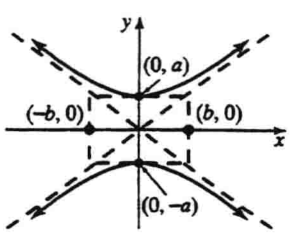
Center: $(0, 0)$

y-intercepts: $(0, \pm a)$

Foci: $(0, \pm c)$,

where $c^2 = a^2 + b^2$

Asymptotes: $y = \pm\dfrac{a}{b}x$



Arithmetic Sequence

$a_1, a_1 + d, a_1 + 2d, a_1 + 3d, \ldots$

Formula for nth term: $a_n = a_1 + (n-1)d$

Sum of n terms: $S_n = \dfrac{n}{2}(a_1 + a_n)$

Geometric Sequence

$a_1, a_1r, a_1r^2, a_1r^3, \ldots$

Formula for nth term: $a_n = a_1r^{n-1}$

Sum of n terms $(r \neq 1)$: $S_n = \dfrac{a_1 - a_1r^n}{1-r}$

Sum of all terms $(|r| < 1)$: $S = \dfrac{a_1}{1-r}$

Binomial Expansion

$(a+b)^2 = a^2 + 2ab + b^2$

$(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$

$(a+b)^4 = a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + b^4$

$(a+b)^n = \sum_{i=0}^{n}\binom{n}{i}a^{n-i}b^i$, where $\binom{n}{i} = \dfrac{n!}{(n-i)!i!}$

Factorial notation: $n! = 1 \cdot 2 \cdot 3 \cdot \cdots \cdot n$

Metric Abbreviations

Length	Volume	Weight
mm millimeter	mL milliliter	mg milligram
cm centimeter	cL centiliter	cg centigram
dm decimeter	dL deciliter	dg decigram
m meter	L liter	g gram
dam dekameter	daL dekaliter	dag dekagram
hm hectometer	hL hectoliter	hg hectogram
km kilometer	kL kiloliter	kg kilogram

English-Metric Conversion

Length	Volume (U.S.)	Weight
1 in. = 2.540 cm	1 pt = 0.4732 L	1 oz = 28.35 g
1 ft = 30.48 cm	1 qt = 0.9464 L	1 lb = 453.6 g
1 yd = 0.9144 m	1 gal = 3.785 L	1 lb = 0.4536 kg
1 mi = 1.609 km		

Length	Volume (U.S.)	Weight
1 cm = 0.3937 in.	1 L = 2.2233 pt	1 g = 0.0353 oz
1 cm = 0.03281 ft	1 L = 1.0567 qt	1 g = 0.002205 lb
1 m = 1.0936 yd	1 L = 0.2642 gal	1 kg = 2.205 lb
1 km = 0.6215 mi		

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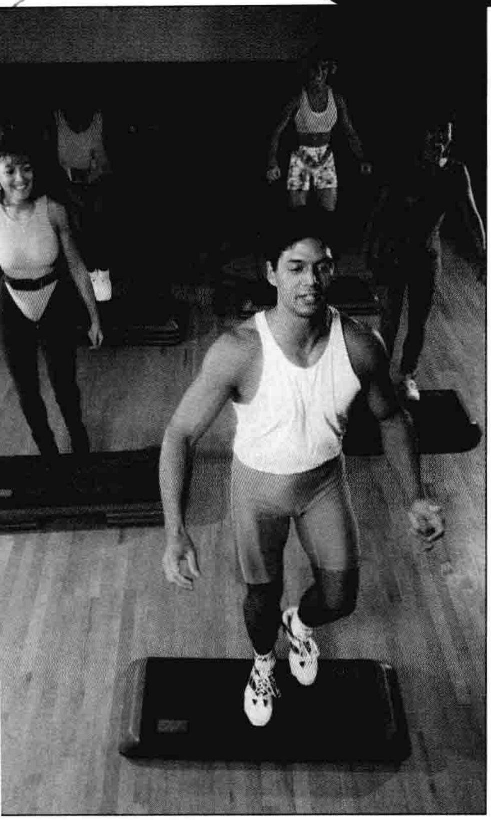
CHAPTER 1

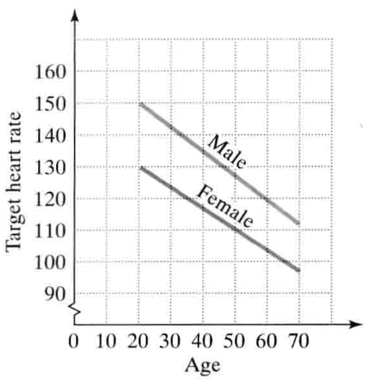
The Real Numbers



Everywhere you look people are running, riding, dancing, and exercising their way to fitness. In the past year more than $25 billion has been spent on sports equipment alone, and this amount is growing steadily.

