New Trends in the History and Philosophy of Mathematics

Edited by
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Preface

This book is the result of a conference held at the University of Roskilde, Denmark, August 6 - 8, 1998. The aim was to provide a forum within which historians of mathematics, philosophers, and mathematicians could exchange ideas and discuss different new approaches in the history and philosophy of mathematics. Hence the conference was called *New Trends in the History and Philosophy of Mathematics*. To quote from the conference abstract:

> During the last decades work in history and philosophy of mathematics has led to several new original views on mathematics. Both new methods and angles of study have been introduced, and old views of, say, the nature of mathematical theories and proofs have been challenged. For instance, disciplines as etnohistorical studies of mathematics and the sociology of mathematics have resulted in several new insights, and classical historians of mathematics are also experimenting with new perspectives. In a similar way philosophy of mathematics has witnessed rather deep changes. Classical foundadional studies have been challenged by new broader perspectives. The purpose of this meeting is to present some of these new ideas on the study of mathematics, its character and the nature of its development.

The editors would like to thank the invited speakers including Prof. Joan Richards (Brown University), Prof. Henk Bos (Universiteit Utrecht), Prof. Donald MacKenzie (University of Edinburgh), Prof. Arthur Jaffe (Harvard University), Prof. Jody Azzouni (Tufts University), Prof. Paulus Gerdes (Universidade Pedagogica, Mozambique), and Prof. Herbert Mehrtens (Technische Universität Braunschweig) for contributing, in the most lucid and encouraging way, to the fulfillment of the conference aim. All the papers but one delivered at the conference are presented in the present proceedings and the editors are grateful to the speakers for making their contributions avaliable for publication.

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http://www.mathnet.ruc.dk

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Finally, the editors are indebeted to Prof. Jørgen Larsen (University of Roskilde), who introduced us to the finer points of typesetting in LaTeX

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Roskilde
August 2002

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Dr. **Henk J.M. Bos** (1940) is professor of History of Mathematics at Utrecht University; he is a member of both the Mathematical Institute and the Institute for History and Philosophy of Science at that University. He has published about Huygens' mathematical and scientific work, about the fundamental concepts of the Leibnizian infinitesimal calculus and about Descartes' geometry. Recently (Dec. 2000) his monograph on the role of the concept of construction in the development of early modern mathematics appeared: Redefining Geometrical Exactness: Descartes' Transformation of the Early Modern Concept of Construction, New York etc. (Springer, 2000).

Henk Bos is editor, together with Jed Buchwald, of the Archive for History of Exact Sciences.

Donald MacKenzie holds a personal chair in Sociology at Edinburgh University, where he has taught since 1975. He is the author of Statistics in Britain, 1865-1930: The Social Construction of Scientific Knowledge (Edinburgh: Edinburgh University Press, 1981), Inventing Accuracy: A Historical Sociology of Nuclear Missile Guidance (Cambridge, Mass.: MIT Press, 1990), Knowing Machines: Essays on Technical Change (Cambridge, Mass.: MIT Press, 1996); Mechanizing Proof: Computing, Risk, and Trust (Cambridge, Mass.: MIT Press, forthcoming). The second of these books won the Ludwik Fleck prize of the Society for Social Studies of Science, and was joint winner of the 1993 Robert K. Merton Award of the American Sociological Association. He has written numerous articles in the sociology and social history of science and technology, and edited, with Judy Wajcman, The Social Shaping of Technology (Milton Keynes, Bucks.: Open University Press, 1985 and 1999).

