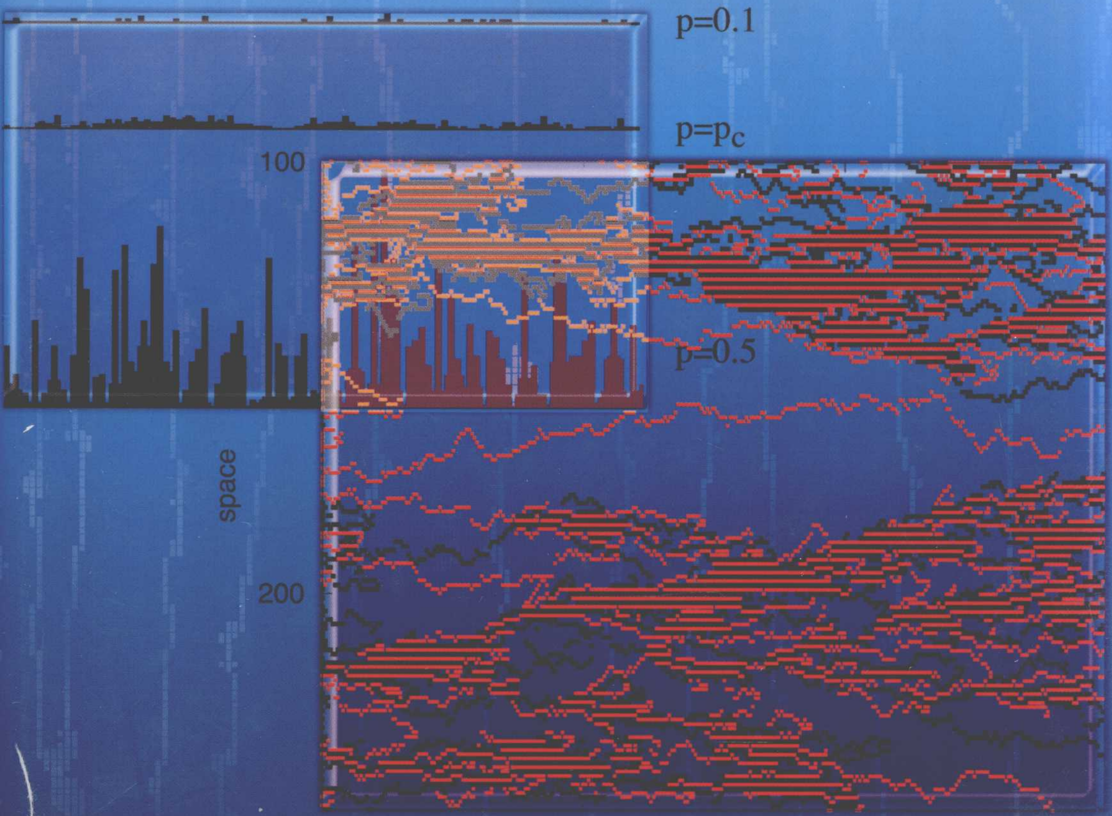
# Universality in Nonequilibrium Lattice Systems

## Theoretical Foundations

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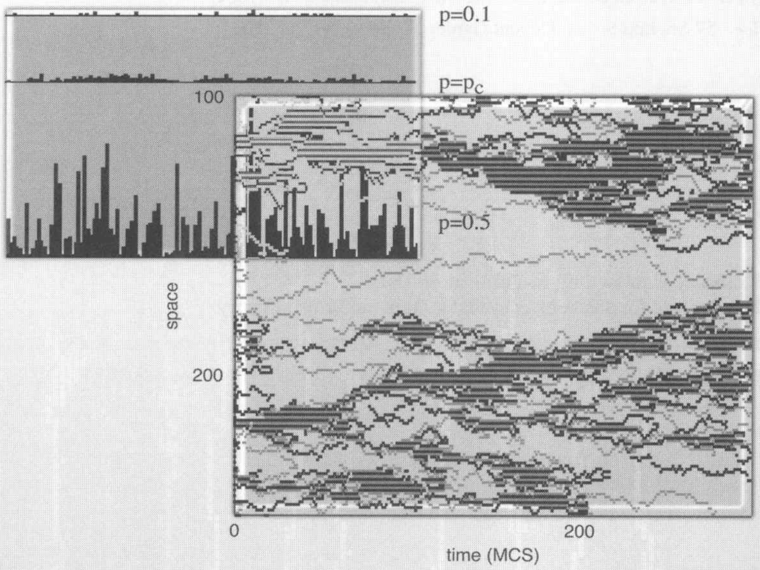


