

THEORETICAL MECHANICS

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for technical schools

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PREFACE

In this text we have devoted considerable attention to a discussion of the basic concepts and propositions of theoretical mechanics and their relationship with reality and engineering practice. The numerous examples, remarks and footnotes to be found in the text are designed to help the student towards a better understanding of the subject and a firmer grasp of the material; they are also aimed at averting false impressions and distorted conceptions as the student masters the subject.

Much care has been taken to develop the student's skills and habits in applying the propositions of mechanics to the solution of specific questions and problems. This aim is served by general suggestions and hints of a methodological nature and also by the 118 typical problems provided with solutions and occupying about 25% of the book.

The textbook offers a full coverage of all aspects of the obligatory portion of the syllabus of theoretical mechanics for mechanical engineering technical schools. It also discusses a number of problems that go beyond the obligatory portion of the syllabus but are important in certain fields. The sections and items that are not obligatory (they constitute about 15% of the whole text) are marked with an asterisk. They may be omitted without any loss of continuity of the remaining material.

All definitions and important conclusions are given in italics and it is hoped that this will serve to fix such propositions more firmly in the mind of the student (after he has fully mastered the contents of the problem at hand, naturally).

A few words are in order on the system of units used in this textbook. The International System of Units (abbrev-

viated SI) is employed throughout. However, since the engineering system of units is still in use, the author has found it expedient to provide basic information about that system as well. As concerns derived units, they are expressed both in the SI and in the engineering system in appropriate sections on dynamics. A relationship is established between the two systems for the purpose of conversion from one system to the other.

INTRODUCTION

Sec. 1. The subject and method of theoretical mechanics

Motion is the mode of existence of matter, its basic inherent property.

By motion in its broad sense we understand not only the displacement of bodies in space but also thermal, chemical, electromagnetic and any other changes and processes, including our consciousness and thought.

Mechanics is the study of the simplest and most readily observable form of motion, mechanical motion.

Mechanical motion is defined as any change, in time, of material bodies relative to one another, and also any change relative to the position of the particles of one and the same material body (its deformation).

Of course, it would be wrong to reduce the great multiplicity of natural phenomena to mechanical motion alone and to explain them on the basis of pure mechanics. Mechanical motion does not by any means exhaust the essence of various forms of motion, but it is always present in every form and must be the first to undergo investigation.

Due to tremendous advances in science and engineering it has become impossible to concentrate in one subject the study of the vast multitude of problems that involve the mechanical motion of diverse material bodies. Today, mechanics is a whole complex of general and special engineering disciplines devoted to the study of the motion of separate bodies and systems of bodies, to the design and analysis of diverse structures, mechanisms, machines, and so forth.

The material bodies involved in these disciplines are extremely diversified, but their motions possess many prop-

erties that are independent of the physical properties of the moving bodies themselves. For instance, one can speak of the velocities of points of a body quite apart from any knowledge of the body itself, whether it be a particle of a fluid or gas, or a piston, a bullet or a spaceship. Again, one can speak of the rotation of a rigid body without going into the question of whether it is a wheel or a planet.

It is these general properties of mechanical motion of material bodies that are studied in theoretical mechanics.

Theoretical mechanics is the science that studies the general laws of mechanical motion of material bodies and establishes general procedures and methods for solving problems involving such motion.

In order to establish the laws of motion common to all material bodies, theoretical mechanics resorts to the device of generalization; this amounts to isolating that which is of primary importance and rejecting all subsidiary circumstances that are of no importance under the conditions at hand. For this reason, in theoretical mechanics we do not consider the motion of the physical bodies that actually exist in nature, but of certain abstract models that reflect only specific common properties of the actual physical bodies. One such model is the material point (particle), another is the absolutely rigid body; these are precisely what are studied in the general course of theoretical mechanics.

All material bodies occupy a definite portion of space, which means they have specific dimensions. Generally speaking, the separate parts of the bodies may not execute the same motions. For example, the points of a pulley at different distances from its axis move in circles of different radii and with different speeds. But the smaller the size of the body, the less the motions of its different parts will differ. In the abstract, we can imagine a material body of infinitesimal proportions.

A material body whose size can be disregarded under the conditions of a given problem is termed a material point (or particle). In theoretical mechanics, a particle is not only some minute particle of a body, but at times can be a body of appreciable size. The main thing is that the dimensions of the body do not play a significant part in the investiga-

tion at hand. For instance, when studying the motion of the earth about the sun, we can regard the earth as a particle, and, since the earth is small compared with its distance from the sun, we can regard all particles of the earth as covering the same distances in that motion. As we shall see later on, any body in translational motion (that is, motion in which all its particles move in the same manner) can be regarded as a material point (particle).

All real bodies undergo deformation under the action of external forces, and so to ensure rigidity and reliability of structures and machines in operation, the material and dimensions of the parts are chosen so that deformations under load are extremely small. In many problems, such insignificant deformations may be ignored and one can regard the distances between the particles of the given body as constant. This brings us to the concept of an absolutely rigid body.

An absolutely rigid body is one in which the distance between any two points remains constant.

In nature, there is no such thing as a material point or an absolutely rigid body. These are merely abstractions that of course do not reflect in toto the properties of specific physical bodies. But one need not attempt to consider all properties, as long as those that are neglected do not appreciably affect the nature of the motion under study.

If at every step we attempted to take into account all the properties of a concrete physical body, the problem would become so complicated that it would be impossible to solve. But if we first establish, through theoretical mechanics, the general laws of motion based on a consideration of the motions of a particle and of absolutely rigid bodies, we can then apply them to concrete physical bodies.

It is only by abstracting oneself from the particular, the individual, that which is peculiar to single objects and phenomena that one is able, in making observations, to establish general regularities, in particular, the general laws of mechanics. This is done by generalizing the results obtained. To illustrate, Galileo started with separate observations of the motion of falling bodies and generalized the results to obtain a general law of motion for all bodies falling in a vacuum.

This, as V. I. Lenin pointed out, is the general path of development of any true science: "From living perception to abstract thought, *and from this to practice*,—such is the dialectic path of the cognition of *truth*, of the cognition of objective reality." (V. I. Lenin, *Collected Works*, Vol. 38, p. 171).

The simplified scheme that we use in theoretical mechanics to replace an actual physical body does not depend only on the properties of that body but also on the type of problem that we are investigating. For instance, when studying the earth's motion about the sun, we can, as mentioned above, assume the earth to be a particle, but that cannot be done if we are engaged in a study of the earth rotating on its axis. This is because the particles of the earth are at different distances from the axis and cover different distances in their motion.

It is customary to divide the course of theoretical mechanics into three basic parts: statics, kinematics and dynamics. *Statics is the study of the rules of composition of forces and the conditions of equilibrium of rigid bodies. Kinematics is the study of motions of bodies from the geometric viewpoint alone, without reference to the forces acting on the bodies. Dynamics is the study of the relationships between the motions of material bodies and the forces acting on them.*

Theoretical mechanics not only makes it possible to explain a number of important phenomena in the surrounding world, but also serves as a scientific foundation for many engineering disciplines. Its methods and procedures are employed in engineering calculations used to design all manner of structures and machines and their operation. The role and significance of theoretical mechanics in engineering in general is constantly growing. The extremely complicated engineering problems that constantly arise in modern technological development, and the organization and expansion of radically new types of production and technical equipment cannot be solved on the basis of experimental findings and practice alone. What is required is scientific foresight and rigorous preliminary calculations based on a profound knowledge of theory, primarily a knowledge of the laws and methods of theoretical mechanics. Thus, besides the important general educational value of