Advanced Series on
Directions in High Energy Physics — Vol. 11

QUANTUM FIELDS ON THE COMPUTER

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World Scientific
Singapore • New Jersey • London • Hong Kong

Published by

World Scientific Publishing Co. Pte. Ltd.
P O Box 128, Farrer Road, Singapore 9128
USA office: Suite 1B, 1060 Main Street, River Edge, NJ 07661
UK office: 73 Lynton Mead, Totteridge, London N20 8DH

Library of Congress Cataloging-in-Publication data is available.

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ISBN 981-02-0939-8
ISBN 981-02-0940-1 (pbk)

Printed in Singapore by General Printing Services Pte. Ltd.

INTRODUCTION

In 1948 Feynman showed that quantum field theory is equivalent to integration. In the more than 40 years since, particle theorists have occupied themselves with various ways to evaluate his sums over paths.

For most of this time the primary approach has been to consider only small perturbations about the free field theory limit, where the integrals become Gaussian. In the last decade, however, there has been an explosion of attempts to attack these integrals more directly, using the vast increases in raw computational power becoming available. These efforts are quite ambitious; indeed, they are attempts at first principle calculations for interacting relativistic quantum field theories.

Since the starting point is an infinite-dimensional integral, approximations are necessary to put it on the computer. The usual approach is to replace the space–time continuum by a discrete lattice in a finite volume. In this way the integrals at least become finite-dimensional, although still generally rather large.

When one is dealing with analytic calculations in the continuum theory, it is particularly difficult to be rigorous. This often allows meaningless calculations involving poorly defined quantities. On the lattice, if one tries to evaluate an undefined number, then the computer quickly complains. This forces an unusual honesty on the lattice gauge theorist.

Of course, one must still ask if the lattice model has anything to do with the continuum world of observation. If one can show that the continuum limit of a lattice theory exists, one might just as well use that as a definition of the continuum theory. If another definition of the continuum model makes sense but is different from the limit of the lattice form, then one must wonder if there is any uniqueness at all to the theory. Indeed, it appears a miracle that in four dimensions only a small class of theories appears to have a reasonable chance of surviving a continuum limit. In more dimensions none are known, while in less a much wider class of renormalizable theories is possible.

In perturbation theory, one quickly encounters the well-known divergences requiring renormalization. The various couplings and masses in the bare Lagrangian are not physical quantities, so predictions for cross sections etc. should be re-expressed in terms of observables, such as the long range electromagnetic force in quantum electrodynamics. These renormalization issues are not unique to perturbation theory, and appear as well in the lattice technique. The language here, however, sometimes appears quite different than in the continuum approach. In a perturbative discussion, it is usual to think of the bare coupling as a parameter which must be adjusted, or renormalized, as the cutoff controlling ultraviolet divergences is removed. Thus the bare coupling is a function of the cutoff. On the lattice, however, we often think of the lattice spacing, or cutoff, as a function of the bare coupling. To take a continuum limit one adjusts the bare coupling to make the lattice spacing small. Owing to asymptotic freedom, for the strong interactions this requires taking the bare coupling to zero.

The purpose of this book is to review the main results of numerical lattice gauge theory. From the initial studies relevant to the confinement problem, lattice calculations have grown into an industry involving hundreds of physicists and tackling a wide spectrum of problems. For a single author to do a thorough job reviewing this diversity would be near-impossible. Instead, we have divided the task and thus hope to have each of the subtopics treated in a reasonably complete manner.

The book consciously focuses on the computational aspects of the problem. While analytic approaches are also crucial to the subject, the results have so far been more limited. We have not attempted a thorough introduction to the basic foundations of lattice field theory. For this there are several reviews in the literature as well as my previous monograph, *Quarks, Gluons, and Lattices* (Cambridge University Press, 1983).

We assume that the reader is at least vaguely familiar with the general formulation of lattice gauge theory, wherein there is a system of group variables $U_{i,j}$ defined on the links $\{i,j\}$ of a four-dimensional hypercubic lattice. For the strong interactions, these elements are from the group SU(3), and we usually have them interact via the standard Wilson action,

$$S = -\sum_p \mathrm{Re}\mathrm{Tr} U_p \,. \tag{1.1}$$

Here U_p is the product of link variables around the elementary square, or plaquette, p, and the sum is over all such plaquettes. The Feynman path integral is then given by

$$Z = \int (dU) e^{-\beta S(U)} \,. \tag{1.2}$$

Here the coupling constant dependence has been parametrized by the variable β. This is related to the more conventional perturbative coupling by

$$\beta = \frac{6}{g_0^2} \,, \tag{1.3}$$

where the subscript 0 on the coupling is to emphasize that this is the bare coupling, which must be renormalized in the continuum limit.

