

# MATHEMATICS

A GOOD BEGINNING

FIFTH EDITION



TROUTMAN  
LICHTENBERG

# MATHEMATICS

## A Good Beginning

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**MATHEMATICS**  
**A Good Beginning**

# *Strategies for Teaching Children*

Fifth Edition



# MGB

## Computer Programs

*This is a description of the MGB programs that come with this book. Macintosh and MS-DOS versions are included to provide you with the greatest flexibility.*



### THE PROBLEM-SOLVING TOOLKIT

This is a database manager that creates and manages databases of word problems and puzzles. Over 100 problems are ready for use, and a special component of the database manager designs and prints unique problem-solving worksheets. The number of databases that can be created and the number of problems in each is limited only by the memory constraints of the user's computer storage. (See Chapter 0, page 21, Chapter 11, pages 388–389, and Chapter 18, pages 544–546.)

### COUNT

This is a colorful early childhood program that teaches number-set-numeral associations and rational counting. (See Chapter 2, pages 70–71.)

### TENS AND ONES; OVER-TEN

This program provides two games. The first game simulates a place-value machine that allows children to build tens and ones models for numbers between 10 and 99. The second game simulates a machine that starts with two numbers between 5 and 10, and with the child's help builds numbers greater than 10. The object is to build place-value readiness and number sense related to learning addition facts greater than 10. (See Chapter 3, page 106.)

### MASTER

Like the TENS and ONES and OVER-TEN programs, this program allows the user to print out an

unlimited number of worksheets containing exercises that rely on place-value models and number-set models. The user can tailor the directions for each worksheet to needs of particular students. (See Chapter 4, page 148, and Chapter 18, page 550.)

### THE ESTIMATION CHALLENGE

This program has four levels of difficulty and sharpens the child's ability to estimate with whole numbers and decimals in computational situations as well as in real-life problem situations. Each level is implemented as a game in which each item is timed, and scoring is based on the number of seconds it takes for the child to respond. High scores for different players are stored, and progress for an individual player is tracked for use in printed progress reports. (See Chapter 5, pages 177–178, and Chapter 10, page 355.)

### ARRAY

This colorful program involves a multiplication machine that requires children to use rectangular arrays to complete multiplication exercises. Children must find the number of rows and the number in each row before they can complete a multiplication sentence. This program is very useful for teaching operation sense for multiplication and builds readiness for accommodating concepts that involve multiplication, such as area and probability. (See Chapter 6, pages 215–216, and Chapter 18, pages 538–540.)

### MULTIPLES

This program helps children develop rational strategies for multiplying and dividing multiples of tens, hundreds, thousands, etc. It is implemented in a game format in which the player tries to beat his or her own score. (See Chapter 7, pages 240–241.)

## **GENIE**

The GENIE who lives in this program has a function machine that manipulates numbers. The child must use the genie's rules in order to guess the numbers and win tokens and prizes. This is a good program to prepare children for number-sentence concepts and associated algebraic concepts. The game has several levels from easy to more difficult as the child progresses. (See Chapter 8, page 276, and Chapter 18, page 541.)

## **PARTS**

This is another game program that requires the child to associate decimal fractions and fractions with a tenths and hundredths model. The player is given a card that will hold 100 heart-shaped stickers. His or her job is to put the closest number of heart stickers on the card without going over the given number. This program is very useful for building number sense for rational numbers. (See Chapter 9, pages 317–318.)

## **SHOPPING MALL**

This program is a simulation in which children act as cashiers in a store. The purchaser selects an item to buy, and the cashier must select the appropriate coins to make change the most efficient way possible. Available coin types vary, so the cashier must use various strategies in order to find the most efficient way to make change. (See Chapter 16, pages 500–501, and Chapter 18, pages 541–542.)