Proponents of exercise claim that it can increase longevity, improve body image, decrease appetite, and generally enhance a person's health. While many sports activities can help you to stay fit, experts have found that aerobic, or dynamic, workouts provide the most fitness benefit. Some of the best aerobic exercises include cycling, running, and even jumping rope. Whatever athletic activity you choose, trainers recommend that you set realistic goals and work your way toward them consistently and slowly. To achieve maximum health benefits, experts suggest that you exercise three to five times a week for 15 to 60 minutes at a time.

There are many different ways to measure exercise. One is to measure the energy used, or the rate of oxygen consumption. Since heart rate rises as a function of increased oxygen, another easier measure of intensity of exercise is your heart rate during exercise. The desired heart rate, or target heart rate, for beneficial exercise varies for each individual depending on conditioning, age, and gender. In Exercises 101 and 102 of Section 1.4 you will see how an algebraic expression can determine your target heart rate for beneficial exercise.



1.1 SETS

In this section

- Set Notation
- Union of Sets
- Intersection of Sets
- Subsets
- Combining Three or More Sets

Every subject has its own terminology, and **algebra** is no different. In this section we will learn the basic terms and facts about sets.

Set Notation

A **set** is a collection of objects. At home you may have a set of dishes and a set of steak knives. In algebra we generally discuss sets of numbers. For example, we refer to the numbers 1, 2, 3, 4, 5, and so on as the set of **counting numbers** or **natural numbers.** Of course, these are the numbers that we use for counting.

The objects or numbers in a set are called the **elements** or **members** of the set. To describe sets with a convenient notation, we use braces, { }, and name the sets with capital letters. For example,

$$A = \{1, 2, 3\}$$

means that set A is the set whose members are the natural numbers 1, 2, and 3. The letter N is used to represent the entire set of natural numbers.

A set that has a fixed number of elements such as {1, 2, 3} is a **finite** set, whereas a set without a fixed number of elements such as the natural numbers is an **infinite** set. When listing the elements of a set, we use a series of three dots to indicate a continuing pattern. For example, the set of natural numbers is written as

$$N = \{1, 2, 3, \ldots\}.$$

The set of natural numbers *between* 4 and 40 can be written

$$\{5, 6, 7, 8, \ldots, 39\}.$$

Note that since the members of this set are *between* 4 and 40, it does not include 4 or 40.

Set-builder notation is another method of describing sets. In this notation we use a variable to represent the numbers in the set. A **variable** is a letter that is used to stand for some numbers. The set is then built from the variable and a description of the numbers that the variable represents. For example, the set

$$B = \{1, 2, 3, \ldots, 49\}$$

is written in set-builder notation as

$$B = \{x \mid x \text{ is a natural number less than } 50\}.$$

↑ The set of numbers ↑ such that ↑ condition for membership

This notation is read as "B is the set of numbers x such that x is a natural number less than 50." Notice that the number 50 is not a member of set B.

The symbol $\in$ is used to indicate that a specific number is a member of a set, and $\notin$ indicates that a specific number is not a member of a set. For example, the statement $1 \in B$ is read as "1 is a member of B," "1 belongs to B," "1 is in B," or "1 is an element of B." The statement $0 \notin B$ is read as "0 is not a member of B," "0 does not belong to B," "0 is not in B," or "0 is not an element of B."

Two sets are **equal** if they contain exactly the same members. Otherwise, they are said to be not equal. To indicate equal sets, we use the symbol $=$. For sets that are not equal we use the symbol $\neq$. The elements in two equal sets do not need to be written in the same order. For example, $\{3, 4, 7\} = \{3, 4, 7\}$ and $\{2, 4, 1\} = \{1, 2, 4\}$, but $\{3, 5, 6\} \neq \{3, 5, 7\}$.

study tip

Find a group of students to work with outside of class. Don't just settle for answers. Make sure that everyone in the group understands the solution to a problem. You really will understand a concept when you can explain it to someone else.

EXAMPLE 1

Set notation

Let $A = \{1, 2, 3, 5\}$ and $B = \{x \mid x \text{ is an even natural number less than } 10\}$. Determine whether each statement is true or false.

a) $3 \in A$ **b)** $5 \in B$ **c)** $4 \notin A$ **d)** $A = N$

e) $A = \{x \mid x \text{ is a natural number less than } 6\}$ **f)** $B = \{2, 4, 6, 8\}$

Solution

a) True, because 3 is a member of set A.

b) False, because 5 is not an even natural number.

c) True, because 4 is not a member of set A.

d) False, because A does not contain all of the natural numbers.

e) False, because 4 is a natural number less than 6, and $4 \notin A$.

f) True, because the even counting numbers less than 10 are 2, 4, 6, and 8. ■

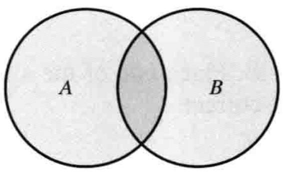
Union of Sets

Any two sets A and B can be combined to form a new set called their union that consists of all elements of A together with all elements of B.

