

Elementary *and*  
Intermediate  
**Algebra**  
Third Edition

Mark Dugopolski

*For DeAnza College*

 College  
Custom  
**Series**

# Elementary and Intermediate Algebra

Third Edition

Mark Dugopolski  
*Southeastern Louisiana University*

For De Anza College



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**Elementary and Intermediate Algebra**

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# INDEX OF SELECTED APPLICATIONS

## Biology/Health/Life Sciences

Basic energy requirement, 386  
Building fitness, 163  
Cigarette usage, 72  
Female target heart rate, 30  
Hazardous to her health, 103  
Heart rate, 193  
Male target heart rate, 30  
Protein and carbohydrates, 235  
Staying fit, 170  
Waist-to-hip ratio, 163, 193

## Business

Advertising dollars, 163  
Allocating resources, 163, 249  
Annual bonus, 282  
Average cost, 295  
Average profit, 295  
Bonus and taxes, 208  
Budget planning, 162  
Choosing a selling price, 82  
Corporate taxes, 208  
Cost, 120, 129, 170, 208  
Depreciation, 171  
Dividing the estate, 83  
Earned income, 170  
Economic impact, 633  
Fortune 500 profits, 83  
Free market, 209  
Going bananas, 626  
Increasing revenue, 490  
Las Vegas vacation, 378  
Less impact, 633, 651  
Making circuit boards, 150  
Marginal cost, 282  
Marginal profit, 282  
Maritime losses, 371  
Maximum profit, 591  
Maximum revenue, 590  
Negative income tax, 332  
Net worth of a bank, 22  
Phase I advertising, 248  
Processing, 357  
Profit function, 129, 482, 498  
Printing costs, 171  
Recovering an investment, 464  
Rental costs, 129, 193  
Rose Bowl bound, 342  
Selling, 358  
Shipping restrictions, 163  
Textbook case, 209  
Ticket prices, 490  
Total economic impact, 637, 651

Total spending, 637  
Toy sales, 83  
Wedding bells, 342  
White-water rafting, 378  
Year-end bonus, 118

## Chemistry

Acid solutions, 82  
Chlorine bleach, 82  
Gas laws, 516  
Increasing acidity, 82  
Increasing the percentage, 83  
Ions for breakfast, 562  
Ions in your veins, 562  
Mixture, 82, 83, 84, 216, 236  
Neuse River pH, 547  
Radioactive decay, 540  
Roanoke River pH, 548  
Stomach acid, 547  
Three solutions, 254  
Tomato juice, 547

## Construction

Area of an inscribed circle, 498  
Building a patio, 579  
Dealing in gravel, 170  
Diagonals, 427  
Dimensions of a frame, 81  
Doorway dimensions, 81  
Fencing, 72  
Great Chicago flood, 72, 323  
Guy wire, 443  
Heavy penalties, 643  
House plans, 288  
Length of a road, 428  
Modern art, 37  
Painting, 358, 378  
Perimeter of a lot, 81  
Pricing plastic, 516  
Reinforcing rods, 516  
Roofing, 357  
Shawn's shed, 84  
Teamwork, 50

## Consumer Applications

Assessed for repairs, 73  
Camaro Z28 depreciation, 129  
Camaro Z28 inflation, 129  
Carpeting costs, 51, 509  
Car shopping, 93  
Charitable contributions, 116  
Comparing copiers, 256  
Constant increase, 633

Cost, 330  
Dealer discounts, 116  
Depreciating Monte Carlo, 139  
Distance between streets, 73  
Fast cat, 171  
Health food mix, 83  
High cost of nursing care, 31  
Increasing salary, 642  
Inflationary spiral, 51  
Listing a house, 82  
Long distance charges, 129  
Measuring risk, 129  
Mixing investments, 254  
Mustang Sally, 82  
Net worth of a family, 22  
Pricing the Crown Victoria, 139  
Saving for retirement, 30, 274  
Selling-price range, 103  
Seven years of salary, 642  
Soaring cost of nursing care, 31  
Social security, 198  
Student loan, 31  
Vehicle cost, 509

## Design

Approach speed, 456  
Cubic coating, 288  
Displacement-length ratio, 498  
Energy efficient, 288  
Fabric design, 633  
Flute reproduction, 599  
Golden rectangle, 472  
House of seven gables, 578  
House plans, 323  
Landing a Piper Cheyenne, 406  
Landing speed and weight, 407, 444  
Landscape design, 330  
Manufacturing a box, 443  
Maximum area, 590, 621  
Open-top box, 472  
Overflow pan, 288  
Pleasing painting, 371  
Sail area-displacement ratio, 499  
Shipping parts, 443  
Spillway capacity, 444  
Swimming pool design, 488  
Volume of a flute, 599

## Environment

An increasing problem, 267  
Air pollution, 183  
Available habitat, 288  
Bitter cold, 22

