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Optimization in Signal and Image Processing

Edited by Patrick Siarry

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Optimization in Signal and Image Processing

Introduction

Engineers constantly encounter technological problems which are becoming increasingly complex. These problems may be encountered in different domains such as transport, telecommunications, genomics, technology for the healthcare sector and electronics. The given problem can often be expressed as one which could be solved by optimization. Within this process of optimization, one or several “objective functions” are defined. The aim of this process is to minimize the “objective function” in relation to all parameters concerned. Apart from problems of optimization, i.e. the problem’s objective function which is part of this topic (e.g. improving the shape of a ship, reducing polluting emissions, obtaining a maximum profit), a large number of other situations of indirect optimization can be encountered (e.g. identification of a model or the learning process of a new cognitive system). When looking at this issue from the angle of available methods used to resolve a given problem, a large variety of methods can be considered. On the one hand, there are “classic methods” that rely purely on mathematics, but impose strict application conditions. On the other hand, digital methods that could be referred to as “heuristic” do not try to find an ideal solution but try to obtain a solution in a given time available for the calculation. Part of the latter group of methods is “metaheuristics”, which emerged in the 1980s. Metaheuristics has many similarities with physics, biology or even ethology. “Metaheuristics” can be applied to a large variety of problems. Success can, however, not be guaranteed. The domain of optimization is also very interesting when it comes to its functions within the field of application. In the domain of optimization, the processing of signals and images is especially varied, which is due to its large number of different applications as well as the fact that it gave rise to specific theoretical approaches such as the Markov fields, to name just one example.

These ideas have influenced the title of this book *Optimization in Signal and Image Processing*. This book has been written for researchers, university lecturers

and engineers working at research laboratories, universities or in the private sector. This book is also destined to be used in the education and training of PhD students as well as postgraduate and undergraduate students studying signal processing, applied mathematics and computer science. It studies some theoretical tools that are used in this field: artificial evolution and the Parisian approach, wavelets and fractals, information criteria, learning and quadratic programming, Bayesian formalism, probabilistic modeling, the Markovian approach, hidden Markov models and metaheuristics (genetic algorithms, ant colony algorithms, cross-entropy, particle swarm optimization, estimation of distribution algorithms (EDA) and artificial immune systems). Theoretical approaches are illustrated by varied applications that are relevant to signals or images. Some examples include: analysis of 3D scenarios in robotics, detection of different aggregates in mammographic images, processing of hand-written numbers, tuning of sensors used in surveillance or exploration, underwater acoustic imagery, face recognition systems, detection of traffic signs, image registration of retinal angiography, estimation of physiological signals and tuning cochlear implants.

Because of the wide variety of different subjects, as well as their interdependence, it is impossible to structure this book – which contains 13 chapters – into distinct divisions, which might, for example, separate traditional methods and metaheuristics, or create a distinction between methods dealing with signals or with imagery. However, it is possible to split these chapters into three main groups:

- the first group (Chapters 1 to 5) illustrates several general optimization tools related to signals and images;
- the second group (Chapters 6 to 10) consists of probabilistic, Markovian or Bayesian approaches;
- the third group (Chapters 11 to 13) describes applications that are relevant for engineering in the healthcare sector, which are dealt with here through the use of metaheuristics.

Chapter 1 deals with the benefits of modelization and optimization in the analysis of images. After the introduction of modelization techniques for complex scenes, the analysis of images has become much more accurate. In particular, traditional means of image analysis, such as the segmentation of an image, need to be revised. Jean Louchet creates a link between two domains that have been developing independently. These are the synthesis and the analysis of images. The synthesis of images relies on a wide range of different modelization techniques which are based on geometrics, depiction and movement. The author shows that some of these techniques can also be used for the analysis of images, which would broaden the possible applications of these techniques. Jean Louchet also shows how artificial evolution can lead to a better exploitation of models, create new methods of

analysis and push back the limits of Hough transform using a stochastic exploration of the model's parameter space.

