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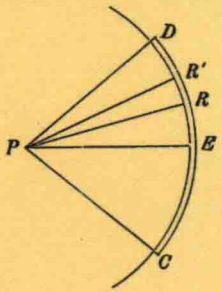
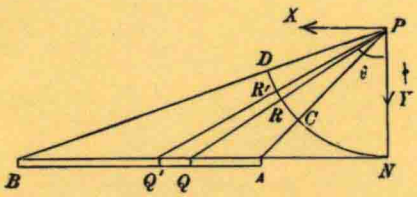
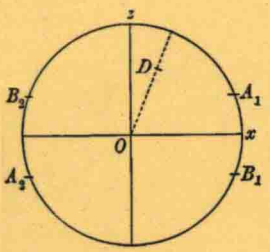
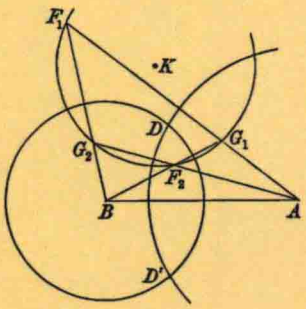
VOLUME 2

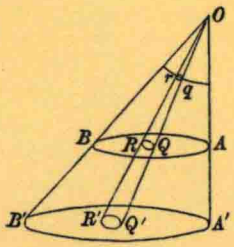
EDWARD JOHN ROUTH











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A Treatise on Analytical Statics

With Numerous Examples

VOLUME 2

EDWARD JOHN ROUTH

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ANALYTICAL STATICS

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A TREATISE ON

ANALYTICAL STATICS

WITH ILLUSTRATIONS TAKEN FROM THE THEORIES OF

ELECTRICITY AND MAGNETISM

BY

EDWARD JOHN ROUTH, Sc.D., LL.D., M.A., F.R.S., ETC.,

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PREFACE.

IN the first edition of this treatise the subject of Attractions was presented only in its gravitational aspect. This limitation was formerly customary, when electricity was less studied than now, but the result has become somewhat unsatisfactory. When lecturing on the subject the Author found that some of the most striking examples of Attraction were those derived from the theory of electricity. While it was impossible wholly to pass these over, it appeared that the interest in them was sensibly diminished if they were discussed without explanations of their meaning. Examples on the attractions of thin layers of matter, subject to what appeared to be arbitrary laws, seemed to have no real applications.

For these reasons a selection has been made of those propositions in Magnetism and Electricity which appeared most forcibly to illustrate the theory of Attraction. These have been joined together, with brief introductions, so as to form a continuous story which could be understood without reference to any other book.

These illustrations have been so far separated from the rest of the volume that any portion of them may be omitted by a reader who desires to confine his attention chiefly to gravitational problems.

Some theorems, which it was not deemed expedient to include in the text, have been shortly discussed in the notes at the end of the volume. These are not always closely connected with the theory of attractions, yet, being natural developments of the text, will probably assist the reader in following the argument.

The general arrangement of the gravitational part of "Attraction" has been only slightly altered. New theorems have, however, been introduced and the demonstrations of some of the old ones simplified.

The second part of this volume is on the stretching and bending of rods. The investigation of the stretching, and consequent thinning, of a rod is founded on Hooke's law. The fact that (with certain restrictions) the stress couple is proportional to the bending is assumed as an experimental result and applied to determine the bending of rods and springs under various circumstances. The problem, when put into this form, is properly included in a treatise on Statics. Although this chapter is not a treatise on the theory of Elasticity, it did not seem proper wholly to omit the theoretical considerations by which the truth of the fundamental law is confirmed. Accordingly some simple examples which had been briefly discussed in the last edition have been retained.

The theory of Astatics occupies the third part of this volume. It was discussed with sufficient fulness in the first edition and only very slight alterations have now been made.

A separate index to each of the three chapters has been given. So many results are included under the head of Attraction that it was found impossible to mention them all without unduly lengthening the list. It was also necessary to classify some theorems only under one heading.

Finally, I desire to express my thanks to Mr J. D. H. Dickson of Peterhouse for the very great assistance he has given me in correcting most of the proof-sheets and for his many valuable suggestions.

EDWARD J. ROUTH.

PETERHOUSE,
December, 1901.

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