Capture-recapture method, 371  
Carbon dioxide emission, 150  
Cleaning up the river, 371  
Depth and flow, 150  
Diversity index, 555, 562  
Factoring in the wind, 406  
Finding river flow, 568  
Going with the flow, 562  
Infestation, 632  
Probability of rain, 216  
Record flood, 562  
Recycling progress, 64  
Solid waste, 64, 267, 342  
Sonic boom, 610  
Thinning eggshells, 183  
Wildlife management, 567  
World energy use, 150

## Geometry

Angle, 236, 243  
Area, 30, 288, 295, 321, 413  
Cardboard box, 71  
Diagonal, 396, 427, 464  
Diameter, 72, 267  
Fish tank, 71  
Height, 71, 72, 443  
Ice sculpture, 71  
Parallelogram, 45  
Parthenon, 45  
Perimeter, 30, 81, 170, 208, 623  
Radius, 72, 183, 396  
Rectangle, 71, 81  
Reflecting pool, 71  
Second base, 71  
Surface area of a cube, 428  
Square, 45, 170, 525  
Tale of two circles, 623  
Triangle, 45, 81, 413  
Volume, 295, 304, 413

## Investment

Average annual return, 428  
Best bond fund, 397  
Best stock fund, 397  
Big saver, 651  
Buying stock, 151  
Chocolate bars, 547  
Comparing investments, 274  
Compound interest, 539, 567  
Diversification, 84, 223, 249  
Doubling time, 547, 567  
Financial independence, 84  
Finding time, 71, 561

Golden years, 562  
Growth rate, 555  
Interest, 71, 208  
Investing, 82  
Outstanding performance, 539  
Overdue loan payment, 397  
Partial year, 540  
Retirement fund, 651  
Saving, 31, 274  
Stocks, 274  
Top stock, 547  
Wealth-building portfolio, 372  
World's largest mutual fund, 651

## Science

Accident reconstruction, 509  
Arecibo Observatory, 591  
Altitude of a satellite, 72  
Below sea level, 22  
Bitter cold, 22  
Comparing wind chills, 406  
Distance to the sun, 267  
Estimating armaments, 72, 372  
Falling objects, 442, 516, 525  
Female femurs, 118  
Heating water, 150  
Highs and lows, 22  
Kepler's third law, 428  
Marine navigation, 609  
Measuring ocean depths, 570  
Musical tones, 633  
Orbit of Venus, 428  
Orbits of the planets, 396  
Popping corn, 446  
Radius of the earth, 72  
Resistance, 378, 516  
Seacoast artillery, 482  
Shock absorbers, 516  
Siege and garrison artillery, 482  
Skeletal remains, 118  
Sound level, 548  
Space travel, 267  
Time, 267, 456  
Using leverage, 516  
World's largest telescope, 591

## Sports

America's Cup, 406  
Bicycle gear ratio, 516  
Boxing match, 487  
Cross-country cycling, 472  
Decathlon champion, 488  
Diving time, 406

First Super Bowl, 83  
Flying high, 482  
Football, 72, 632  
Foul ball, 464  
Maximum height, 183, 590  
Maximum sail area, 396  
Maximum sailing speed, 406  
Mixed doubles, 83  
Ping pong, 488  
Pole vaulting, 456  
Putting the shot, 482  
Sailboat speed, 428  
Sailboat stability, 427  
Sky diving, 406  
Super Bowl contender, 216  
Tennis, 82, 322  
Time of flight, 488  
Velocity of a pop up, 130  
World records, 111

## Statistics/Demographics

Above the poverty level, 562  
AIDS, 384  
Average price, 364  
Average speed, 364  
Bachelor's degrees, 93  
Below the poverty level, 562  
Big family, 651  
California growin', 397  
Campaigning for governor, 371  
Civilian labor force, 570  
Explosive situation, 364  
Fastest airliner, 37  
Golden years, 330  
Higher education, 103, 343  
Imports and exports, 567  
Life expectancy, 274, 282, 330  
Logistic growth, 548  
Master's degrees, 94  
Population growth, 444, 540, 548  
Predicting heights of preschoolers, 663  
Public school enrollment, 64, 349  
Racial balance, 364  
Rising costs of health care, 443  
Senior citizens, 103  
Teacher's average salary, 64  
The golden state, 349  
Total construction, 171  
Weighted average, 94

# DEFINITIONS, RULES, AND FORMULAS

## Subsets of the Real Numbers

Natural Numbers =  $\{1, 2, 3, \dots\}$

Whole Numbers =  $\{0, 1, 2, 3, \dots\}$

Integers =  $\{\dots, -3, -2, -1, 0, 1, 2, 3, \dots\}$

Rational =  $\left\{\frac{a}{b} \mid a \text{ and } b \text{ are integers with } b \neq 0\right\}$