## **HIGHLOW**

This is a game of chance in which players must guess the position of a number in a sequence based on what they know about whole numbers and probability. The game has two levels of play. In one level, the player is allowed to choose from among three daffy computer opponents, and he or she must outwit the opponent to win the game. In the other level, the player plays alone. High scores are kept, and the player's progress is tracked and can be printed. This is a good program for sharpening probability concepts. (See Chapter 17, pages 525–526.)

## **SPREADSHEET SIMULATOR**

This is a simple program that simulates a spreadsheet program. It allows children to enter names for items and numbers for quantities to solve complex problems involving quantities of data. Several templates are provided to illustrate its use. Example problems are also provided that require children to use thoughtful strategies to build the spreadsheets. (See Chapter 18, pages 546–548.)

## **CERTIFICATE**

This program allows the user to print mathematics stories or certificates. The user can choose from among nine borders that range from sophisticated to amusing. The user can tailor either the mathematics story or the certificate as desired. (See Chapter 18, pages 550–551, and Chapter 20, page 611.)

# Preface

This fifth edition of *Mathematics: A Good Beginning* is a major refinement and extension of previous editions, but it maintains the original focus and character. The fifth edition includes many new features, chapters, and topics. It amplifies the philosophy and recommendations of the National Council of Teachers of Mathematics (NCTM) and is closely aligned with the current growing movement to restructure the way mathematics is taught in schools. This edition is solidly grounded in the research on how children learn mathematics and how teachers develop attitudes, beliefs, and knowledge that are prerequisite to successful teaching. Retained components of the previous edition have been completely rewritten to provide greater depth and clarity. In essence, we think of this new edition as something far more than a textbook. We have made every attempt to provide an instructional program that will serve a multitude of curriculum needs that spring from a variety of circumstances.

Teaching in today's world is tremendously complex. Any successful program must go beyond the limits of mathematics pedagogy and content to reflect the real world in which teaching takes place. New features are included in this edition that will enable the teacher to consider the learning of mathematics within many contexts. For example, how does a multicultural environment affect the teaching process? Within this process, how do we reward the learning of mathematics? How can interdisciplinary themes and cooperative learning help children develop abilities that will enhance future employment opportunities?

Another solid feature of this book is the attention that is given to technological trends. We have gone the distance by providing substantial stand-alone computer programs that come packaged with the book—both Macintosh and MS-DOS versions. These programs will not only enhance the development of teaching skills but will also provide familiar computing tools that can be directly exported into the classroom.

An important point: Life is not intended to be simply functional. The mathematics environment in

which children learn should reflect their creative, cultural, and esthetic needs. So another important feature of this program is to reflect the artistic and literary world of children. This book is devoted to the nurturing of happy creative students, no matter what their ages.

## New Chapters

Several new chapters have been included. Chapter 0, "Problem-Solving: A Way of Life," is an entirely new chapter devoted to problem-solving. It presents a six-step model that identifies many strategies for engaging in mathematical inquiry at each step. The intent is to help readers acquire problem-solving abilities and problem-solving confidence they may not have gained in their prior education. Elements from this chapter are integrated throughout the book in discussions, activities for children, and special sections entitled "The Problem-Solving Connection," and "A Problem-Solving Challenge for You." The goal is to create an environment in which teachers learn to present mathematical ideas in the context of stimulating mathematical inquiry. A special Problem-Solving Book Marker (Material Sheet 1) summarizes the problem-solving model and is included in Appendix C. Readers can remove it and use it as a study aid.

"Planning Learning Experiences: From the Child's Perspective" is another new chapter that replaces Chapter 0 found in earlier editions. Now Chapter 1, this chapter builds a model for mathematics instruction based on constructivism and a developmental view of learning. Here, we demonstrate strategies for preparing and sequencing learning experiences that enable children to construct mathematical concepts while establishing confidence and positive attitudes. A chart that summarizes elements of the model is provided in the chapter text and also as Material Sheet 3 (Appendix C). The chart can be used throughout the book to create original activities for children, to evaluate or expand activities provided in the text, and to evaluate and rewrite activities obtained from other sources.