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INTRODUCTION

The History and philosophy of mathematics can be viewed as two different scholarly disciplines investigating the complex problem of the development and nature of mathematics through different kinds of questions and with different methods. Traditionally, the practitioners of the two fields have viewed, understood and explained the nature of mathematics from very different perspectives. In history of mathematics the primary method has been to trace the origin of modern mathematical ideas through an internalistic analysis. In philosophy of mathematics the most dominant standpoint until recently can be traced back to the foundational crisis in mathematics at the beginning of the 20th century. The consequence of this for the philosophy of mathematics was a focus on the question of what the foundations of mathematics should be and the question was analysed within the field of mathematical logic. Even though this foundational style in the philosophy of mathematics has influenced historians of mathematics the two fields have until recently lived side by side as two different enterprises with almost no interaction at all. Historians and philosophers respectively seemed only to have had the subject matter - mathematics - as their common ground. But during the last decade a series of new critical discussions about the development, the nature, and the significance of mathematics have emerged almost simultaneously within the two disciplines themselves caused by self-examinations of the goals and methods of these enterprises. It has been a characteristic feature of these discussions that historians and philosophers as well as working mathematicians have taken part in them. These discussions are indications of a process of change, and they have opened up a variety of interesting perspectives on the understanding of the development and the nature of mathematics and brought along several new tendencies in the history and philosophy of mathematics.

Regarding the history of mathematics the last decades have witnessed the beginning of critical voices raised from inside the field itself. They have pointed to several methodological questions and urged the historians of mathematics to take a critical view of the whole endavour. The new trends, which have emerged in recent years, can be seen as answers to these methodological questions. There are only very few papers dealing explicitly with the methodology of the history of mathematics.[1] Characteristic of these papers is that they all show a tension between the internalist and the externalist approach to the history of mathematics. In fact, the most single, long-standing debate in the historiography of mathematics is the externalism - internalism debate which has haunted the field during the last three decades. In his essay "Genies, Ideen, Institutionen, mathematische Werkstätten: Formen der Mathematikgeschichte" published in 2000 Moritz Epple discusses about the still unsettled controversy between externalism and internalism.[2]

The question is - simply put - this: should the history of mathematics should be studied from inside the discipline itself with a focus only on the development of mathematical ideas or should the history of mathematics be studied from a broader perspective that also takes external factors into accounts. In a report from a meeting in 1991 on "Critical Problems" in the history of science Joan Richards wrote about the historians of mathematics, that they "[...] were actively engaged in drawing an ever sharper line to divide themselves into these two groups [internalism/externalism]." Richards' conclusion on the subject at that time was "that the division between these two camps is not only *a* but *the* critical problem in the history of mathematics."[3] A reason for this tension can be found in the composition of people working in the field. The ideal historian of mathematics should be a mathematician as well as a full-fledged historian. Some of the practitioners identify more with one of the fields and the two groups often have very different ways of working with the historical text. Mathematicians often tend to stress internal factors to the point of neglecting all external influences, whereas historians tend to embed their histories in a broader cultural, institutional, philosophical, and/or political context but tend to care less about the strictly logical conceptual development of the mathematical techniques. As parallel to this distinction between historians and mathematicians is the distinction between whig and anti-whig historical writing. The mathematician, studying the history of his or her subject will often judge the past with the standards of the present. He or she will typically find that earlier mathematics is similar in content to modern mathematics although different in form. He or she may want to find "the royal road to me".[4] The historian on the other hand will typically look for differences in the mathematics of different times and in different locations, and explore historical changes in mathematics, without using modern ideas as a yardstick.

Even though this dichotomy between internalism and externalism can be singled out in the historiography of mathematics historians of mathematics are finding new ways of understanding and treating the historical sources that

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[1]The following four have inspired this introduction (Aspray and Kitcher 1988), (Richards 1995), (Rowe 1996), (Epple 2000).

[2](Epple 2000).

[3](Richards 1995, p. 123-24).

[4](Grattan-Guinness 1990, p. 157).

