INTRODUCTORY
Algebra

Richelle M. Blair

Introductory Algebra

Richelle M. (Rikki) Blair
Lakeland Community College

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Preface

From the Author

Students enter Introductory Algebra with a variety of mathematics experiences, some positive and some negative. The goal of this textbook is to engage all students and develop their algebraic thinking and confidence in learning mathematics. Standards-based and traditional content have been blended by actively involving students in real-world problem–solving, reasoning, and skill development. The instructional strategies in this textbook are supported by research on the science of learning and teaching mathematics with the goal of addressing multiple learning styles of students and different teaching styles of faculty. Content and instructional strategies have been chosen based on what we know about how students best learn and how faculty best teach.

Focus on Student Learning

Research shows that developing an understanding of the concepts of equivalence and variable are key to students developing an understanding of algebraic systems. In this textbook, students will review arithmetic as needed and develop the following:

- An understanding of equivalence
- The ability to use variables in a variety of ways
- Problem-solving skills with real-world problems
- Confidence in solving mathematics problem situations
- Quantitative literacy and algebraic skills to be applied in other courses

Recent developments in the science of learning emphasize the importance of students being involved and taking control of their own learning. Active student learning activities have been shown to increase the degree to which students transfer learning to new concepts.[1] Learning new concepts involves connecting previously learned information to new information, reflecting, and transferring that knowledge to new problem situations. Each section of this textbook presents student activities to create an active learning environment and build cognitive connections. In addition, students are asked to do the following:

- Read, write, explore, discuss, reflect, and do mathematics as active learners
- Complete true/false, matching, and reasoning questions
- Analyze data and solve real-world problems
- Create 3 × 5-inch study cards of important definitions, formulas, and examples
- Complete a mathematics dictionary

Focus on Standards-Based Teaching

An instructor using this textbook is seen not only as an expert mathematician and problem solver, but also as a facilitator and manager of student learning in an active classroom, employing a variety of instructional activities. This textbook embraces the

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[1]Bransford, Brown, and Cocking (Eds.). (2000). *How People Learn—Brain, Mind, Experience and School*. National Academy Press, Washington, D.C.

standards of the American Mathematical Association of Two-Year Colleges,[2] the National Council of Teachers of Mathematics,[3] and the Mathematical Association of America[4] by employing a variety of standards-based strategies:

- Activities for students to work (learn) together
- Activities for interactive lecturing
- Step-by-step examples
- Organizing, analyzing, and graphing real-world data
- Connecting mathematics to other disciplines
- Optional problems using a graphing calculator to explore, visualize, observe patterns, and generalize and confirm solutions, if desired

Organization and Features of the Textbook

Each section of each chapter is organized into three sub-sections: *Getting Started*, *Discussing Mathematics*, and *Doing Mathematics*. The innovative keystone feature of the textbook is the student activities in the Getting Started section. These activities and problems are designed to help the student review previously learned concepts and connect that knowledge to new concepts in the section. The Getting Started section can be used in a variety of ways, depending on the desired approach chosen by the instructor. Getting Started is followed by Discussing Mathematics, an explanation of concepts and examples, and Doing Mathematics, a selection of practice problems. The entire textbook is written in a comfortable and welcoming tone and style that students can read on their own.
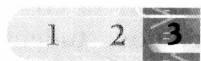
Using the textbook is as easy as 1-2-3:
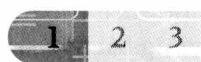
Getting Started presents student activities to be used as classroom group work, lecture discussions, or assigned as homework for the next day.

Discussing Mathematics presents definitions, examples of key concepts, and helpful hints.

Doing Mathematics presents practice problems.

Supplements for the Instructor

INSTRUCTOR'S SOLUTIONS MANUAL

(ISBN 0-321-27957-3) This supplement contains complete, worked-out solutions to all of the exercises in the text.

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[2]AMATYC. (1995). *Crossroads in Mathematics: Standards for Introductory College Mathematics Before Calculus*. Memphis, TN: AMATYC. These standards are in the process of being revised and are to be released in November 2006.
[3]NCTM. (2000). *Principles and Standards for School Mathematics*. Reston, VA: NCTM.
[4]MAA. (2004). *Undergraduate Programs and Courses in the Mathematical Sciences: CUPM Curriculum Guide 2004*. Washington, DC: MAA.

PRINTED TEST BANK/INSTRUCTOR'S RESOURCE GUIDE

(ISBN 0-321-27958-1) This supplement contains several tests per chapter, teaching notes correlated to each chapter, and a variety of other helpful resources for this text.