The original chapter on problem solving, Chapter 10, has been replaced by Chapter 11, “Believe, Think, Then Solve: Building Problem-Solving Environments for Children.” In this new chapter, techniques are characterized that enable teachers to build a problem-solving base for mathematical instruction. A model is given that not only emphasizes the major steps in the problem-solving process, but also demonstrates how to teach specific mathematical thinking skills and encourage the development of meta-cognitive skills. We admit that problem-solving can be done alone, but we underscore that it is much more exciting when done with others. We must always keep in mind that mathematical inquiry is not solely for the gifted but must be a rudimentary component of education for all children. In this chapter, many concepts set forth in the NCTM recommendations are thoroughly illustrated—mathematical inquiry, mathematical reasoning, mathematics as communication, and mathematical patterns.

Chapter 19, “Encouraging Student Growth: Assessment and Diagnosis,” represents a major update of the diagnostic chapter in previous editions. This chapter encompasses in-depth coverage of alternate methods of assessment as it relates to diagnosing, preventing, and remediating mathematical difficulties.

## New and Expanded Features

### MiniChapters

American society is complex, and teaching involves far more than knowing how to teach a particular subject area. An effective teacher in this society must deal with an overwhelming number of issues and conditions that go far beyond the scope of mathematics and the conception of the traditional classroom. To help you face this reality, we include eight MiniChapters. These short chapters (only 2 or 3 pages) deal with important topics or issues of general importance that should not get buried in, or overshadowed by, the main chapters in the book. They express sophisticated ideas that have specific relevance for teaching mathematics at any level, and include rich bibliographies that can be used to achieve a variety of professional goals. The topics included are constructivism; cooperative learning and effective interaction; meta-cognition and mathematics; using commercial manipulatives; mathematics as an interdisciplinary theme; multiculturalism and mathematics; teaching children of poverty; and mathematics, equity, and gender. These MiniChapters are intended to aid both the instructors of this course as well as the prospective teachers.

### Recurring Topics and Themes

Several new or expanded recurring themes are included in this edition. These topics give continuity from chapter to chapter and provide many opportunities to consider ideas and concepts in different contexts.

***Problem-Solving Challenges for You.*** A section entitled “A Problem-Solving Challenge for You” is included in almost every chapter of the book. We believe that elementary and middle school teachers can’t be expected to teach something they don’t appreciate and can’t do themselves, so these sections include problems for the prospective or in-service teacher to solve. Specific problem-solving strategies are suggested, and cooperative work is encouraged. These sections, along with many other elements, reinforce and extend ideas introduced in Chapter 0, “Problem-Solving: A Way of Life.”

***What NCTM Says.*** A discussion of the NCTM Standards and how they are reflected in the content of the given chapter is presented at the beginning of each chapter where relevant. We believe that prospective teachers should become immediately aware of our professional organizations, prevailing philosophy for mathematics instruction, and the current movement that is devoted to restructuring the mathematics curriculum for elementary and middle school. Hopefully, this will enable prospective and in-service teachers to shed outmoded conceptions of teaching and learning that they may have acquired from practices observed in their own education.

***Teachers in the Real World.*** Many veteran teachers in the field have several years of experience and have earned the admiration and respect of their supervisors, peers, and students. Each of these teachers could write their own book on how to teach. They are the real researchers. So, in this edition, we celebrate these unsung artists, their knowledge, and their expertise in sections entitled “Teachers in the Real World.” In these sections, we feature practicing teachers and give their special observations and teaching conclusions. Their candid remarks provide insight into what happens when the doors of real classrooms open and close. For prospective teachers, the integration of conclusions of practicing teachers into their college coursework provides authenticity and credibility.

***Ideas for the Middle School.*** Basic materials in the text have been extended to cover topics and methods suitable for the middle school. Instead of writing a chapter on middle school mathematics, we include a new section, “Ideas for the Middle

School,” in most chapters. This enables the prospective or in-service teacher to view the scope of things and to construct a view of where children are going and where they should have been. This feature expands NCTM recommendations for grades 5–8.

