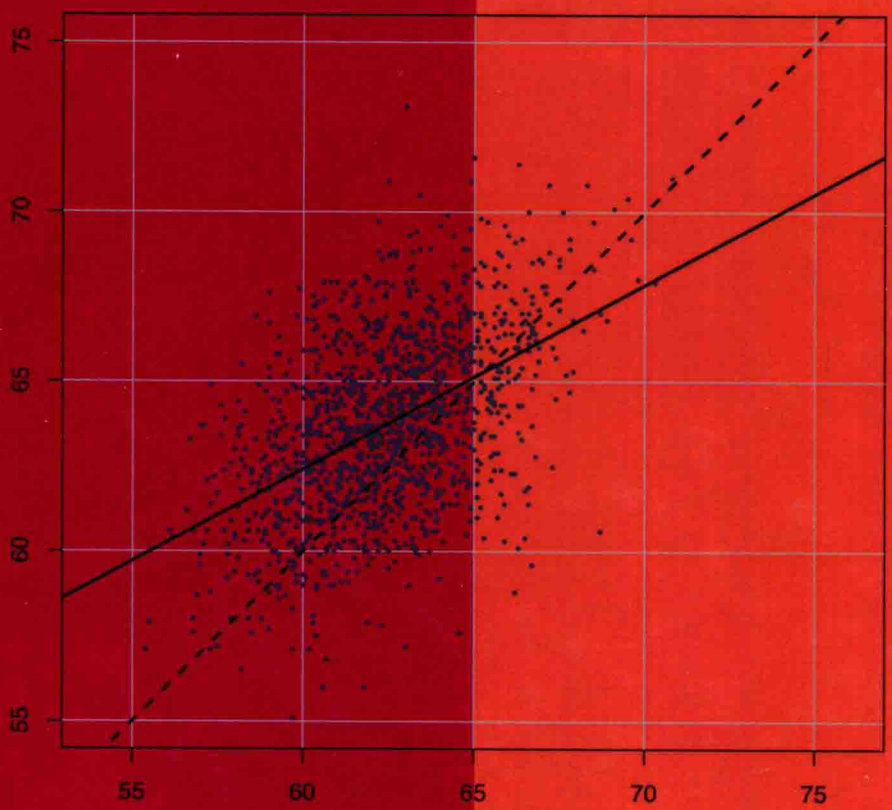
Wiley Series in Probability and Statistics

Applied Linear Regression

SANFORD WEISBERG

FOURTH EDITION



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Applied Linear Regression

Fourth Edition

SANFORD WEISBERG

School of Statistics
University of Minnesota
Minneapolis, MN

WILEY

Published by John Wiley & Sons, Inc., Hoboken, New Jersey
Published simultaneously in Canada

Library of Congress Cataloging-in-Publication Data:

Weisberg, Sanford, 1947–
Applied linear regression / Sanford Weisberg, School of Statistics, University of Minnesota, Minneapolis, MN.—Fourth edition.
pages cm
Includes bibliographical references and index.
ISBN 978-1-118-38608-8 (hardback)
1. Regression analysis. I. Title.
QA278.2.W44 2014
519.5′36–dc23

2014026538

Printed in the United States of America

10 9 8 7 6 5 4 3 2 1

Applied Linear Regression

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A complete list of the titles in this series appears at the end of this volume.

To Carol, Stephanie,
and
the memory of my parents

Preface to the Fourth Edition

This is a *textbook* to help you learn about applied linear regression. The book has been in print for more than 30 years, in a period of rapid change in statistical methodology and particularly in statistical computing. This fourth edition is a thorough rewriting of the book to reflect the needs of current students. As in previous editions, the overriding theme of the book is to help you learn to do data analysis using linear regression. Linear regression is a excellent model for learning about data analysis, both because it is important on its own and it provides a framework for understanding other methods of analysis.

This edition of the book includes the majority of the topics in previous editions, although much of the material has been rearranged. New methodology and examples have been added throughout.