go beyond this split of internalism – externalism. In 1996 Rowe pointed out that "the ever widening range of interests among scholars and students alike reflects one of the major trends now taking place in the history of mathematics. This trend is linked with a shift from a relatively narrow, Eurocentric vision of a monolithic body of mathematical knowledge to a broader, multilayered picture of mathematical activity embedded in a rich variety of cultures and periods."[5] One of these scholars is Moritz Epple who in (Epple 2000, p. 143) argues "dass echte historische Arbeit erst mit der Frage der Kausalität beginnt." In his study of the history of topology he discusses the method of causally coherent historical narratives, which is inspired by Hayden White's "Meta-History."[6] Epple concludes that "it is the weave of concrete scientific action rather than an abstract life of mathematical ideas that historians need to analyse in a detailed and realistic way if they wish to adhere to the goal of producing a causally coherent account of developments like the emergence of topology."[7] A consequence of this approach is that the internalism – externalism dichotomy disappears because in order to produce such a narrative it is often the case "that the historian must insist on *both* the objective character of mathematical knowledge *and* the fact that this knowledge was constructed in a fabric of social and communicative actions."[8] Epple points to the fact that external approaches to the history of mathematics are rare, but suggests that further historical studies may reveal that various aspects of the technological developments during the Second World War may have had a deeper influence on the development of 20th century mathematics than it is usually assumed. Approaches in this directions are the papers by Dahan-Dalmedica and Kjeldsen for the case of USA,[9] and Epple and Remmert for the case of Germany.[10] Epple also refers to papers by MacKenzie, Daston and Brian,[11] which shows that in the history of statistics external factors have been taken into account to a large extent.[12] Epple's historiographical method encourages all kinds of different approaches to the history of mathematics; as an example he refers to the study of Donald MacKenzie from 1999, of the history of the proof for the four color problem,[13] which is an example of a sociohistory of a mathematical proof. When historians begin to look for the causality of events the field opens widely and blurs the distinction between internalism and externalism.

[5](Rowe 1996, p. 4).
[6](Epple 1998, p. 307).
[7](Epple 1998, p. 307).
[8](Epple 1998, p. 307).
[9](Dahan-Dalmedico 1996), (Kjeldsen 2000).
[10](Epple and Remmert 2000).
[11](MacKenzie 1981), (Daston 1988) and (Brian 1994).
[12](Epple 2000, p. 144).
[13](MacKenzie 1999).

As far as the interaction between history of mathematics and the philosophy of mathematics is concerned Richards points to the danger she finds in adopting within the history of mathematics the view of foundationalism in the philosophy of mathematics. Because, Richards argues, this separates mathematics from "the messiness of human history", and will cause the historian to miss a great deal of the diversity in which mathematics develops.[14] Instead she argues for historical studies that incorporate the human element of mathematics since a historical approach where the practice of mathematics is in focus will give a much richer history of mathematics. This means that Richards welcomes the new so-called *maverick trend* in the philosophy of mathematics and she invites the historian of mathematics to "part company with the philosopher or the sociologist" to apply a "rich array of approaches and methodologies" to the history of mathematics.[15]

One of the debates about the nature of mathematics that has also defined to a wide extent the history of mathematics is the cumulative nature of mathematics. However, some historical studies from the last two decades have lead to a new understanding of changes in mathematics. As an example of this Richards refers to the study by Henk Bos of geometric construction problems in the seventeenth century.[16] The result of Bos' historical analysis of the changing approaches to geometric construction problems shows that with Descartes' method of geometry the goals of the subject became superfluous. Bos' work also shed lights on the role that 'challenging problems' play in order for a mathematical field to attract the interest of the mathematicians and thereby in keeping it alive.[17] Daston's and Richards' work on projective geometry show that also other factors than challenging problems can influence the level of activity in field.[18] They show that a reason behind the decline of interest in projective geometry by the end of the nineteenth century was due to changes in the larger philosophical and institutional matrix of mathematical practice.[19] This is an example where the motivation for working with a subject was ruled more by the surrounding society, than by mathematics itself. Bos' work also points towards the significance of other than mathematical factors responsible for the development of mathematics. He shows that the geometry of Descartes is closely linked to Descartes' philosophical thinking. Richards views Bos' work as an indication of "the importance of balancing technical with contextual forces in assessing the development of mathematics."[20]

[14](Richards 1995, p. 127).
[15](Richards 1995, p. 130).
[16](Bos 1990).
[17](Richards 1995).
[18](Daston 1986), (Richards 1986).
[19](Richards 1995, p. 131).
[20](Richards 1995, p. 132).