**Guides for Preventative Teaching.** At the end of most chapters, we have provided a chart identifying major concepts that children should learn and the difficulties they encounter when doing so. The information in these charts will facilitate planning learning experiences that prevent mathematical difficulties and will help teachers recognize individual student’s difficulties on the spot.

**Calculator Activities.** Many stimulating calculator activities have been included in almost every chapter. These activities build around the main philosophy of the book: That children should discover and construct their own knowledge and teachers should orchestrate. These activities amplify the recommendations of the NCTM.

**Computer Corners.** In most chapters, there is a section entitled “The Computer Corner.” Descriptions of newly designed computer programs that accompany the book are provided. Ideas for successfully integrating computing into teaching are also described. In each instance, the descriptions illustrate computing concepts that have particular relevance to learning materials provided in the chapter. These sections not only discuss how computing can be integrated into the mathematics curriculum, but also illustrate tasks that computers can accomplish more effectively than teachers acting alone. All the computer programs described are included with the book and can be copied for instructional needs of those who own the book. This software represents an extremely valuable resource for prospective teachers. (See the list of MGB Computer Programs, page xiv.)

### New Activities

Many activities for children have been rewritten and many new activities have been added. Some of the activities are classified by theme so that it is easy to translate their use to the prospective teacher’s classroom. For example, the *Problem of the Day* is a special theme that is revisited throughout the book. This theme illustrates a way to integrate problem-solving into daily teaching. Other themes suggest the building of learning centers, corners, or bulletin boards. The main focus of providing the abundance of activities is not to furnish a bag full of tricks or gimmicks. Our intention is to mirror appropriate models for translating theory into practice

and to give specific examples of how to communicate effectively with children at their level. All the activities in this book have been fielded with children and revised based on related outcomes.

### New Material Sheets

Many new material sheets have been included in Appendix C to augment the activities. These material sheets provide prospective teachers with good models from which to learn mathematics and also provide valuable resources for teaching. In many cases, we favor these materials over commercial materials because they can be customized to fit specific needs, they carry the stamp of originality, they are inexpensive, and they can be made almost immediately. Teacher-made aids are not static, they can be sculpted to fit the imaginations and needs of the specific moment. The material sheets along with the activities in the book demonstrate how teachers can become resourceful and creative when building activities for children. Adults, like children, learn by doing. They, too, must construct knowledge. By designing, building, altering, and extending learning materials, they build ownership and a strong incentive to use manipulatives. By wrestling with the connections between content and pedagogy, they build strong cognitive bases from which to model mathematical ideas. One of the most important tasks that faces teachers daily is finding activities that will challenge children. Most will also agree that they spend a lot of time searching for ideas and creating such activities. We think teachers should be provided with good models, should begin to develop related skills, and should begin to collect resources before leaving college. After years of observation and participation, we conclude that it is the norm, not the exception, that commercial materials are generally not in sufficient supply for all learners in a school environment. It is the responsibility of teacher-training programs to provide viable alternatives that encourage the stamp of originality.

### New Practice Exercises

The ThinkTank exercises included at the end of each chapter have been significantly expanded. There are now more types of exercises and more exercises of a given type. The exercises have also been categorized, so that instructors or students can pick and choose exercises to suit their study needs. Some of the many special categories of exercises are as follows: extending mathematical tools, preparing for better teaching, diagnosing difficulties, forming good study habits, creating original *problems*, aligning with NCTM, explaining mathematical ideas, making mathematical connections, making educational decisions, and so on. The objectives of these

exercises are not just to review and memorize the chapter's content, but to organize, extend, and transfer learning in productive ways.