In Chapter 2 Pierre Collet and Jean Louchet present the so-called "Parisian" approach of evolutionary algorithms and how these algorithms are used in applications when processing signals and images. Evolutionary algorithms are reputed to take a long time to perform calculations. The authors, however, show that it is possible to improve the performance of these algorithms by – if possible – splitting the problem into smaller sub-problems. When using the "Parisian" approach to analyze a scene, the objects which have been modified by genetic operators are not the vectors of the parameters that determine a complete model of an image. These objects are elementary entities which only make sense when merged together as a representative model of the scene that will be studied. In other words, a problem cannot be represented by a single individual but by several individuals, or even the entire population. The "Parisian" approach is successfully used in the field of robotics when analyzing 3D scenes via stereovision. The so-called "Fly algorithm" allows for the detection of obstacles in real time and much more quickly than when using traditional approaches. Other visual applications based on models can be processed by evolutionary methods. Here, the authors discuss the identification of models of mechanical systems based on sequences of images.

Chapter 3 deals with the use of wavelets and fractals when analyzing signals or images. The application of these techniques is becoming increasingly frequent in natural science as well as in the study and research carried out in the scientific fields of engineering and economics. Abdeljalil Ouahabi and Djedjiga Ait Aouit show that multifractal analysis and the exploitation of techniques of multiresolution based on the concept of wavelets lead to a local as well as global description of the signal's singularities. On a local level, the criterion of punctual regularity (rugosity) based on Hölder's inequality can be characterized by the decrease of the wavelets' coefficients of the analyzed signal. On a global level, the distribution of a signal's singularities can be estimated by global measures when using the auto-similarity of multifractals. In other words, the spectrum of singularities is obtained when localizing the maxima of the module of the wavelet transform of a signal. The authors give two examples of the aims and applications of this formalism. One example in the healthcare sector is a multifractal analysis which allows for the detection of different aggregates in mammographic images. The second example is fracture mechanics. In this field the formalism described above is used to study the resistance of materials.

Chapter 4 deals with the information criteria and their applications when processing signals and images. Here, the model of a random signal should be optimized. An information criterion is a description or formulation of an objective

function that should be minimized. The information criteria are an improvement on the traditional technique of the maximum likelihood. This improvement is due to the focus being shifted towards simultaneous research on the optimal number of free parameters in the model as well as the ideal values for these parameters. Christian Olivier and Olivier Alata first give a general overview of the main information criteria as well as the relevant literature. The majority of the criteria were introduced for research using 1D auto-regressive (1D AR) models. In Chapter 4, this case is illustrated by an application that involves the segmentation of natural images. The information criteria were then transferred to the 2D AR model. Two applications resulted from this. These are the modelization of the image's texture and the unsupervised segmentation of textured images. The authors then look at the extension of the information criteria to other models based on parameters. These are a mix of Gauss's laws n -D, which are here applied to unsupervised classification as well as Markov's modes. Last but not least, this chapter deals with the application of information criteria in the case of non-parametrical problems, such as the estimation of distribution via histograms or the search for antiderivatives that carry a maximum amount of information depending on the form of the information. The information criteria finally offer a means to justify the choice of parameters which are linked to a large number of problems when processing signals or images. The information criterion deals with a high number of observations. This is why the time required to carry out the calculation might be high (particularly in an unsupervised context). Dynamic algorithms, however, are able to reduce the number of operations that need to be carried out.