Irrational =  $\{x \mid x \text{ is not rational}\}$

## Properties of the Real Numbers

For all real numbers  $a$ ,  $b$ , and  $c$

$a + b = b + a$ ;  $a \cdot b = b \cdot a$  Commutative

$(a + b) + c = a + (b + c)$ ;  $(ab)c = a(bc)$  Associative

$a(b + c) = ab + ac$ ;  $a(b - c) = ab - ac$  Distributive

$a + 0 = a$ ;  $1 \cdot a = a$  Identity

$a + (-a) = 0$ ;  $a \cdot \frac{1}{a} = 1$  ( $a \neq 0$ ) Inverse

$a \cdot 0 = 0$  Multiplication property of 0

## Absolute Value

$$|a| = \begin{cases} a & \text{for } a \geq 0 \\ -a & \text{for } a < 0 \end{cases}$$

$$\sqrt{x^2} = |x| \text{ for any real number } x.$$

$$|x| = k \Leftrightarrow x = k \text{ or } x = -k \quad (k > 0)$$

$$|x| < k \Leftrightarrow -k < x < k \quad (k > 0)$$

$$|x| > k \Leftrightarrow x < -k \text{ or } x > k \quad (k > 0)$$

(The symbol  $\Leftrightarrow$  means "if and only if.")

## Interval Notation

$$(a, b) = \{x \mid a < x < b\} \quad [a, b] = \{x \mid a \leq x \leq b\}$$

$$(a, b] = \{x \mid a < x \leq b\} \quad [a, b) = \{x \mid a \leq x < b\}$$

$$(-\infty, a) = \{x \mid x < a\} \quad (a, \infty) = \{x \mid x > a\}$$

$$(-\infty, a] = \{x \mid x \leq a\} \quad [a, \infty) = \{x \mid x \geq a\}$$

## Exponents

$$a^0 = 1 \quad a^{-1} = \frac{1}{a}$$

$$a^{-r} = \frac{1}{a^r} = \left(\frac{1}{a}\right)^r \quad \frac{1}{a^{-r}} = a^r$$

$$a^r a^s = a^{r+s} \quad \frac{a^r}{a^s} = a^{r-s}$$

$$(a^r)^s = a^{rs} \quad (ab)^r = a^r b^r$$

$$\left(\frac{a}{b}\right)^r = \frac{a^r}{b^r} \quad \left(\frac{a}{b}\right)^{-r} = \left(\frac{b}{a}\right)^r$$

## Roots and Radicals

$$a^{1/n} = \sqrt[n]{a} \quad a^{m/n} = (\sqrt[n]{a})^m = \sqrt[n]{a^m}$$

$$\sqrt[n]{ab} = \sqrt[n]{a} \cdot \sqrt[n]{b} \quad \sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}$$

## Factoring

$$a^2 + 2ab + b^2 = (a + b)^2$$

$$a^2 - 2ab + b^2 = (a - b)^2$$

$$a^2 - b^2 = (a + b)(a - b)$$

$$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$$

$$a^3 + b^3 = (a + b)(a^2 - ab + b^2)$$

## Rational Expressions

$$\frac{a}{b} + \frac{c}{b} = \frac{a + c}{b}$$

$$\frac{a}{b} - \frac{c}{b} = \frac{a - c}{b}$$

$$\frac{ac}{bc} = \frac{a}{b}$$

$$\frac{a}{b} + \frac{c}{d} = \frac{ad + bc}{bd}$$

$$\frac{a}{b} \cdot \frac{c}{d} = \frac{ac}{bd}$$

$$\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \cdot \frac{d}{c}$$

If  $\frac{a}{b} = \frac{c}{d}$ , then  $ad = bc$ .

## Quadratic Formula

The solutions to  $ax^2 + bx + c = 0$  with  $a \neq 0$  are

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

## Distance Formula

The distance from  $(x_1, y_1)$  to  $(x_2, y_2)$  is

$$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

## Midpoint Formula

The midpoint of the line segment with endpoints  $(x_1, y_1)$  and  $(x_2, y_2)$  is

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

## Slope Formula

The slope of the line through  $(x_1, y_1)$  and  $(x_2, y_2)$  is

$$\frac{y_2 - y_1}{x_2 - x_1} \quad (\text{for } x_1 \neq x_2)$$

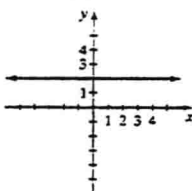
## Linear Function

$f(x) = mx + b$  with  $m \neq 0$

Graph is a line with slope  $m$ .