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**UNIVERSALITY IN NONEQUILIBRIUM LATTICE SYSTEMS**
**Theoretical Foundations**



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# Preface

Universal scaling behavior is an attractive feature in statistical physics because a wide range of models can be classified purely in terms of their collective behavior due to a diverging correlation length. Scaling phenomenon has been observed in many branches of physics, chemistry, biology, economy etc., most frequently by critical phase transitions and surface growth. Nonequilibrium phase transitions appear, for example, in models of

- origin of life [Cardozo and Fontanari (2006b)]
- biological control systems [Kiyono *et al.* (2005)]
- enzyme biology [Berry (2003)]
- brain [Werner (2007)]
- population [Albano (1994)]
- spatiotemporal intermittency [Jabeen and Gupte (2005)]
- socio-physics [Baronchelli *et al.* (2007)]
- epidemics [Ligget (1985); Mollison (1977)]
- catalysis [Ziff *et al.* (1986); yin Hua (2004)]
- itinerant electron systems [Feldman (2005)]
- cooperative transport [Havlin and ben Avraham (1987); Chowdhury *et al.* (2000)]
- plasma physics [Knapek *et al.* (2007)]
- stock-prize fluctuations and markets [Bouchaud and Georges (1990); Kiyono *et al.* (2006)]
- collision of solids [Kun and Hermann (1999)]

The concept of self-organized critical (SOC) phenomena has been introduced some time ago [Bak *et al.* (1987)] to explain the frequent occurrence of scaling laws experienced in nature. The term SOC usually refers to a mechanism of slow energy accumulation and fast energy redistribution, driving a system toward a critical state. The prototype of SOC systems is the sand-pile model in which particles are randomly dropped onto a two dimensional lattice and the sand is redistributed by fast avalanches. Therefore, in SOC models, instead of tuning the parameters, an inherent mechanism is responsible for driving it to criticality. SOC mechanism has been proposed to model

- earthquakes [Bak and Tang (1989); Sornette (1989)]
- the evolution of biological systems [Bak and Sneppen (1993)]
- solar flare occurrence [Lu and Hamilton (1991)]
- fluctuations in confined plasma [Politzer (2000)]
- snow avalanches [Faillettaz *et al.* (2004)]
- rainfall [Peters *et al.* (2002)]

However, SOC critical classes can be shown to be equivalent to those of the ordinary critical ones by identifying the control parameters and the boundary conditions properly [Vespignani and Zapperi (1997); Dickman *et al.* (1998); Dickman (2002a)], therefore I shall discuss the critical behavior of SOC models briefly in Sect. 6.10.

Diverging correlation length — necessary to change the global symmetry at a second order phase transition point — and scaling may also occur away from the critical point. Naturally, in a fully ordered state (below the ordering temperature) the correlation length is infinite. If the interactions of the system is such that reaching this state requires diverging time, one finds dynamical scaling near that point. This happens usually in case of multi-particle, reaction-diffusion systems in the ordered phase (experimentally observed in [Kroon *et al.* (1993)]). In quantum matter, near absolute zero temperature thermal equilibration can be obstructed in the case of topological ordered ground states, where only the slow dynamical relaxation of defects pairs — via annihilation-diffusion — can occur [Chamon (2005)]. By quenching magnets to zero, temperate domain coarsening occurs by power laws since topological defects such as interfaces or vortices slow down the dynamics [Bray (1994)].