TESTGEN WITH QUIZMASTER

(ISBN 0-321-27959-X) TestGen enables instructors to build, edit, print, and administer tests. Featuring a computerized bank of questions developed to cover all text objectives, TestGen is available on a dual-platform Windows/Macintosh CD-ROM.

Supplements for the Student

STUDENT'S SOLUTIONS MANUAL

(0-321-39954-4) This supplement contains complete, worked-out solutions to the odd-numbered exercises in the text.

ADDISON-WESLEY MATH TUTOR CENTER

The Addison-Wesley Math Tutor Center is staffed by qualified mathematics instructors who provide students with tutoring on examples and odd-numbered exercises from the textbook. Tutoring is available via toll-free telephone, toll-free fax, e-mail, or the Internet. White Board technology allows tutors and students to actually see problems worked while they "talk" in real time over the Internet during tutoring sessions. For more information, go to www.aw-bc.com/tutorcenter.

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Arithmetic Review Skills Test

This test is intended to assess skills at the beginning of the course.
Do not use a calculator on this test.

Round the given number to the indicated place-value. (R. 1)

1. Round to the nearest ones place: 549.89

    (a) 549   (b) 550   (c) 549.9   (d) 500

2. Round to the nearest tenths place: 150.523

    (a) 150   (b) 150.5   (c) 150.52   (d) 200

Find the numerator of the fraction to make the two fractions equivalent. (R. 1)

3. $\dfrac{2}{5} = \dfrac{}{30}$

    (a) 12   (b) 4   (c) 6   (d) 2

Rewrite the mixed number as an improper fraction. (R. 1)

4. $4\dfrac{1}{5}$

    (a) $\dfrac{11}{5}$ (b) $\dfrac{21}{5}$ (c) $\dfrac{21}{4}$ (d) $\dfrac{11}{4}$

Rewrite the improper fraction as a mixed number. (R. 1)

5. $\dfrac{43}{7}$

    (a) $6\dfrac{1}{7}$ (b) $7\dfrac{6}{7}$ (c) $5\dfrac{6}{7}$ (d) $7\dfrac{1}{7}$

Reduce the fraction to lowest terms. (R. 1)

6. $\dfrac{18}{48}$

    (a) $\dfrac{6}{16}$ (b) $\dfrac{1}{4}$ (c) $\dfrac{3}{8}$ (d) $\dfrac{9}{24}$

Determine the number that is closest to the exact arithmetic answer by estimating. (R. 1)

7. Is 10.11×9 closer to 90 or closer to 900?

    (a) 90   (b) 900

8. Is $\dfrac{35.7}{5}$ closer to 7 or closer to 0.7?

    (a) 7   (b) 0.7

Multiply or divide the fractions and reduce to lowest terms. (R.2)

9. $\dfrac{28}{3} \cdot \dfrac{15}{7}$

   (a) $\dfrac{420}{21}$ (b) $\dfrac{43}{10}$ (c) 20   (d) 22

10. $1\dfrac{4}{7} \cdot 2\dfrac{3}{4}$

   (a) $\dfrac{32}{11}$ (b) 2   (c) $4\dfrac{1}{3}$ (d) $\dfrac{121}{28}$

11. $\dfrac{15}{7} \div \dfrac{27}{35}$

   (a) $\dfrac{50}{34}$ (b) $\dfrac{525}{189}$ (c) $\dfrac{405}{245}$ (d) $\dfrac{25}{9}$

12. $1\dfrac{4}{7} \div 5\dfrac{2}{3}$

   (a) $\dfrac{33}{68}$ (b) $\dfrac{187}{21}$ (c) $\dfrac{33}{119}$ (d) $\dfrac{33}{24}$

Add or subtract the fractions and reduce to lowest terms. Write answers that are improper fractions as mixed numbers. (R.2)

13. $\dfrac{7}{5} + \dfrac{3}{5}$

   (a) $\dfrac{21}{25}$ (b) 2   (c) $2\dfrac{1}{3}$ (d) 1

14. $\dfrac{8}{5} + \dfrac{2}{7}$

   (a) $\dfrac{10}{12}$ (b) $\dfrac{5}{6}$ (c) $1\dfrac{31}{35}$ (d) $\dfrac{10}{35}$

15. $\dfrac{11}{8} - \dfrac{2}{5}$

   (a) $\dfrac{9}{3} = 3$ (b) $3\dfrac{1}{40}$ (c) $\dfrac{39}{40}$ (d) $1\dfrac{1}{40}$

Perform the operations and reduce to lowest terms. Follow Order of Operations. Write answers that are improper fractions as mixed numbers. (R.2)