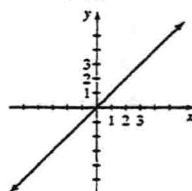
A constant function

$$f(x) = 2$$



The identity function

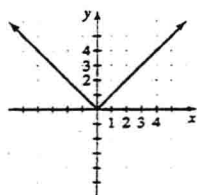
$$f(x) = x$$





### Absolute Value Function

$$f(x) = |x|$$

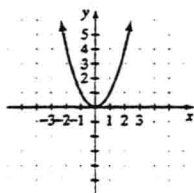


### Quadratic Function

$$f(x) = ax^2 + bx + c \text{ with } a \neq 0$$

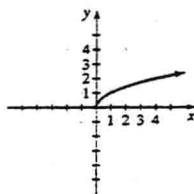
Graph is a parabola.

$$f(x) = x^2 \text{ (the squaring function)}$$



### Square-Root Function

$$f(x) = \sqrt{x}$$

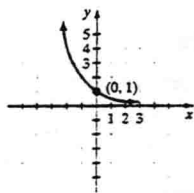
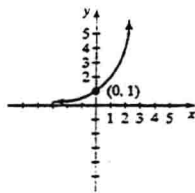


### Exponential Function

$$f(x) = a^x \text{ for } a > 0 \text{ and } a \neq 1$$

One-to-one property:  $a^m = a^n \leftrightarrow m = n$

$$f(x) = a^x \text{ for } a > 1 \quad f(x) = a^x \text{ for } 0 < a < 1$$



### Logarithmic Function

$$f(x) = \log_a(x) \text{ for } a > 0 \text{ and } a \neq 1$$

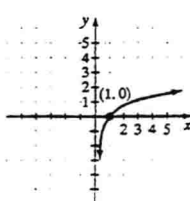
One-to-one property:  $\log_a(m) = \log_a(n) \leftrightarrow m = n$

$$\text{Base-}a \text{ logarithm: } y = \log_a(x) \leftrightarrow a^y = x$$

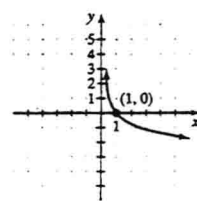
$$\text{Natural logarithm: } y = \ln(x) \leftrightarrow e^y = x$$

$$\text{Common logarithm: } y = \log(x) \leftrightarrow 10^y = x$$

$$f(x) = \log_a(x) \\ \text{for } a > 1$$



$$f(x) = \log_a(x) \\ \text{for } 0 < a < 1$$



### Properties of Logarithms

$$\log_a(a) = 1$$

$$\log_a(1) = 0$$

$$\log_a(a^M) = M$$

$$a^{\log_a(M)} = M$$

$$\log_a(MN) = \log_a(M) + \log_a(N)$$

$$\log_a\left(\frac{M}{N}\right) = \log_a(M) - \log_a(N)$$

$$\log_a(M^N) = N \cdot \log_a(M)$$

$$\log_a\left(\frac{1}{N}\right) = -\log_a(N)$$

$$\log_a(M) = \frac{\log_b(M)}{\log_b(a)} = \frac{\ln(M)}{\ln(a)} = \frac{\log(M)}{\log(a)}$$

### Interest Formulas

$A$  = amount and  $P$  = principal

Compound interest:  $A = P(1 + i)^n$ , where  $n$  = number of periods and  $i$  = interest rate per period

Continuous compounding:  $A = Pe^{rt}$ , where  $r$  = annual interest rate and  $t$  = time in years

### Variation

$$\text{Direct: } y = kx \quad (k \neq 0)$$

$$\text{Inverse: } y = \frac{k}{x} \quad (k \neq 0)$$

$$\text{Joint: } y = kxz \quad (k \neq 0)$$

### Straight Line

$$\text{Slope-intercept form: } y = mx + b$$

$$\text{Slope: } m \quad \text{y-intercept: } (0, b)$$

$$\text{Point-slope form: } y - y_1 = m(x - x_1)$$

$$\text{Slope: } m \quad \text{Point: } (x_1, y_1)$$

$$\text{Standard form: } Ax + By = C$$

$$\text{Horizontal: } y = k \quad \text{Vertical: } x = k$$

## Parabola

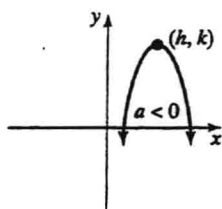
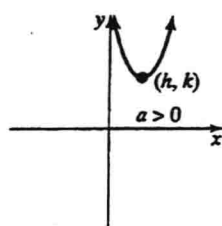
$$y = a(x - h)^2 + k \quad (a \neq 0)$$

Vertex:  $(h, k)$

Axis of symmetry:  $x = h$

Focus:  $(h, k + p)$ , where  $a = \frac{1}{4p}$

Directrix:  $y = k + p$



## Ellipse

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \quad (a > b > 0)$$

Center:  $(0, 0)$

Foci:  $(\pm c, 0)$ ,

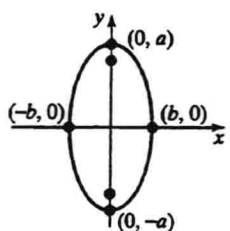
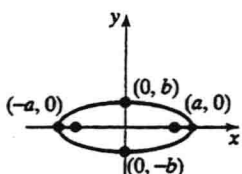
where  $c^2 = a^2 - b^2$