At the beginning of the 20th century philosophers of mathematics were quite concerned with certainty. This, of course, was a consequence of the foundational crisis. The well-known paradoxes in set theory seemed to threaten the consistency of mathematics. Hilbert expressed this very strongly

> Confronted by these paradoxes, Dedekind and Frege completely abandoned their point of view and retreated. ··· So violent was this reaction that even the most ordinary and fruitful concepts and the simplest and most important deductive methods of mathematics were threatened and their employment was on the verge of being declared illicit.[21]

On the other hand, methods of set theory and other abstract and advanced mathematical fields led to new epoch-making ideas and results - ideas and results very few mathematicians wanted to lose. Hilbert expressed it poetically in the following way

> No one shall drive us out of the paradise which Cantor has created for us.[22]

So, during the beginning of the 20th century new abstract methods and theories - based on transfinite methods - had appeared which led to new exciting results. But this development was loaded down with concerns about the justification of these methods. It is therefore understandable that philosophers of mathematics saw it as a main issue to justify these new methods, either by restricting them, and by that reducing the power of new mathematical tools, or by finding some way of justifying or proving the consistency of mathematics. Philosophy of mathematics became foundational studies.

It is well-known that - mainly due to Gödel's incompleteness results - it is impossible to prove the consistency of mathematics within a formal framework. Even after it became clear that it was impossible to establish the consistency of mathematics foundational studies continued to dominate the philosophy of mathematics, and these studies have led to new fruitful results both within mathematics itself and within the philosophy of mathematics. For instance, Hilbert's proof theory has developed into a powerful mathematical theory and Hilbert's original foundational programme has been modified in realistic ways and as such produced abundantly many results of great philosophical value.[23] In a similar way, restrictive programmes like Brouwer's intuitionism has developed into interesting mathematical and philosophical theories, as for instance constructive theories of the continuum.[24]

However, in the second part of the 20th century the focus of philosophy of mathematics changed. The question of certainty is no longer the only central

[21] (Hilbert 1926, p. 190).
[22] (Hilbert 1926, p. 190.)
[23] See, for instance, the survey papers (Feferman 1988), (Sieg 1988), and (Simpson 1988)
[24] We are thinking of topos theory and synthetic differential geometry as developed in (Kock 1981), (Moerdijk and Reyes 1991), and (MacLane and Moerdijk 1992)

issue. It is still an important question. But it is realized that mathematics is a cultural activity where human beings are shaping new ideas, developing new methods, producing theorems and proofs, etc. As all other human activities it is fallible. One cannot expect complete certainty especially when the scope of mathematics is constantly developing. Foundational questions are still important, but there are other kinds of problems which philosophy of mathematics must face.

During the first half of the 20th century the main focus of philosophy of mathematics was on the products of mathematics, i.e., theorems and proofs. Within this scope it was natural to concentrate on the quality of the products, namely their justification and potential use. In this sense philosophy of mathematics resembled the philosophy of natural sciences. Reichenbach's classical distinction between the logic of discovery and logic of justification seemed to be accepted. Epistemology and philosophy of science were mainly occupied with justification, whereas questions about the process of discovery were matters of psychology and therefore outside the scope of philosophy.

As is well-known this attitude changed in the 50thies and 60thies. One big event which indicated this change was the publication in 1962 of Kuhn's seminal work *The Structure of Scientific evolutions*.[25] This work, together with similar works by philosophers like Norwood Russell Hanson[26] and Stephen Toulmin,[27] had a radical impact on philosophy of science. From these and other works it became evident that the distinction between the logic of discovery and the logic of justification could not pass muster. In order to understand philosophically the nature of science it was necessary to consider the total process of scientific production and not only the final products.

This change in philosophy of science did not to the same extent take place within philosophy of mathematics. One of the first publications that initiated a similar development in philosophy of mathematics was Imre Lakatos' dissertation *Proofs and Refutations*[28] which appeared in 1976. Lakatos discussed in details the development of Euler's polyhedron theorem[29] from being a simple hypothesis about polyhedrons to being a well-established theorem in modern topology.

A main idea in Lakatos' work is that first versions of theorems are conjectures, and that mathematicians try to establish or reject assumed theorems by either giving proofs or constructing counterexamples. A first proof may uncover questionable assumptions which may lead to counterexamples as in

[25](Kuhn 1970)
[26](Hanson 1958).
[27](Toulmin 1972).
[28](Lakatos 1976).
[29]For every polyhedron of genus zero (i.e., no holes) the Euler characteristic is equal to two:

$$V - E + F = 2$$

where V is the number of vertices, E the number af edges, and F the number of faces.