## Ancillaries

### Computer Programs

A collection of completely rewritten software programs that illustrate concepts covered in the text is supplied with this book. These programs reinforce effective practices for using computers in the classroom. They illustrate how teachers can integrate computers into the instructional environment by using computers to direct, enhance, and manage aspects of instruction. Teaching concepts, promoting problem-solving, developing estimation skills, preparing teaching masters and transparencies are just a few of the topics covered. These programs have been developed for both the Macintosh and MS-DOS computer platforms. They come packaged with the book and may be copied for an instructor's or prospective teacher's professional use. Complete program documentation is included on the disks. Descriptions of the programs can be found in the list of MGB Computer Programs on page xiv.

### Instructor's Resource Book

This book is available to instructors who adopt the text. It contains:

- ✓ Assessment items of many types
- ✓ Transparency masters for each chapter
- ✓ Suggested outlines for course delivery that have different focuses

- ✓ A list of sources for public domain and shareware software
- ✓ An outline of how to obtain software and other resources using Internet, a telecommunication utility available to educators
- ✓ A set of mathematics discovery (laboratory) activities for elementary and middle school teachers that extend strategies covered in the text (Each activity is designed to provide a single 1–2 hour lab session in which prospective and in-service teachers complete hands-on experiences related to concepts that they might teach.)

## Helpful Appendixes

**Appendix A.** This appendix provides selected answers to many of the exercises in the ThinkTank exercises to facilitate self-evaluation of progress.

**Appendix B.** In this appendix, we provide a list of the NCTM Standards indicating concepts and skills to receive emphasis and those to receive less emphasis than in prior years. These Standards are referred to in the sections entitled “What NCTM Says” at the beginning of most chapters.

**Appendix C.** This appendix provides the material sheets that go with the activities in this book. These are black-line masters that can be used to make manipulative materials for learning mathematical concepts and reinforcing mathematical skills.

## Annotated Table of Contents

Chapter 0 Problem-Solving: <i>A Way of Life</i>	This chapter is devoted to prospective or in-service teachers. The chapter provides a problem-solving excursion that sets the mood for the rest of the book. We go back in time to explore some problem types that prospective teachers should have encountered in their pre-college school experiences.
MiniChapter 1 Constructivism: <i>A View of Learning</i>	The constructivist view of learning is described, and a list of implications for teaching mathematics is provided.
Chapter 1 Planning Learning Experiences: <i>From the Child's Perspective</i>	In this chapter the focus is on learning. We maintain that teachers can guide, aid, or be a partner in learning; however, they can't dictate learning, no matter how good they are at showing and telling. We examine how certain beliefs and attitudes the prospective teacher may have can impact the way in which instruction is delivered and how learning is affected. We demonstrate a learning and teaching model based on the premise that children learn mathematics by seeing and doing, by constructing knowledge from the inside, and by incorporating new information with schemata that already exist.  (Annotated Table of Contents continues)