$$\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1 \quad (a > b > 0)$$

Center:  $(0, 0)$

Foci:  $(0, \pm c)$ ,

where  $c^2 = a^2 - b^2$



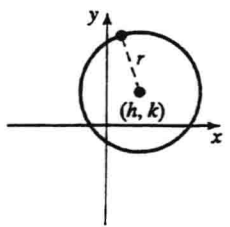
## Circle

$$(x - h)^2 + (y - k)^2 = r^2 \quad (r > 0)$$

Center:  $(h, k)$  Radius:  $r$

$$x^2 + y^2 = r^2$$

Center:  $(0, 0)$  Radius:  $r$



## Hyperbola

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

Center:  $(0, 0)$

x-intercepts:  $(\pm a, 0)$

Foci:  $(\pm c, 0)$ ,

where  $c^2 = a^2 + b^2$

Asymptotes:  $y = \pm \frac{b}{a}x$

$$\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$$

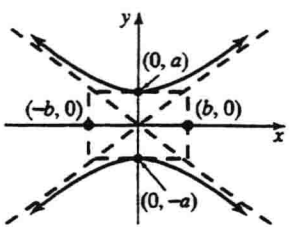
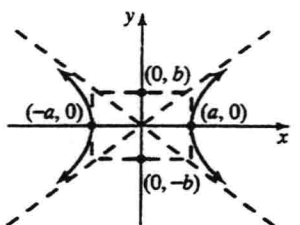
Center:  $(0, 0)$

y-intercepts:  $(0, \pm a)$

Foci:  $(0, \pm c)$ ,

where  $c^2 = a^2 + b^2$

Asymptotes:  $y = \pm \frac{a}{b}x$



## Arithmetic Sequence

$$a_1, a_1 + d, a_1 + 2d, a_1 + 3d, \dots$$

Formula for  $n$ th term:  $a_n = a_1 + (n - 1)d$

Sum of  $n$  terms:  $S_n = \frac{n}{2}(a_1 + a_n)$

## Geometric Sequence

$$a_1, a_1r, a_1r^2, a_1r^3, \dots$$

Formula for  $n$ th term:  $a_n = a_1r^{n-1}$

Sum of  $n$  terms ( $r \neq 1$ ):  $S_n = \frac{a_1 - a_1r^n}{1 - r}$

Sum of all terms ( $|r| < 1$ ):  $S = \frac{a_1}{1 - r}$

## Binomial Expansion

$$(a + b)^2 = a^2 + 2ab + b^2$$

$$(a + b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$$

$$(a + b)^4 = a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + b^4$$

$$(a + b)^n = \sum_{i=0}^n \binom{n}{i} a^{n-i} b^i, \text{ where } \binom{n}{i} = \frac{n!}{(n-i)!i!}$$

Factorial notation:  $n! = 1 \cdot 2 \cdot 3 \cdot \dots \cdot n$

## Metric Abbreviations

| Length        | Volume        | Weight       |
|---------------|---------------|--------------|
| mm millimeter | mL milliliter | mg milligram |
| cm centimeter | cL centiliter | cg centigram |
| dm decimeter  | dL deciliter  | dg decigram  |
| m meter       | L liter       | g gram       |
| dam dekameter | daL dekaliter | dag dekagram |
| hm hectometer | hL hectoliter | hg hectogram |
| km kilometer  | kL kiloliter  | kg kilogram  |

## English-Metric Conversion

| Length           | Volume (U.S.)   | Weight           |
|------------------|-----------------|------------------|
| 1 in. = 2.540 cm | 1 pt = 0.4732 L | 1 oz = 28.35 g   |
| 1 ft = 30.48 cm  | 1 qt = 0.9464 L | 1 lb = 453.6 g   |
| 1 yd = 0.9144 m  | 1 gal = 3.785 L | 1 lb = 0.4536 kg |
| 1 mi = 1.609 km  |                 |                  |

| Length            | Volume (U.S.)    | Weight            |
|-------------------|------------------|-------------------|
| 1 cm = 0.3937 in. | 1 L = 2.2233 pt  | 1 g = 0.0353 oz   |
| 1 cm = 0.03281 ft | 1 L = 1.0567 qt  | 1 g = 0.002205 lb |
| 1 m = 1.0936 yd   | 1 L = 0.2642 gal | 1 kg = 2.205 lb   |
| 1 km = 0.6215 mi  |                  |                   |



# C O N T E N T S

## Preface

xiii

## Chapter 1

### The Real Numbers

1

- 1.1 Sets 2
- 1.2 The Real Numbers 7
- 1.3 Operations on the Set of Real Numbers 13
- 1.4 Evaluating Expressions 23
- 1.5 Properties of the Real Numbers 31
- 1.6 Using the Properties 38
  - Collaborative Activities 44
  - Chapter Summary 45
  - Enriching Your Mathematical Word Power 47
  - Review Exercises 48
  - Chapter 1 Test 50

## Chapter 2

### Linear Equations and Inequalities in One Variable

53

- 2.1 Linear Equations in One Variable 54
- 2.2 Formulas 65
- 2.3 Applications 73
- 2.4 Inequalities 84
- 2.5 Compound Inequalities 94
- 2.6 Absolute Value Equations and Inequalities 104
  - Collaborative Activities 112
  - Chapter Summary 113
  - Enriching Your Mathematical Word Power 115
  - Review Exercises 115
  - Chapter 2 Test 119
  - Making Connections 120