Note that these definitions emphasize an analogy with statistical mechanics. Solving the four-dimensional space–time field theory is equivalent to evaluating the thermodynamics of a four-dimensional classical system with Hamiltonian given by the action. As in condensed matter physics, it is not Z itself which is of primary interest, but the correlations between observables in the equilibrium ensemble. Correlation lengths are the inverse masses of physical particles, and coupling constants come from correlations between multiple fields.

Just as with the continuum formulation, the lattice action carries an invariance under gauge transformations. The Wilson approach has the remarkable property that this invariance remains an exact local symmetry. Indeed, if we introduce an arbitrary group element g_i associated with every site on our lattice, the action (1.1) is unchanged if we replace U_{ij} with $g_i U_{ij} g_j^{-1}$. Thus all issues related to gauge fixing can be discussed on the lattice as well as in the continuum. Indeed, this is necessary for a perturbative treatment. Nevertheless, Wilson's compact formulation makes the theory well defined even without a gauge choice.

From perturbative analysis we obtain some rather remarkable information on the continuum limit of the lattice theory. Indeed, the phenomenon of asymptotic freedom tells us that the coupling defined at increasingly shorter distances goes to zero logarithmically with the scale. As the bare coupling is an effective coupling at the scale of the lattice cutoff, it must be taken to zero as the lattice spacing is reduced. In this process we have the remarkable phenomenon of dimensional transmutation, wherein the dimensionless coupling constant is replaced by the scale of this logarithmic behavior. In the end, the pure gauge theory contains no dimensionless couplings, and all mass ratios should be determined. This is one of the main goals of lattice calculations: to determine the properties of the physical hadrons from fundamental principles and with no adjustable parameters beyond the quark masses.

We begin the book with two chapters on some of the primary physical results of these efforts. First T. DeGrand reviews the status of calculating hadronic masses. This in some sense represents the particle physicist's most fundamental desire: to predict the observed spectrum of nature. The second chapter turns to some experimental predictions that have not yet been verified. Here R. Gavai treats the behavior of hadronic matter at finite physical temperatures, and the predicted transition to a quark–gluon plasma. Indeed, it is the lattice approach which has given the best estimates of the temperature and properties of this transition.

The next two chapters push lattice methods beyond the gauge theory of the strong interactions. A particularly important topic for the near future is the Higgs boson, the study of which is a primary goal for the SSC. Lattice methods have been applied here, and given rather stringent nonperturbative bounds on the possible masses for this as-yet-undiscovered particle. This is the topic of the chapter by A. Hasenfratz. Extending these ideas to include the fermionic contributions to the standard model of weak interactions, R. Shrock then reviews chiral and Yukawa models. This represents an area where the lattic formulation is not yet fully understood; in particular there are fundamenta difficulties in formulating a theory with fermions interacting in a chiral man ner. Nevertheless, such work is essential for understanding weak interactio phenomenology on a nonperturbative level.

We then turn to two chapters on the basic numerical algorithms being use for lattice calculations. A. Sokal provides a detailed discussion on the basi Monte Carlo methods for lattice simulations, concentrating on bosonic fields He points out the advantages and pitfalls in the various schemes, and cover recent develpments in overcoming computational limitations as the continuun limit is approached. I then present a discussion on the modifications of thes methods developed to study quark fields. Here the anticommuting nature o the fundamental objects creates severe new difficulties, and probably the bes approach is yet to be found.

For us to have confidence in any predictions for quantum field theory, th results must behave in a well-understood manner as one removes the cutoff. A the core of these analyses lies the renormalization group. Indeed, the verifica tion of the appropriate scaling behaviors is an essential step towards confidenc and future improvements in the basic lattice approach. These ideas form th basis of R. Gupta's chapter.

In the final chapter, C. Bernard and A. Soni review the use of lattic approaches to calculate the previously unknown hadronic corrections to weal interaction processes. This is a crucial step in our ability to comprehend th underlying fundamental weak forces. Indeed, these and related calculation represent the current dominant use of lattice gauge methods to describe rea phenomena in particle physics.