16. $\dfrac{17}{2} - \dfrac{7}{2} \cdot \dfrac{4}{3}$

   (a) $-2\dfrac{3}{4}$ (b) $3\dfrac{5}{6}$ (c) $6\dfrac{2}{3}$ (d) $13\dfrac{1}{3}$

17. $\left(\dfrac{11}{2} - \dfrac{7}{2}\right) \cdot \dfrac{4}{3}$

    (a) $5\dfrac{1}{3}$ (b) $1\dfrac{1}{3}$ (c) $2\dfrac{2}{3}$ (d) $6\dfrac{2}{3}$

Perform the operations. (R.3)

18. $142.56 + 35.47$

    (a) 1780.3   (b) 17.803   (c) 1.7803   (d) 178.03

19. $142.56 - 35.47$

    (a) 107.09   (b) 10,709   (c) 1070.9   (d) 0.10709

20. 142.56×35.47

    (a) 50.56603   (b) 5056.6032   (c) 50566.03   (d) 0.5056

21. $46.08 \div 4.5 \times 15.45$

    (a) 1.58208   (b) 15.8208   (c) 158.208   (d) 1582.08

Perform the operations. (R.3)

22. What is 40% of 250?

    (a) 1   (b) 10   (c) 100   (d) 1000

23. 20 is what percent of 400?

    (a) 1%   (b) 5%   (c) 50%   (d) 5.5%

Solve the following proportions. (R.4)

24. $\dfrac{3}{5} = \dfrac{x}{20}$

    (a) $x = 60$ (b) $x = 6$ (c) $x = 4$ (d) $x = 12$

25. $\dfrac{x}{32} = \dfrac{3}{4}$

    (a) $x = 8$ (b) $x = 12$ (c) $x = 24$ (d) $x = 4$

Convert the given unit of measurement to the unit of measurement specified using the equivalencies or approximate equivalencies below. Round to the nearest hundredth. (R.4)

    1 yard (yd)   = 3 feet (ft)
    1.09 yard (yd) $\approx$ 1 meter (m)
    1 mile $\approx$ 1.61 kilometers (km)

26. Convert 100 kilometers = _____ miles

    (a) 6.211 miles   (b) 62.11 miles   (c) 1.61 miles   (d) 16.1 miles

27. Convert 4.6 meters = _____ yards

(a) 5.01 yards   (b) 50.1 yards   (c) 4.6 yards   (d) 1.09 yards

28. Write 2^3 in expanded and simplified (standard) form. (R.5)

(a) $2 \times 3 = 6$ (b) $2 \times 2 \times 2 = 8$ (c) $3 \times 3 = 9$ (d) $3 \times 2 \times 2 = 12$

29. Convert 1.02×10^{-2} from scientific notation to standard form. (R.5)

(a) 0.00102   (b) 0.00102   (c) 0.0102   (d) 0.102

30. Convert 2,530,000 from standard form to scientific notation. (R.5)

(a) 25.3×10^5 (b) 2.53×10^5 (c) 25.3×10^6 (d) 2.53×10^6

Reflection: Which four problems on this Skills Test did you find the most difficult? Review the sections in Chapter R related to those difficult problems.

Contents

Symbols, Real Numbers, and Problem Solving

> *The equal symbol was first used by Robert Recorde (1510–1558). A translation of what he wrote: I will set a pair of parallel lines (as the equal sign) because no two things can be more equal.*

1.1 Symbols and Notations

OBJECTIVES

- State multiple meanings of mathematics symbols.
- Express relationships between real numbers.
- Use objects as placeholders for real numbers.

1 2 3 Getting Started

In this section, we explore the meaning of various mathematics symbols, beginning with the = symbol. In transitioning from arithmetic to algebra, it should be recognized that the equal symbol has two interpretations.

- The equal symbol can represent an instruction to "get an answer": $6 + 2 = ?$.
- The equal symbol can communicate that each side of the equation represents the same quantity and that both sides are equivalent: $2 + 5 - 1 = 3 \cdot 2$.

A mathematical statement that includes the equal symbol, =, is called an equation. We will call everything to the left of the equal symbol the "left-hand side" of the equation and everything to the right of the equal symbol the "right-hand side" of the equation.

1. Perform the indicated operations. The equal symbol in these problems is an instruction to "get an answer."

(a) $3 + 4 =$ _____

(b) $\dfrac{11}{5} - \dfrac{4}{5} =$ _____

(c) $7 \times 3 =$ _____

(d) $14 \div 2 =$ _____

2. To explore the concept of equivalency, think about how many ways $1.00 may be obtained with different combinations of coins. List two combinations of pennies, nickels, dimes, and quarters that total $1.00 in the table. Multiply the number of coins by the value of each coin. Add the total value of each type of coin and write an equation showing your total as $1.00. It is not necessary to use each type of coin in every combination.