## Annotated Table of Contents (continued)

<p>Chapter 2  <b>Getting Ready for a Good Beginning:  <i>Learning Pre-Number Concepts</i></b></p>	<p>This revised chapter is about children between ages 4 and 7 and their development of number ideas. The content of the chapter is based on the premise that children must experience a lot of seeing, doing, and saying before reading and writing numerals will have meaning for them.</p>
<p><i>MiniChapter 2</i>  <b>Cooperative Learning:  Effective Interaction</b></p>	<p>In this MiniChapter, we examine cooperative learning and discuss positive outcomes that can result from this type of instructional grouping. We provide guidelines for establishing cooperative learning groups and describe roles that different members of a group may assume.</p>
<p>Chapter 3  100s, 10s, 1s, . . . The Best Yet:  <b><i>Our Base-Ten Numeration System</i></b></p>	<p>The way we represent numbers is examined analytically so that a teacher can develop ideas for relating these concepts to children. We emphasize the consistent use of aids, models, stories, or explanations from one learning experience to another. We demonstrate creative story-telling techniques that are used to connect a set of related concepts. The logical extension to decimal fractions is examined also. Activities for problem-solving and special topics are recommended for the middle school.</p>
<p><i>MiniChapter 3</i>  <b>Meta-Cognition and Mathematics</b></p>	<p>In this MiniChapter, we explore some cognitive techniques that can be used to help children organize, reflect upon, and control mental processes. Purposeful use of these techniques can facilitate the ability to systematically approach problem-solving and mathematical justification.</p>
<p>Chapter 4  <b>Addition and Subtraction of  Whole Numbers:  <i>Constructing Meaning</i></b></p>	<p>Earlier versions of this book contained one chapter on addition and subtraction. We have completely revised that chapter and broken it into two parts, Chapters 4 and 5, in order to demonstrate stronger relationships between addition and subtraction and to provide more extensive work in problem-solving, using calculators, and mental arithmetic. In this chapter, we emphasize learning experiences that enable children to construct meaning for addition and subtraction. We stress the relationship that exists between addition and subtraction and the importance of using number sentences (equations) to communicate mathematical ideas. We introduce the use of the calculator to solve problems with numbers greater than 10, even though computation is explored in Chapter 5. Activities for mental arithmetic and number puzzles are demonstrated.</p>
<p><i>MiniChapter 4</i>  <b>Using Commercial Manipulatives</b></p>	<p>In this MiniChapter, we describe many types of commercial manipulatives and provide sources for obtaining them. We discuss the relationship between teacher-made materials and commercial materials and indicate ways to use both types of materials effectively.</p>
<p>Chapter 5  <b>Addition and Subtraction  Algorithms for Whole Numbers:  <i>Building, Understanding,  Applying, and Estimating</i></b></p>	<p>In this revised chapter, we extend ideas presented in Chapter 4 to meaningful procedures for computing and estimating with addition and subtraction of whole numbers. We demonstrate strategies for helping children to develop operation sense and to construct algorithms for each operation. We offer models and stories that organize concepts they have previously learned, and extend these concepts to new understandings. The development of computational procedures is extended through the development of estimation procedures. Throughout the chapter, we offer high-interest activities that stimulate mental reflection, mathematical thinking, and problem-solving. We encourage the use of calculators and computers.</p>
<p><i>MiniChapter 5</i>  <b>Mathematics as an Interdisciplinary  Theme</b></p>	<p>In this MiniChapter, we discuss the benefits of interdisciplinary learning and provide many ideas for getting started. Specific ideas from science, sports, art, health, history, business, travel, plant and animal life, human growth, and the world of work are offered as points of departure. Sources for obtaining information are also listed, and a rich bibliography for gathering many examples and ideas is included.</p>

(Annotated Table of Contents continues)