Chapter 5, written by Gaëlle Loosli and Stéphane Canu, deals with an aspect of optimization that can currently be encountered within signals and images, for example in shape recognition, i.e. learning processes. More precisely, the chapter focuses on the formulation of learning as a problem in convex quadratic programming on a large scale (several million variables and constraints). This formulation was obtained by the "nucleus methods", which emerged about a decade after neural networks. Its main aim is linked to the fact that the solution in question is often "parsimonious", i.e. more than 99% of all unknown variables are zero. Using this specific feature enables learning algorithms to solve large scale quadratic programming problems within a reasonable amount of time. The best-performing methods, known as "active constraints", work "off-line". In other words, they are able to determine the exact solution of a given problem if they are provided with all the data that is used in the learning process. To carry out an "online" learning process, a method of iterative stochastic optimization is used, which allows us to obtain an approximate solution. This chapter describes one of the methods which is part of the "support vector machine" (SVM) type. The efficiency of this technique is illustrated by results of experiments which focused on the problem of recognizing handwritten numbers.

Chapter 6 deals with the problem of planning within time and space the use of sensors with the aim of optimizing the exploration and surveillance of a specific zone; given the rather low number of available sensors as well as their capacity, this zone is large. Due to the problem being rather extensive, exact methods cannot be used. An approximate solution can, however, be obtained with the help of metaheuristics. In this case, Frédéric Dambreville, Francis Celeste and Cécile Simonin, the authors of this chapter, recommend the use of “cross-entropy”. This method was initially created to evaluate the probability of rare events and has been adapted to “difficult optimization” problems (many local minima need to be considered). The solution is obtained with the help of a probability law that continually approaches the global optimum. This method is applied to the problem of planning sensors via *a priori* modeling mainly under the form of different groups of probability laws, of possible planning policies. In this chapter, three examples are explained in detail. The first example looks at how to ideally array search units in the context of military operations. The aim is to maximize the probability of locating the target which does not move but is hidden. In the second example, cross-entropy is used for an exploratory mission. The movement of the vehicle needs to be planned based on maps that show the environment. The third example is the problem of optimal control in an environment where only certain parts of the environment can be observed. Cross-entropy is particularly useful when dealing with data that are very difficult to formalize. Optimization via cross-entropy therefore means to “learn” an optimal strategy.

The topic of Chapter 7 is linked to that of the previous chapter. Chapter 7 deals with a surveillance system such as a maritime patrol aircraft that needs to locate a moving target. In order to do this, all resources, i.e. passive as well as active sensors (e.g. a radar), need to be used. Passive measures do not involve any cost. However, they only determine the direction of the target. Active measures provide much more information since they can evaluate the distance to the target. These measures, however, need to be used sparsely because of their cost (emitting a wave) and with discretion. The author of this chapter, Jean-Pierre Le Cadre, gives a general outline of the problem of optimal and temporal repartition when using active measures. He furthermore describes the general mathematical tools (e.g. multilinear algebra) that allow for the analysis of this problem. The study focuses on the explicit calculation of objective functions while expressing the quality of the estimation (or tracking) of the trajectory’s location by using non-linear observations of state. First of all, this chapter examines the case of targets that contain a determined trajectory. Their movement is rectilinear and uniform, or in other words the target is “maneuvering”. When dealing with certain types of approximations, the problem of convex optimization comes into play. This problem can easily be resolved. The author also looks at the stochastic evaluation of this case. He shows that it is possible to directly calculate the objective function of a target of Markovian trajectory without having to use simulations.

Chapter 8 deals with segmentation methods of images which exploit both the Markovian modeling of images and the Bayesian formalism. For every image under observation there is an infinite number of combinations of objects that can be associated with it. These combinations of objects represent, or in other words create, the image. To reduce the number of possible solutions that should be integrated in the stage of segmentation, prior local or global knowledge is required. The aim of Markovian modeling lies precisely in its capacity to locally describe global properties. Due to the equivalence between Markov's field and Gibbs's distribution, the optimal segmentation can be obtained by the minimization of a function linked to energy. Christophe Collet, the author of this chapter, applies this formalism to the context of underwater acoustic imagery. To detect small objects on the seabed, the author exploits images that have been taken by a lateral multibeam sonar. The images that were obtained were distorted by noise. A segmentation of good quality therefore requires the nature of noise to be taken into consideration during the process of image modeling. This chapter shows different examples of application. These are the segmentation of sonar images into two different groups (shadow, reflections of the seabed) or segmentation into three different groups (shadow, seabed and echo). Due to the third group, echo, physics, which forms the basis of the creation of sonar images, is also taken into consideration. Two other examples are the differentiation between manufactured and natural objects, as well as the subdivision of the seabed into different regions (sand, mid-ocean ridges, dunes, stones and rocks). All tasks linked to detection and classification are first of all united in the fact that the function of energy, which integrates the prior knowledge required to obtain a solution, needs to be minimized. The technique used for this optimization is a deterministic method or a genetic algorithm, depending on whether an initial good quality solution is available or not.