Rough surfaces and interfaces can also exhibit temporal and spatial scaling if the correlation length and time diverges. They are ubiquitous in nature and from a technological point of view the control of their roughness is becoming critical for applications in fields such as micro-electronics, image formation, surface coating or thin film growth [Chow (2000)]. Understanding the fundamental laws driving tumor development is one of the biggest challenges of contemporary science. Internal dynamics of a tumor reveals itself in a number of phenomena, one of the most obvious ones being the growth [Brutovsky *et al.* (2006); Martins *et al.* (2007)].

Earlier most of the models were investigated on regular lattice type of systems (approximating a smooth field theory by continuum limit). It is well known that for systems with translational symmetry (like a lattice) the influence of underlying structure becomes negligible at the critical point (i.e. when the correlation length is much larger than the cell spacing). Note that in systems with non-integer (fractal) dimensions the translational symmetry is replaced by discrete scale invariance and the topological details of the generating cell are present at any scale. As the consequence, log-periodic corrections to scaling — described by complex exponents — occur [Sornette (1998)], which I shall not discuss here. The advantage of lattice realizations is that they are simpler to handle than models in continuum space, e.g., they sometimes allow for exact results and are easier to be implemented in a computer. Furthermore, a bunch of emerging techniques may now be applied to lattice systems, including nonequilibrium statistical field theory. A general amazing result from these studies is that lattice models often capture the essentials of

- social organisms [Antal *et al.* (2001); Washenberger *et al.* (2007)]
- epidemics [Hinrichsen (2007a)]
- glasses [Chamon (2005)]
- electrical circuits
- transport [Ez-Zahraouy *et al.* (2006)]
- hydrodynamics [Díez-Minguito *et al.* (2005)]
- colloids, computational neuro-science [Furtado and Copelli (2006)]
- botany [K. A and Peak (2006)]

In the past few years, the interest focused on the research of complex scale-free networks [Albert and Barabási (2002); Barabási (2002); Dorogovtsev and Mendes (2003)]. Recently the dynamics and the phase transitions of network systems came under study [Aldana and Larralde (2004)]. Contrary to the regular lattices universality in network models is not so well defined, usually it depends on the underlying topology, therefore I shall not discuss it in this book. A very recent analytical study has shown that the asymptotics of random walks on uncorrelated random networks is essentially the same as that for regular Bethe lattices [Samukhin *et al.* (2007)].

Dynamical extensions of static universality classes — established in equilibrium — are the simplest nonequilibrium models systems, but beyond that critical phenomena, with classes have been explored so far [Marro and

Dickman (1999); Grassberger (1996); Hinrichsen (2000a)]. While the theory of phase transitions is quite well understood in thermodynamic equilibrium, its research in nonequilibrium is rather new. In general phase transitions, scaling and universality retain most of the fundamental concepts of equilibrium models. The basic ingredients affecting universality classes are again the collective behavior of systems, the symmetries, the conservation laws and the spatial dimensions as described by renormalization group (RG) theory. Besides these, several new factors have also been identified recently. Low dimensional systems are of primary interest because the fluctuation effects are relevant, hence the mean-field type of description is not valid. In the past decades, this field of research grew very quickly and now we are faced with a zoo of models, universality classes, strange notations and abbreviations. This book aims to help newcomers as well as researchers to navigate in the literature by systematically reviewing most of the explored universality classes. I define models by their field theory (when it is available), show their symmetries or other important features and list the critical exponents and scaling relations.

Nonequilibrium systems can be classified into two categories:

(a) Systems which do have a hermitian Hamiltonian and whose stationary states are given by the proper Gibbs-Boltzmann distribution. However, they are prepared in an initial condition which is far from the stationary state and sometimes, in the thermodynamic limit, the system may never reach true equilibrium. These nonequilibrium systems include, for example, phase ordering systems, spin glasses, glasses etc. I show the scaling behavior of the prototypes of such systems in the second chapter (Out of Equilibrium Classes). These are defined by the addition of simple dynamics to static models. The initial condition can be regarded as a boundary condition in the time direction, hence these models exhibit strong resemblance to static models near surfaces. They are discussed in Chapter 2.