## Annotated Table of Contents (continued)

<p>Chapter 6  <b>Multiplication and Division of Whole Numbers: Constructing Meaning</b></p>	<p>Earlier versions of this book included one chapter on multiplication and division of whole numbers. We have completely revised that chapter and broken it into two parts, Chapters 6 and 7, in order to demonstrate stronger relationships between multiplication and division and to provide more extensive work in problem-solving, using calculators, and facilitating mental arithmetic. In this chapter, we emphasize learning experiences that enable children to develop operation sense by constructing meaning for multiplication and division of whole numbers. Throughout the chapter, we emphasize the close relationship between multiplication and division and stress the importance of using number sentences (equations) to communicate mathematical ideas.</p>
<p>Chapter 7  <b>Multiplication and Division Algorithms for Whole Numbers: Building, Understanding, Estimating, and Applying</b></p>	<p>In this revised chapter, we extend the ideas presented in Chapter 6 to meaningful procedures for computing and estimating. We develop estimation techniques, emphasize problem-solving, and encourage the use of calculators and computers.</p>
<p>MiniChapter 6  <b>Multiculturalism and Mathematics</b></p>	<p>In this MiniChapter on multiculturalism, we discuss issues that evolve in our society that may not occur in countries whose populations are more homogeneous. We consider the impact on the classroom and the effects on curriculum. We provide problem-solving scenarios to help teachers develop awareness and sensitivity to characteristics of different cultures. An extended bibliography is provided for extended study.</p>
<p>Chapter 8  <b>Some Theory About Numbers: Factors, Multiples, Primes, and Composites</b></p>	<p>This chapter has been extensively revised to present ideas from number theory that provide essential background that supports operations on rational numbers and many concepts children will encounter in pre-algebra and algebra. We encourage teachers to promote a spirit of inquiry by having children explore a variety of ways to represent numbers, by having them construct patterns, and by having them make conjectures based on the patterns they have observed. While the emphasis at this stage is on motivation, the suggested activities abound in mathematical ideas.</p>
<p>MiniChapter 7  <b>Teaching Children of Poverty</b></p>	<p>In this chapter, we describe the conditions of children in poverty and illustrate how children's learning may be affected by such conditions. We provide role-playing activities in which the prospective or in-service teacher develops insight, abilities to empathize, and abilities to provide safe learning environments for children who come from impoverished environments.</p>
<p>Chapter 9  <b>Not All Numbers Are Whole Numbers: Representing, Adding, and Subtracting Rational Numbers</b></p>	<p>This chapter has been significantly extended to include more material on the development of number sense, operation sense, problem-solving, and mathematical reasoning. It covers representing, adding, and subtracting rational numbers. Major decimal and fractional systems have been arranged to illustrate how these topics may be presented simultaneously. Important relationships that exist between the fractional and decimal systems for representing rational numbers are made explicit.</p>
<p>Chapter 10  <b>Security Is Knowing Why: Multiplying and Dividing Rational Numbers</b></p>	<p>This chapter has been extensively revised to include many more opportunities for problem-solving, critical thinking, and making mathematical connections with rational numbers. We develop the meaning of multiplication and division with rational numbers, placing the accent on the interpretations for these operations. Our strategy is to explore situations that are simple, to discover patterns, and to draw generalizations.</p>

(Annotated Table of Contents continues)

## Annotated Table of Contents (continued)

<p>Chapter 11  <b>Believe, Think, Then Solve:</b>  <i>Building Problem-Solving Environments for Children</i></p>	<p>This new chapter is written from the child's point of view. We identify specific thinking abilities and meta-cognitive abilities that children should develop in order to perform the complex task of mentally retrieving and organizing information or processes to solve a problem. These abilities relate to methods of inquiry, mathematical reasoning, and making mathematical connections. It has been many years since Polya introduced his model for problem-solving, and much research has been done that allows educators to gain insights into the thinking abilities that encourage and amplify the use of his model. This chapter presents a model that integrates findings from research and classroom practices and Polya's model. The chapter is rich with activities that have been tried with children and can be exported directly to the prospective teacher's classroom. This chapter has unique sections called "Stop and Reflect" in which the reader is encouraged to reinforce certain abilities immediately following related descriptions and examples.</p>
<p>Chapter 12  <b>The Shape of Things:</b>  <i>Geometric Figures and Relationships</i></p>	<p>This chapter summarizes the basic geometric ideas that a teacher in the elementary school should know. The purposes of this chapter include (1) helping to pinpoint and review specific ideas that often get lost in more rigorous mathematics courses concerned with precise terminology and deductive methods; (2) providing the background for the discussions in Chapters 14, 15, and 16; and (3) providing a concise but comprehensive reference for classroom teachers.</p>
<p>Chapter 14  <b>Seeing Is Believing:</b>  <i>Constructing Geometric Ideas</i></p>	<p>In this chapter, we describe how children between ages 2 and 12 construct geometric concepts, and we present many related activities. We also discuss Pierre and Dina van Hiele's model for the progressive development of geometric ideas and provide implications for teaching.</p>
<p>Chapter 15  <b>Before You Teach Measurement:</b>  <i>Attributes of Measurement</i></p>	<p>This chapter presents ideas that the teacher should consider before beginning to design strategies for teaching measurement to children. We stress the importance of measurement as a means of mathematically describing our environment and consider important characteristics of measurement itself. We introduce the metric system of measurement as the standard system of units to be used by elementary school children, and we suggest physical referents for these units. Concepts that are prerequisites to teaching formal procedures for measurement and the importance of readiness activities are given substantial coverage.</p>
<p>Chapter 16  <b>Sizing It Up:</b>  <i>The Measurement of Attributes</i></p>	<p>This chapter has been considerably revised. It concentrates on methods of presenting measurement ideas to children in the elementary school. The topics discussed include measurement of length, area, volume, weight, angles, time, and temperature. The methods for each of these topics are intended to help children become familiar with measuring units, to provide exploratory experiences, and to develop concepts informally through active participation in measurement situations. The decision to use only metric units in the two chapters on measurement was a careful and deliberate one. The schools must lead in the voluntary conversion to the metric system in the United States, and teachers of mathematics should assume leadership roles in this conversion. In addition, the metric system is easier to teach than the customary system and it extends naturally from place-value concepts. Also, important measurement concepts do not get lost in descriptions of many different and inconsistently defined units. We realize, however, that people who first learned our customary units and then need to learn metric units must be treated gently.</p>
<p>Chapter 17  <b>Making Numbers Count:</b>  <i>Organizing, Representing, and Interpreting Data</i></p>	<p>Incorporating the recommendations of the NCTM, this revised chapter presents statistics and probability ideas that are appropriate for elementary school children. We demonstrate the design of high-interest experiences that make use of familiar language, that draw from real life or interdisciplinary situations, and that can be tailored to fit the needs of different levels of learners. We suggest that children participate in the selection of problems and that they collect, interpret, and organize data from which they formulate tenable hypotheses.</p>