Chapter 9 was written by Sébastien Aupetit, Nicolas Monmarché and Mohamed Slimane and describes the use of hidden Markov models (HMM) for the recognition of images. Hidden Markov models are statistical tools which allow for the modeling of stochastic phenomena. This type of phenomenon may, for example, consist of several sequences of images. Images of the same sequence are taken from different angles but show the same scene, e.g. a person's face. After a learning phase, HMM is prepared for the process of recognition. During this learning phase several sequences of images, let us say four sequences of four photographs each showing the faces of four different people are processed. When confronted with a new photograph of a face, HMM is able to distinguish which person is shown in the picture from the four previous pictures. At the same time, the risk of HMM making a mistake is minimized. More precisely, a discrete HMM corresponds to the modeling of two stochastic processes. The first process is hidden and perfectly modeled by a discrete Markov chain while the second observed process is dependent on the state of the first process, i.e. the hidden process. This chapter focuses on learning processes, a crucial aspect of HMM. It provides an overview of the main

criteria of existing learning processes and the possible solutions for HMM learning processes. Furthermore, the principles of three metaheuristics inspired by biology and population-based are also addressed by the authors and analyzed in light of HMM learning processes. These three metaheuristics are a genetic algorithm, ant colony algorithm and particle swarm optimization (PSO). Several versions of these types of metaheuristics (which are different to one another because of the mechanisms which are implemented, or simply due to the settings of the respective methods) are examined and tested in great detail. These tests are carried out on a set of test images as well as samples of literature. The chapter emphasizes the fact that results can be improved if metaheuristics used for learning processes are combined with a method dedicated to local optimization.

In Chapter 10 Guillaume Dutilleul and Pierre Charbonnier use different metaheuristics inspired by biology for the automatic detection of traffic signs. The aim is to make an inventory of road signs currently used in the French secondary road network. The data used are images that have been collected by vehicles inspecting the roads that are part of the respective network. The application does not face any real time constraint. However, the application needs to be robust when faced with changes in the conditions under which the images are collected. Problems might occur due to differences in light, backlighting, worn out or partially hidden traffic signs. The method that has been proven to be successful includes the technique of “deformable models”. This technique consists of a mathematical model, a prototype of which the object research is carried out upon. This model’s shape can be manipulated and changed to such an extent that it is adapted to the respective image that should be analyzed. The quality of this adjustment and to what extent manipulation can be accepted are, in the case of Bayesian formalism, respectively measured by a likelihood and an *a priori*. The problem of localizing an object therefore comes down to the problem of optimization in the sense of a maximum *a posteriori*. The residual value of a minimized objective function gives an indication of the effective presence of the object in the scene which is to be analyzed. In practice, the presence of numerous local minima justifies the use of metaheuristics. The authors have carried out experiments with three different techniques in the field of metaheuristics. These are an evolutionary strategy, PSO and a method of clone selection (the latter is relevant to a more general field of “artificial immune systems”). The performance of automatic detection is compared to a number of different algorithms when dealing with a sequence of traffic signs. (For these test images the real data had already been obtained manually.)