## Chapter 3

### Graphs and Functions in the Cartesian Coordinate System

121

- 3.1 Graphing Lines in the Coordinate Plane 122
- 3.2 Slope of a Line 131
- 3.3 Three Forms for the Equation of a Line 140

|     |  |     |
|-----|--|-----|
| 3.4 | Linear Inequalities and Their Graphs   | 151 |
| 3.5 | Relations and Functions                | 164 |
| 3.6 | Graphs of Functions                    | 171 |
|     | Collaborative Activities               | 184 |
|     | Chapter Summary                        | 185 |
|     | Enriching Your Mathematical Word Power | 187 |
|     | Review Exercises                       | 188 |
|     | Chapter 3 Test                         | 194 |
|     | Making Connections                     | 197 |

## Chapter 4

## Systems of Linear Equations

199

|     |  |     |
|-----|--|-----|
| 4.1 | Solving Systems by Graphing and Substitution   | 200 |
| 4.2 | The Addition Method                            | 209 |
| 4.3 | Systems of Linear Equations in Three Variables | 217 |
| 4.4 | Solving Linear Systems Using Matrices          | 224 |
| 4.5 | Cramer's Rule for Systems in Two Variables     | 231 |
| 4.6 | Cramer's Rule for Systems in Three Variables   | 237 |
| 4.7 | Linear Programming                             | 244 |
|     | Collaborative Activities                       | 250 |
|     | Chapter Summary                                | 250 |
|     | Enriching Your Mathematical Word Power         | 252 |
|     | Review Exercises                               | 253 |
|     | Chapter 4 Test                                 | 255 |
|     | Making Connections                             | 256 |

## Chapter 5

## Exponents and Polynomials

257

|     |  |     |
|-----|--|-----|
| 5.1 | Integral Exponents and Scientific Notation               | 258 |
| 5.2 | The Power Rules  | 268 |
| 5.3 | Addition, Subtraction, and Multiplication of Polynomials | 275 |
| 5.4 | Multiplying Binomials                                    | 283 |
| 5.5 | Division of Polynomials                                  | 289 |
| 5.6 | Factoring Polynomials                                    | 296 |
| 5.7 | Factoring $ax^2 + bx + c$                                | 305 |
| 5.8 | Factoring Strategy                                       | 310 |
| 5.9 | Solving Equations by Factoring                           | 316 |
|     | Collaborative Activities                                 | 323 |
|     | Chapter Summary  | 324 |
|     | Enriching Your Mathematical Word Power                   | 327 |
|     | Review Exercises   | 328 |
|     | Chapter 5 Test   | 330 |
|     | Making Connections                                       | 332 |

**Chapter 6****Rational Expressions****333**

- 6.1 Properties of Rational Expressions 334
- 6.2 Multiplication and Division 343
- 6.3 Addition and Subtraction 350
- 6.4 Complex Fractions 358
- 6.5 Solving Equations Involving Rational Expressions 365
- 6.6 Applications 372
  - Collaborative Activities 379
  - Chapter Summary 379
  - Enriching Your Mathematical Word Power 381
  - Review Exercises 382
  - Chapter 6 Test 385
  - Making Connections 386

**Chapter 7****Rational Exponents and Radicals****387**

- 7.1 Rational Exponents 388
- 7.2 Radicals 397
- 7.3 Operations with Radicals 407
- 7.4 More Operations with Radicals 413
- 7.5 Solving Equations with Radicals and Exponents 418
- 7.6 Complex Numbers 429
  - Collaborative Activities 437
  - Chapter Summary 437
  - Enriching Your Mathematical Word Power 440
  - Review Exercises 440
  - Chapter 7 Test 444
  - Making Connections 446

**Chapter 8****Quadratic Equations and Inequalities****447**

- 8.1 Factoring and Completing the Square 448
- 8.2 The Quadratic Formula 457
- 8.3 More on Quadratic Equations 465
- 8.4 Quadratic and Rational Inequalities 473
  - Collaborative Activities 483
  - Chapter Summary 484
  - Enriching Your Mathematical Word Power 485
  - Review Exercises 486
  - Chapter 8 Test 488
  - Making Connections 490

**Chapter 9****Additional Function Topics****491**

- 9.1 Combining Functions 492
- 9.2 Inverse Functions 499
- 9.3 Variation 510
- 9.4 The Factor Theorem 517
  - Collaborative Activities 522
  - Chapter Summary 522
  - Enriching Your Mathematical Word Power 524
  - Review Exercises 524
  - Chapter 9 Test 526
  - Making Connections 527

**Chapter 10****Exponential and Logarithmic Functions****529**

- 10.1 Exponential Functions 530
- 10.2 Logarithmic Functions 541
- 10.3 Properties of Logarithms 548
- 10.4 Solving Equations 556
  - Collaborative Activities 563
  - Chapter Summary 564
  - Enriching Your Mathematical Word Power 565
  - Review Exercises 565
  - Chapter 10 Test 568
  - Making Connections 569