(b) Systems without a hermitian Hamiltonian defined by transition rates, which do not satisfy the detailed balance condition (the local time reversal symmetry is broken). They may or may not have a steady state and even if they have one, it is not a Gibbs one. Such models can be created by combining different dynamics or by generating currents in them externally. In some cases, their critical behavior is insensitive to such changes and these are discussed in Chapter 2. There are also systems, which are not related to equilibrium models, in the simplest case these are lattice Markov processes of interacting particle systems [Ligget (1985)]. These are referred here as "genuinely nonequilibrium systems" and are discussed in the rest of the work.

The discussion of latter type of classes is split into five parts: In Chapter 3, transition classes of models possessing fluctuating ordered states are provided. There are not too many of them known yet, but the exploration of the phase transitions of current driven systems attracts much interest nowadays. In Chapter 4, phase transition classes of models with absorbing states are presented. These are usually reaction-diffusion (RD) type of models, but sometimes they are defined by spin systems or via a coarse grained Langevin equation. In Chapter 5, I briefly touch on the point of discontinuous nonequilibrium phase transitions and tricritical phenomena, especially because dynamical scaling may occur in such nonequilibrium cases. In Chapter 6, I list known classes, which occur by combinations of basic genuine class processes. These models are coupled, multi-component RD systems. While the former three chapters are related to critical phenomena near to extinction, in Chapter 7, I discuss universality classes in systems where site variables are non-vanishing in surface growth models. The bosonic field theoretical description is applicable for them. I point out mapping between growth and RD systems when it is known.

I define a critical universality class by the complete set of exponents at the phase transition. Therefore, different dynamics split up the basic static classes of homogeneous systems. I emphasize the role of symmetries and boundary conditions which affect these classes. I also point out very recent evidence, according to which in low-dimensional systems, symmetries are not necessarily the most relevant factors of universality classes. Although the systems covered here might prove to be artificial to experimentalists or to application-oriented people they constitute the fundamental blocks of understanding of nonequilibrium critical phenomena. Note that even the understanding of models so simple runs into tremendous difficulties very often.

I shall not discuss the critical behavior of quantum systems [Rácz (2002)] and just briefly mention experimental realizations. However, it is well known that a quantum phase transition (occurring at $T = 0$ due to quantum fluctuations) in $d$ space dimensions can be mapped onto a classical (finite temperature) transition at $d + Z$ dimensions (where $Z$ (see Sect. 1.3) is the dynamical exponent, and $Z = 1$ in space-time isotropic systems). The effect of boundary conditions in static models is reviewed elsewhere. The detailed discussion of the applied methods is also omitted due to the lack of space, although in Sect. 1.6 I give a brief introduction to the field theoretical approach in the first chapter. This section shows the formalism for defining nonequilibrium models. This is necessary to express the symmetry relations affecting critical behavior. The detailed discussion of renormalization group solutions is not provided in this book, and only mean-field theoretical derivations and scaling arguments are shown in some important cases.

Researchers from other branches of science are provided with a kind of catalog of classes in which they can identify their models and find corresponding theories. The classification of phase transition classes of one-component, bosonic reaction-diffusion models has been attempted very recently [Elgart and Kamenev (2006)]. This is based on topological portraits of "zero energy" lines of the reaction Hamiltonians in the phase-space, similarly to the Ginzburg-Landau potential minima in the case of equilibrium systems. Although the predictions of this scheme is not always in agreement with the results of other methods, especially in case of non-bosonic models (with topological site restrictions) it gives a constructive, organizational view for the zoo of nonequilibrium models and classes. I discuss this new method in Sect. 1.6.1 and the corresponding phase portraits will be shown in the field theoretical introduction of the reaction-diffusion classes.