The majority of metaheuristics were initially created for the processing of problems that arise when dealing with discrete optimization. Chapter 11, written by Johann Dréo, Jean-Claude Nunes and Patrick Siarry, looks at their adaptation to applications with continuous variables, which are encountered frequently, especially in the field of signals and images. The techniques suggested in the literature for this

adaptation are linked to each specific form of metaheuristics. These techniques cannot be generalized, i.e. it is not possible to apply these techniques to another application. Furthermore, no metaheuristic, whether it is continuous or discrete, is the ideal technique, i.e. most efficient, for all possible sorts of problems. This is why hybrid methods, which combine different forms of metaheuristics or metaheuristics with downhill simplex techniques, often need to be used. This chapter describes two “continuous metaheuristics”. These are an ant colony algorithm and EDA. Furthermore, a local technique, which is frequently used in continuous cases to refine the search within a “promising valley” of solutions, is Nelder and Mead’s downhill simplex method. These methods are used for image registration in the field of retinal angiography. Before a doctor can actually interpret a sequence of images, the problem of inevitable eye movement during the procedure needs to be dealt with. In the example given in this chapter, image registration is carried out by using only translatory motions between different images. Metaheuristics were found to be particularly appropriate for image registration in angiography with a high resolution. The time required for calculations only increases a little when increasing the resolution of images.

Chapter 12, written by Amine Naït-Ali and Patrick Siarry, describes the introduction of a genetic algorithm used for the estimation of physiological signals, the Brainstem Auditory Evoked Potentials (BAEP). BAEP is an electric signal which is generated by the auditory system as a response to acoustic stimulation. Studying this signal allows for the detection of pathologies such as acoustic neuroma. Measuring BAEP is, however, a problem as this signal is of a very low energy and covered by electric noise that stems from spontaneous electric activity of the cerebral cortex (these signals can be measured using electroencephalograms (EEG)). To identify a patient’s effective BAEP, several hundred signals need to be exploited. These signals are obtained as a result of acoustic stimulation. They also have to be synchronized before being simply added to one another in order to eliminate the noise. The synchronization process is expressed in the form of an optimization problem in which unknown variables are the random delays of different signals. Here, the problem is solved with the help of a genetic algorithm. The authors show that a significant acceleration of this technique can be obtained when creating a model for the variation law of these delays. This can, for example, be performed using a set of sinusoids.

Chapter 13, written by Pierre Collet, Pierrick Legrand, Claire Bourgeois-République, Vincent Péan and Bruno Frachet, presents an evolutionary algorithm that allows for the adjustment of parameters for a cochlear implant. This adjustment is carried out in interaction with the patient using the device. This type of implant enables deaf people, whose cochlear plate is still intact, to hear. The device works as follows: a group of electrodes is implanted into the patient’s cochlear plate. These electrodes stimulate the auditory nerve. The electrodes are connected to a digital

signal processor (DSP) that receives the sound as signals through a microphone situated next to the patient's ear. The parameters of DSP need to be adjusted in a way that reconstructs the patient's auditory ability to a point that he/she might even be able to understand spoken language. Adjusting these parameters is usually undertaken by a human and becomes increasingly complicated as technology progresses. A current implant consists of 20 electrodes and several hundred parameters. The effort for adjusting these parameters is dependent on the patient's ability to understand spoken language. This is why this study looks at the performance of an interactive evolutionary algorithm which should take over the task of adjusting the parameters of a cochlear implant. There are a large number of difficulties that lie within this application. These are the subjective evaluation of every single patient, the quality of every single solution produced by the algorithm, the necessity of a rapid convergence of the algorithm in order to strictly limit the amount of solutions to be evaluated by the patient (as every evaluation takes a few minutes) as well as the fact that the search space is very broad. This chapter presents experiments undertaken by the authors with the help of a small number of patients following a methodical protocol. The first results are promising. They show the disadvantages of manual adjustment in cochlear implants which is increasing because the number of available electrodes is currently increasing.

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