**Chapter 11****Nonlinear Systems and the Conic Sections****571**

- 11.1 Nonlinear Systems of Equations 572
- 11.2 The Parabola 580
- 11.3 The Circle 593
- 11.4 The Ellipse and Hyperbola 600
- 11.5 Second-Degree Inequalities 610
  - Collaborative Activities 616
  - Chapter Summary 616
  - Enriching Your Mathematical Word Power 619
  - Review Exercises 619
  - Chapter 11 Test 624
  - Making Connections 625

**Chapter 12****Sequences and Series****627**

- 12.1 Sequences 628
- 12.2 Series 634
- 12.3 Arithmetic Sequences and Series 638
- 12.4 Geometric Sequences and Series 643

|      |  |     |
|------|--|-----|
| 12.5 | Binomial Expansions                    | 652 |
|      | Collaborative Activities               | 656 |
|      | Chapter Summary                        | 657 |
|      | Enriching Your Mathematical Word Power | 658 |
|      | Review Exercises                       | 659 |
|      | Chapter 12 Test                        | 661 |
|      | Making Connections                     | 662 |

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**Appendix****A-1**

|   |                                   |     |
|---|-----------------------------------|-----|
| A | Geometry Review                   | A-1 |
| B | Table of Squares and Square Roots | A-4 |
| C | Common Logarithms                 | A-5 |
| D | Answers to Selected Exercises     | A-7 |

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**Index****I-1**

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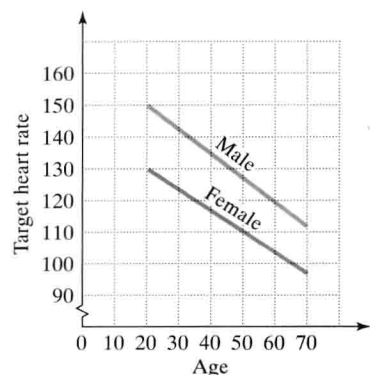


## The Real Numbers

**E**verywhere you look people are running, riding, dancing, and exercising their way to fitness. In the past year more than \$25 billion has been spent on sports equipment alone, and this amount is growing steadily.

Proponents of exercise claim that it can increase longevity, improve body image, decrease appetite, and generally enhance a person's health. While many sports activities can help you to stay fit, experts have found that aerobic, or dynamic, workouts provide the most fitness benefit. Some of the best aerobic exercises include cycling, running, and even jumping rope. Whatever athletic activity you choose, trainers recommend that you set realistic goals and work your way toward them consistently and slowly. To achieve maximum health benefits, experts suggest that you exercise three to five times a week for 15 to 60 minutes at a time.

There are many different ways to measure exercise. One is to measure the energy used, or the rate of oxygen consumption. Since heart rate rises as a function of increased oxygen, another easier measure of intensity of exercise is your heart rate during exercise. The desired heart rate, or target heart rate, for beneficial exercise varies for each individual depending on conditioning, age, and gender. In Exercises 101 and 102 of Section 1.4 you will see how an algebraic expression can determine your target heart rate for beneficial exercise.



## 1.1 SETS

## In this section

- Set Notation
- Union of Sets
- Intersection of Sets
- Subsets
- Combining Three or More Sets

Every subject has its own terminology, and **algebra** is no different. In this section we will learn the basic terms and facts about sets.

## Set Notation

A **set** is a collection of objects. At home you may have a set of dishes and a set of steak knives. In algebra we generally discuss sets of numbers. For example, we refer to the numbers 1, 2, 3, 4, 5, and so on as the set of **counting numbers** or **natural numbers**. Of course, these are the numbers that we use for counting.

The objects or numbers in a set are called the **elements** or **members** of the set. To describe sets with a convenient notation, we use braces,  $\{ \}$ , and name the sets with capital letters. For example,

$$A = \{1, 2, 3\}$$

means that set  $A$  is the set whose members are the natural numbers 1, 2, and 3. The letter  $N$  is used to represent the entire set of natural numbers.

A set that has a fixed number of elements such as  $\{1, 2, 3\}$  is a **finite** set, whereas a set without a fixed number of elements such as the natural numbers is an **infinite** set. When listing the elements of a set, we use a series of three dots to indicate a continuing pattern. For example, the set of natural numbers is written as

$$N = \{1, 2, 3, \dots\}.$$

The set of natural numbers *between* 4 and 40 can be written

$$\{5, 6, 7, 8, \dots, 39\}.$$

Note that since the members of this set are *between* 4 and 40, it does not include 4 or 40.