Another very recent advance in this field is the recognition of more general scale transformations than mere rescaling. Similar to the conformal invariance in equilibrium systems, the concept of local scale invariance (LSI) has been introduced [Henkel (2002)]. I provide LSI scaling exponents and forms determined recently for some basic models and discuss their limitations.

Besides scaling exponents and scaling relations, there are many other interesting features of universality classes like scaling functions, extremal statistics, finite size effects, fluctuation-dissipation theory etc., which I do not discuss in this review. Still, I believe the material shown provides a useful frame for orientation in this huge field. There is no general theory of nonequilibrium phase transitions, hence a widespread overview of known classes can help theorists deduce the relevant factors determining universality classes.

There exists some recent, similar reviews in the literature. One of them is by [Marro and Dickman (1999)], which gives a pedagogical introduction to driven lattice gas systems and to fundamental particle systems with absorbing states. The other one [Hinrichsen (2000a)] focuses more on basic absorbing state phase transitions, methods and experimental realizations. A more technically detailed review on universal scaling of basic models exhibiting absorbing phase transitions supplements the second one [Lübeck

(2004)], focusing on scaling functions, external fields, crossovers and upper-critical behavior.

This book is based on a former review article by the author, which considered nonequilibrium universality classes systematically [Ódor (2004a)]. That overview aimed to give a comprehensive overview of known nonequilibrium dynamical classes, incorporating surface growth classes, classes of spin models, percolation and multi-component system classes and damage spreading behavior. Relations and mappings of the corresponding models were pointed out. Now effects of anisotropy, boundary conditions, long-range interactions, external fields and disorder are shown more systematically for each class. The crossover between classes is emphasized by boldface letters. This book provides an updated and extended review (as a result of lack of the size limitation of an article). The extensions includes local scale invariance, phase space topologies, non-perturbative renormalization group etc.

To help navigating in the text and in the literature I provided a list of the most common abbreviations in the appendix. Naturally this review cannot be complete and I apologize for any references I have inadvertently omitted.

*Géza Ódor*

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Support from the Hungarian Research Fund OTKA (Grant No. T-046129) during the preparation of this book is gratefully acknowledged.

# Contents

# Chapter 1

# Introduction

## 1.1 Critical exponents of equilibrium (thermal) systems

In this section I briefly summarize the definition of well known critical exponents of homogeneous equilibrium systems and show some scaling relations [Fisher (1967); Kadanoff *et al.* (1967); Stanley (1971); Ma (1976); Amit (1984)]. The basic thermal exponents (denoted by subscript '$H$' to avoid confusion with the some nonequilibrium ones defined later) are defined via the scaling laws:

$$c_H \propto \alpha_H^{-1} \left( (|T - T_c|/T_c)^{-\alpha_H} - 1 \right) \ , \tag{1.1}$$

$$m \propto (T_c - T)^{\beta} \ , \tag{1.2}$$

$$\chi \propto |T - T_c|^{-\gamma} \ , \tag{1.3}$$

$$m \propto H^{1/\delta_H} \ , \tag{1.4}$$

$$G_c^{(2)}(r) \propto r^{2-d-\eta_\perp} \ , \tag{1.5}$$

$$\xi \propto |T - T_c|^{-\nu_\perp} \ . \tag{1.6}$$

Here $c_H$ denotes the specific heat, $m$ the order parameter, $\chi$ the susceptibility and $\xi$ the correlation length. Note that the anomalous dimension exponent in the spatial two-point correlation scaling-law is denoted by $\eta_\perp$ in nonequilibrium systems — where $\perp$ corresponds to perpendicular to the time direction — therefore in the time being I shall use this notation. The presence of another degree of freedom besides the temperature $T$, like a (small) external field (labeled by $H$), leads to other interesting power laws when $H \to 0$. The $d$ present in the expression of two-point correlation function $G_c^{(2)}(r)$ is the space dimension of the system.

Some laws are valid both to the right and to the left of the critical point; the values of the relative proportionality constants, or *amplitudes*,