	Pennies	Nickels	Dimes	Quarters	Write an equation representing the TOTAL of $1.00
EXAMPLE	$5 \times \$0.01$	$3 \times \$0.05$	$3 \times \$0.10$	$2 \times \$0.25$	$5 \times \$0.01 + 3 \times \$0.05 + 3 \times \$0.10 + 2 \times \$0.25 = \$1.00$
(a)					
(b)					

3. The = symbol is used to indicate that two quantities are "equivalent." The ≠ symbol is used between two expressions that are "not equivalent." Complete each statement by writing the appropriate symbol, either = or ≠, on the line between the two quantities.

(a) $15 + 15 + 4$ _____ 9×4 (b) $3\dfrac{1}{2}$ _____ 3.5

(c) $50 \div 5$ _____ $7 + 5$ (d) $\dfrac{1}{2}$ _____ $\dfrac{4}{8}$

4. A balanced scale is a way to visualize equivalency between quantities. When the quantity on the left side is equivalent to the quantity on the right side, the scale is *balanced*. Fill in the box in each problem with the number that will keep the scale balanced.

(a) $14 + 3$ ___ □   (b) □ ___ 10×5
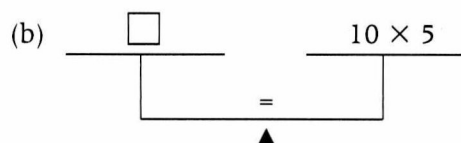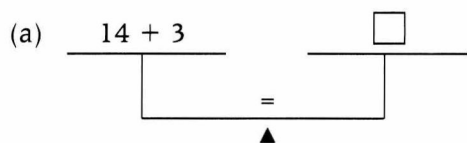
The set of real numbers can be visualized on the real number line. Positive numbers are written to the right of zero and negative numbers to the left of zero. The arrows on each end of the real number line indicate that the line continues forever.

$$-\infty \quad -4 \quad -3 \quad -2 \quad -1 \quad 0 \quad 1 \quad 2 \quad 3 \quad 4 \quad +\infty$$
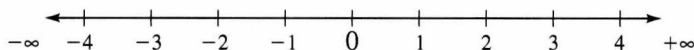
When a number is *negative*, the − symbol is always written. When a number is *positive*, the + symbol may or may not be written. The farther a number is to the right on the number line, the larger the number. The farther a number is to the left on the number line, the smaller the number.

5. Some mathematics symbols have multiple meanings. The $+$ symbol can be an instruction to *add* or it may denote that a number is *greater than zero* (a positive number). Write (A) if the $+$ symbol is an instruction to add. Write (P) if the $+$ symbol denotes a positive number.

(a) $2 + 4$ _____

(b) $10 - (+3)$ _____

(c) $+12 - 4$ _____

(d) $1 + 3$ _____

6. The $-$ symbol can be an instruction to *subtract* or it may denote that a number is *less than zero* (a negative number). Write (S) if the $-$ symbol is an instruction to subtract. Write (N) if the $-$ symbol denotes a negative number.

(a) $200 + (-6)$ _____

(b) $7 - 3$ _____

(c) $(-2) + 6$ _____

(d) $15 - 8$ _____

7. The $>$ and $<$ symbols are also used to express relationships between quantities. The $>$ symbol means "greater than" and the $<$ symbol means "less than." Complete each statement by writing the appropriate symbol, either $<$ or $>$, on the line between the two quantities.

(a) 10 _____ $5 + 3$

(b) $\dfrac{1}{2}$ _____ 8

(c) -3 _____ -6

(d) 0 _____ -1

8. Multiple symbols exist to denote multiplication and division. Write whether the indicated operation in each problem is multiplication (M) or division (D).

(a) 10×5 _____

(b) $(10)(5)$ _____

(c) $5\overline{)10}$ _____

(d) $10 \div 5$ _____

(e) $10/5$ or $\dfrac{10}{5}$ _____

(f) $10 \cdot 5$ _____

9. Other mathematics symbols and notations have specific meanings and act as a kind of shorthand notation. Numbers or operations are understood but not always written explicitly.

(a) Does the number $5\dfrac{3}{4}$ mean $5 + \dfrac{3}{4}$ or $5 \times \dfrac{3}{4}$?

(b) An **exponent** is a symbol that denotes repeated multiplication. The expression 3^4 is read "3 to the fourth power." The exponent, 4, is an instruction to multiply the **base**, 3, four times:

$$3^4 = \underbrace{3 \cdot 3 \cdot 3 \cdot 3}_{4 \text{ times}} = 81.$$

Rewrite the expression 5^3 without an exponent.