We have enjoyed putting this book together, and hope it will be useful i clarifying what is known and pointing out directions for new research. Lattic field theory is a large subject, and necessarily some topics have been left out We have tried to be reasonably up to date, but in any rapidly evolving fiel the true excitement lies in unanticipated new developments.

Michael Creut

CONTENTS

TECHNIQUES AND RESULTS FOR LATTICE QCD SPECTROSCOPY

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ABSTRACT: This article describes how to compute the masses of hadrons in lattice simulations of QCD. I discuss the most important theoretical and calculational techniques for finding the masses of mesons, baryons, and glue-balls. Part of this discussion is pedagogic and is intended for use by newcomers to this field. A second section describes some interesting open problems in spectroscopy: wave functions, resonances, orbitally excited states. I review recent developments in this subject (through Autumn 1991).

I. INTRODUCTION

This chapter of the book discusses how to compute the masses of the ordinary elementary particles in lattice simulation of Quantum Chromodynamics. (Most of the techniques which I discuss can be applied to other models in particle and condensed matter physics.) This is a problem of great current interest to the lattice gauge community as shown by the many large computer simulations devoted in whole or in part to the calculation of spectroscopy.

There are several reasons why this problem is important. First of all, we think QCD is the theory of the strong interactions. We cannot have confidence in our ability to compute in QCD if we cannot use it to calculate at least the low lying spectrum of the theory. We also wish to calculate hadronic matrix elements, either for their own sake or as ingredients to calculations which go beyond the strong interactions– perhaps to constrain parameters of the Standard Model. A necessary (but not sufficient) condition for trusting lattice calculations of matrix elements of complicated operators is the knowledge that matrix elements of simple operators (like the Hamiltonian) are under control. Thus we need to see good spectroscopy calculations first.

Actually, I believe that the goal of calculating the complete low-lying spectrum of QCD is presently unrealistic. The techniques at our disposal are too primitive. Lattice methods cannot compete with continuum models which are not QCD but are QCD-inspired which give an essentially perfect fit to light quark spectroscopy.[1] However, calculations of matrix elements are much more model dependent. Since we believe that the lattice "model" is more fundamental than the other approaches, we want to use it to compute matrix elements. We want these calculations to be reasonably reliable. Poor qualitative agreement of QCD spectroscopy with experiment will indicate that these calculations are not trustworthy.

Lattice gauge theory simulations allow us to vary physical parameters (quark masses, number of flavors of sea quarks) in a way that experiment cannot. In principle, this gives us more information on confinement than we would get from real data. One even hopes that the behavior of QCD's with zero, 2 or 3 or 4 degenerate flavors of quarks are quite different from each other, and from the real world, in which all quark masses are different! (For future reference, simulations with zero flavors of dynamical fermions are done dropping the fermion determinant from the functional integral. This approximation is called the "quenched approximation.") However, present day simulations are presently limited by computer power and algorithms to unphysical values of the dynamic quark mass, and unphysical numbers of flavors or degeneracies. This means that if one's goal is to make a direct comparison of a simulation to the real world, one must make an extrapolation. It's important to remember that the extrapolation is not part of the lattice simulation. It certainly involves its own physical assumptions whose validity is independent of the validity of the simulation. One should try to keep extrapolation issues as separate as possible from simulation issues.

While the goal of this chapter is spectroscopy, a large component of the discussion will involve the study of wave functions of quarks and gluons inside hadrons. I believe that the major advances in spectroscopy have come through our ability to model and use realistic wave functions (mainly as interpolating fields), and that essentially all future progress in this field will center on the study of wave functions.

The outline of this review is as follows: In Sec. II I will present an overview of the basic techniques one uses in a lattice spectroscopy simulation, hopefully done at a level suitable for a beginner. Sec. III is devoted to a set of interesting open questions in lattice spectroscopy simulations. Secs. IV and V are reviews of recent progress in glueball and quark spectroscopy. Finally I make a few concluding remarks in Sec. VI. If you want to get an overview of the ingredients you "have to know" to do a lattice simulation, read Sec. II. If you want to see how interesting physics questions collide with lattice techniques, skip Sec. II and read Sec. III. If the year is later than 1994, ignore Secs. IV and V completely–they are (hopefully) obsolete.