**Set-builder notation** is another method of describing sets. In this notation we use a variable to represent the numbers in the set. A **variable** is a letter that is used to stand for some numbers. The set is then built from the variable and a description of the numbers that the variable represents. For example, the set

$$B = \{1, 2, 3, \dots, 49\}$$

is written in set-builder notation as

$$B = \{x \mid x \text{ is a natural number less than } 50\}.$$

$\uparrow \quad \uparrow \quad \uparrow$   
 The set of numbers   such that   condition for membership

This notation is read as “ $B$  is the set of numbers  $x$  such that  $x$  is a natural number less than 50.” Notice that the number 50 is not a member of set  $B$ .

The symbol  $\in$  is used to indicate that a specific number is a member of a set, and  $\notin$  indicates that a specific number is not a member of a set. For example, the statement  $1 \in B$  is read as “1 is a member of  $B$ ,” “1 belongs to  $B$ ,” “1 is in  $B$ ,” or “1 is an element of  $B$ .” The statement  $0 \notin B$  is read as “0 is not a member of  $B$ ,” “0 does not belong to  $B$ ,” “0 is not in  $B$ ,” or “0 is not an element of  $B$ .”

Two sets are **equal** if they contain exactly the same members. Otherwise, they are said to be not equal. To indicate equal sets, we use the symbol  $=$ . For sets that are not equal we use the symbol  $\neq$ . The elements in two equal sets do not need to be written in the same order. For example,  $\{3, 4, 7\} = \{3, 4, 7\}$  and  $\{2, 4, 1\} = \{1, 2, 4\}$ , but  $\{3, 5, 6\} \neq \{3, 5, 7\}$ .

## study tip

Find a group of students to work with outside of class. Don't just settle for answers. Make sure that everyone in the group understands the solution to a problem. You really will understand a concept when you can explain it to someone else.





**helpful hint**

To remember the meaning of "intersection," think of the intersection of two roads. At the intersection you are on both roads.

**Intersection of Sets**

If  $A$  and  $B$  are sets, the **intersection** of  $A$  and  $B$ , denoted  $A \cap B$ , is the set of all elements that are in both  $A$  and  $B$ . In symbols,

$$A \cap B = \{x \mid x \in A \text{ and } x \in B\}.$$

It is possible for two sets to have no elements in common. A set with no members is called the **empty set** and is denoted by the symbol  $\emptyset$ . Note that  $A \cup \emptyset = A$  and  $A \cap \emptyset = \emptyset$  for any set  $A$ .

**CAUTION**

The set  $\{0\}$  is not the empty set. The set  $\{0\}$  has one member, the number 0. Do not use the number 0 to represent the empty set.

**EXAMPLE 3****Intersection of sets**

Let  $A = \{0, 2, 3\}$ ,  $B = \{2, 3, 7\}$ , and  $C = \{7, 8\}$ . List the elements in each of the following sets.

a)  $A \cap B$

b)  $B \cap C$

c)  $A \cap C$

**Solution**

a)  $A \cap B$  is the set of all numbers that are in both  $A$  and  $B$ . So  $A \cap B = \{2, 3\}$ .

b)  $B \cap C = \{7\}$

c)  $A \cap C = \emptyset$

**EXAMPLE 4****Membership and equality**

Let  $A = \{1, 2, 3, 5\}$ ,  $B = \{2, 3, 7, 8\}$ , and  $C = \{6, 7, 8, 9\}$ . Place one of the symbols  $=$ ,  $\neq$ ,  $\in$ , or  $\notin$  in the blank to make each statement correct.

a)  $5 \underline{\hspace{1cm}} A \cup B$

b)  $5 \underline{\hspace{1cm}} A \cap B$

c)  $A \cup B \underline{\hspace{1cm}} \{1, 2, 3, 5, 7, 8\}$

d)  $A \cap B \underline{\hspace{1cm}} \{2\}$

**Solution**

a)  $5 \in A \cup B$  because 5 is a member of  $A$ .

b)  $5 \notin A \cap B$  because 5 must belong to *both*  $A$  and  $B$  to be a member of  $A \cap B$ .

c)  $A \cup B = \{1, 2, 3, 5, 7, 8\}$  because the elements of  $A$  together with those of  $B$  are listed. Note that 2 and 3 are members of both sets but are listed only once.

d)  $A \cap B \neq \{2\}$  because  $A \cap B = \{2, 3\}$ .

**Subsets**

If every member of set  $A$  is also a member of set  $B$ , then we write  $A \subseteq B$  and say that  $A$  is a **subset** of  $B$ . See Fig. 1.3. For example,

$$\{2, 3\} \subseteq \{2, 3, 4\}.$$

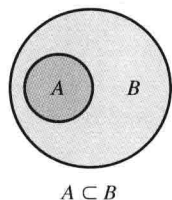
If set  $A$  is not a subset of  $B$ , we write  $A \not\subseteq B$ .

**CAUTION**

To claim that  $A \not\subseteq B$ , there *must* be an element of  $A$  that does *not* belong to  $B$ . For example,

$$\{1, 2\} \not\subseteq \{2, 3, 4\}$$

because 1 is a member of the first set but not of the second.

**FIGURE 1.3**