- Even more emphasis is placed on graphics. The first two editions stressed graphics for diagnostic methods (Chapter 9) and the third edition added graphics for understanding data before any analysis is done (Chapter 1). In this edition, *effects plots* are stressed to summarize the fit of a model.
- Many applied analyses are based on understanding and interpreting parameters. This edition puts much greater emphasis on parameters, with part of Chapters 2–3 and all of Chapters 4–5 devoted to this important topic.
- Chapter 6 contains a greatly expanded treatment of testing and model comparison using both likelihood ratio and Wald tests. The usefulness and limitations of testing are stressed.
- Chapter 7 is about the variance assumption in linear models. The discussion of weighted least squares has been been expanded to cover problems of ecological regressions, sample surveys, and other cases. Alternatives such as the bootstrap and heteroskedasticity corrections have been added or expanded.
- Diagnostic methods using transformations (Chapter 8) and residuals and related quantities (Chapter 9) that were the heart of the earlier editions have been maintained in this new edition.

- The discussion of variable selection in Chapter 10 has been updated from the third edition. It is designed to help you understand the key problems in variable selection. In recent years, this topic has morphed into the area of *machine learning* and the goal of this chapter is to show connections and provide references.
- As in the third edition, brief introductions to nonlinear regression (Chapter 11) and to logistic regression (Chapter 12) are included, with Poisson regression added in Chapter 12.

Using This Book

The website for this book is `http://z.umn.edu/alr4ed`.

As with previous editions, this book is not tied to any particular computer program. A primer for using the free `R` package (R Core Team, 2013) for the material covered in the book is available from the website. The primer can also be accessed directly from within `R` as you are working. An optional published companion book about `R` is Fox and Weisberg (2011).

All the data files used are available from the website and in an `R` package called `alr4` that you can download for free. Solutions for odd-numbered problems, all using `R`, are available on the website for the book[1]. You cannot learn to do data analysis without working problems.

Some advanced topics are introduced to help you recognize when a problem that looks like linear regression is actually a little different. Detailed methodology is not always presented, but references at the same level as this book are presented. The bibliography, also available with clickable links on the book's website, has been greatly expanded and updated.

Mathematical Level

The mathematical level of this book is roughly the same as the level of previous editions. Matrix representation of data is used, particularly in the derivation of the methodology in Chapters 3–4. Derivations are less frequent in later chapters, and so the necessary mathematics is less. Calculus is generally not required, except for an occasional use of a derivative. The discussions requiring calculus can be skipped without much loss.

ACKNOWLEDGMENTS

Thanks are due to Jeff Witmer, Yuhong Yang, Brad Price, and Brad's Stat 5302 students at the University of Minnesota. New examples were provided by April Bleske-Rechek, Tom Burk, and Steve Taff. Work with John Fox over the last few years has greatly influenced my writing.

For help with previous editions, thanks are due to Charles Anderson, Don Pereira, Christopher Bingham, Morton Brown, Cathy Campbell, Dennis Cook,

[1]All solutions are available to instructors using the book in a course; see the website for details.

Stephen Fienberg, James Frane, Seymour Geisser, John Hartigan, David Hinkley, Alan Izenman, Soren Johansen, Kenneth Koehler, David Lane, Michael Lavine, Kinley Larntz, Gary Oehlert, Katherine St. Clair, Keija Shan, John Rice, Donald Rubin, Joe Shih, Pete Stewart, Stephen Stigler, Douglas Tiffany, Carol Weisberg, and Howard Weisberg.

Finally, I am grateful to Stephen Quigley at Wiley for asking me to do a new edition. I have been working on versions of this book since 1976, and each new edition has pleased me more that the one before it. I hope it pleases you, too.

SANFORD WEISBERG

St. Paul, Minnesota
September 2013

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CHAPTER 1

Scatterplots and Regression

Regression is the study of dependence. It is used to answer interesting questions about how one or more predictors influence a response. Here are a few typical questions that may be answered using regression:

- Are daughters taller than their mothers?
- Does changing class size affect success of students?
- Can we predict the time of the next eruption of Old Faithful Geyser from the length of the most recent eruption?
- Do changes in diet result in changes in cholesterol level, and if so, do the results depend on other characteristics such as age, sex, and amount of exercise?
- Do countries with higher per person income have lower birth rates than countries with lower income?
- Are highway design characteristics associated with highway accident rates? Can accident rates be lowered by changing design characteristics?
- Is water usage increasing over time?
- Do conservation easements on agricultural property lower land value?

In most of this book, we study the important instance of regression methodology called *linear regression*. This method is the most commonly used in regression, and virtually all other regression methods build upon an understanding of how linear regression works.

As with most statistical analyses, the goal of regression is to summarize observed data as simply, usefully, and elegantly as possible. A theory may be available in some problems that specifies how the response varies as the values

Applied Linear Regression, Fourth Edition. Sanford Weisberg.