(Annotated Table of Contents continues)



## Annotated Table of Contents (continued)

<p>Chapter 18 Computers and Mathematics Instruction: <i>Status and Direction</i></p>	<p>In this revised chapter, we discuss the implications of microcomputer technology for mathematics instruction. We emphasize specific computer uses in relationship to general categories that will continue to have potential as technological advancements occur. We define broad categories for instructional computing and demonstrate programs for each. We also explore generic software tools such as graphics programs, spreadsheets, and databases, and identify the relevance of these programs to the mathematical curriculum. Before we leave the chapter, we discuss important issues related to instructional computing concerning equity, appropriate uses of computers at different age levels, and ergonomic considerations. The chapter has an extensive bibliography that identifies sources for obtaining good educational software.</p>
<p>Chapter 19 Encouraging Student Growth: <i>Assessment and Diagnosis</i></p>	<p>This chapter (formerly, Chapter 12) has been revised to infuse substantial ideas related to alternate methods of assessment with those of identifying children's mathematical difficulties. We also provide more exercises in which teachers acquire abilities to recognize mathematical difficulties and associated errors. Our motto is, "Prevention over remediation." We encourage teachers to become aware of the difficulties children frequently experience and to plan learning activities that will help children avoid these difficulties. Since the next best thing to prevention is remediation, we also provide materials that help teachers develop skills for quickly spotting and remediating difficulties.</p>
<p>MiniChapter 8 Mathematics, Equity, and Gender</p>	<p>In this MiniChapter we discuss issues related to providing equal opportunities in the classroom for all learners. We provide strategies for creating environments where all learners can gain confidence and experience success.</p>
<p>Chapter 20 The End...Your Beginning: <i>Toward Efficient Instruction</i></p>	<p>In this chapter, we briefly outline techniques for keeping records, and we provide strategies for arranging and managing the classroom. Rarely is a new teacher expected to plan a total instructional program, since most school districts have established programs. The purpose of this chapter is to prepare teachers and provide them with resources to adjust their school district's programs to their own individual styles and the needs of their students.</p>

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*Andria P. Troutman*

*Betty K. Lichtenberg*

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