Union of Sets

If A and B are sets, the **union** of A and B, denoted $A \cup B$, is the set of all elements that are either in A, in B, or in both. In symbols,

$$A \cup B = \{x \mid x \in A \text{ or } x \in B\}.$$



$A \cup B$

FIGURE 1.1

In mathematics the word "or" is always used in an inclusive manner (allowing the possibility of both alternatives). The diagram in Fig. 1.1 can be used to illustrate $A \cup B$. Any point that lies within circle A, circle B, or both is in $A \cup B$. Diagrams (like Fig. 1.1) that are used to illustrate sets are called **Venn diagrams.**

EXAMPLE 2

Union of sets

Let $A = \{0, 2, 3\}$, $B = \{2, 3, 7\}$, and $C = \{7, 8\}$. List the elements in each of the following sets.

a) $A \cup B$ **b)** $A \cup C$

helpful hint

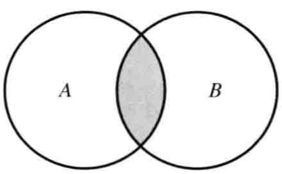
To remember what "union" means think of a labor union, which is a group formed by joining together many individuals.

Solution

a) $A \cup B$ is the set of numbers that are in A, in B, or in both A and B.

$$A \cup B = \{0, 2, 3, 7\}$$

b) $A \cup C = \{0, 2, 3, 7, 8\}$ ■



$A \cap B$

FIGURE 1.2

Intersection of Sets

Another way to form a new set from two known sets is by considering only those elements that the two sets have in common. The diagram shown in Fig. 1.2 illustrates the intersection of two sets A and B.

helpful hint

To remember the meaning of "intersection," think of the intersection of two roads. At the intersection you are on both roads.

Intersection of Sets

If A and B are sets, the **intersection** of A and B, denoted $A \cap B$, is the set of all elements that are in both A and B. In symbols,

$$A \cap B = \{x \mid x \in A \text{ and } x \in B\}.$$

It is possible for two sets to have no elements in common. A set with no members is called the **empty set** and is denoted by the symbol $\varnothing$. Note that $A \cup \varnothing = A$ and $A \cap \varnothing = \varnothing$ for any set A.

CAUTION The set $\{0\}$ is not the empty set. The set $\{0\}$ has one member, the number 0. Do not use the number 0 to represent the empty set.

EXAMPLE 3 **Intersection of sets**

Let $A = \{0, 2, 3\}$, $B = \{2, 3, 7\}$, and $C = \{7, 8\}$. List the elements in each of the following sets.

a) $A \cap B$ **b)** $B \cap C$ **c)** $A \cap C$

Solution

a) $A \cap B$ is the set of all numbers that are in both A and B. So $A \cap B = \{2, 3\}$.

b) $B \cap C = \{7\}$ **c)** $A \cap C = \varnothing$ ■

EXAMPLE 4 **Membership and equality**

Let $A = \{1, 2, 3, 5\}$, $B = \{2, 3, 7, 8\}$, and $C = \{6, 7, 8, 9\}$. Place one of the symbols $=$, $\neq$, $\in$, or $\notin$ in the blank to make each statement correct.

a) 5 _____ $A \cup B$ **b)** 5 _____ $A \cap B$

c) $A \cup B$ _____ $\{1, 2, 3, 5, 7, 8\}$ **d)** $A \cap B$ _____ $\{2\}$

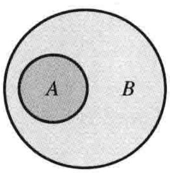
Solution

a) $5 \in A \cup B$ because 5 is a member of A.

b) $5 \notin A \cap B$ because 5 must belong to *both* A and B to be a member of $A \cap B$.

c) $A \cup B = \{1, 2, 3, 5, 7, 8\}$ because the elements of A together with those of B are listed. Note that 2 and 3 are members of both sets but are listed only once.

d) $A \cap B \neq \{2\}$ because $A \cap B = \{2, 3\}$. ■



$A \subseteq B$

FIGURE 1.3

Subsets

If every member of set A is also a member of set B, then we write $A \subseteq B$ and say that A is a **subset** of B. See Fig. 1.3. For example,

$$\{2, 3\} \subseteq \{2, 3, 4\}.$$

If set A is not a subset of B, we write $A \not\subseteq B$.

CAUTION To claim that $A \not\subseteq B$, there *must* be an element of A that does *not* belong to B. For example,

$$\{1, 2\} \not\subseteq \{2, 3, 4\}$$

because 1 is a member of the first set but not of the second.