Recent Advances in Fluid Mechanics

RECENT ADVANCES IN FLUID MECHANICS

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RECENT ADVANCES IN FLUID MECHANICS

Dedicated to Prof. N. Rudraiah

PREFACE

This Festschrift is published as a tribute to Professor N. Rudraiah on the occasion of his 65th birthday. It is a very happy occasion for his students, colleagues and friends in India and abroad. Every one of them could think of him as an affectionate teacher, researcher, accomplished (applied) mathematician and committed educational administrator.

Professor Rudraiah was born at Bellave near Tumkur in Karnataka state, India, on 18 August 1932. He obtained his Honors and Masters degrees in Mathematics from the Central College, University of Mysore in 1955 and 1957 respectively. He pursued his graduate work in Applied Mathematics at the University of Toronto (1959–60) and obtained his Master's degree in 1960. Then he joined the Department of Applied Mathematics, University of Western Ontario, London, Canada and obtained his PhD degree under the guidance of Professor J.H. Blackwell in 1964.

On his return to India in 1964 he became a member of the faculty in the Department of Mathematics at the Central College, Bangalore, which was one of the best known colleges in the country, devoted to teaching and research in different branches of science, including mathematics. Professor Rudraiah initiated research activities in fluid mechanics, particularly in magnetohydrodynamics, as soon as he returned to India from Canada in 1964, with a Research Project on magnetohydrodynamics sponsored by the Council of Scientific and Industrial Research (CSIR), India.

Professor Rudraiah established a good research group working on different aspects of fluid mechanics like MHD generator, lubrication problems, internal gravity waves, heat and mass transfer, flow through and past porous media, and biomechanics. The work done by him and his research group did not happen by chance or by decree. The development of these topics was the natural result of an important and relatively recent shift in the priorities of industrial developments in India. In recognition of the research work carried out by him and his group at the Department of Mathematics, Bangalore University, in the area of fluid mechanics, the University Grants Commission (UGC) sanctioned special assistance for Mathematics under the Departmental Special Assistance (DSA) Programme with Thrust Area in Fluid Mechanics for a period of five years in the first instance, starting in 1981. On account of the excellent work done under the DSA Programme, the University Grants Commission extended the Programme up to 1998, and also sanctioned the COSIST Programme in 1993 for a period of five years in the first instance. In 1981 Professor Rudraiah was instrumental in establishing a Centre for Fluid Mechanics which attracted a number of university scholars and professors from different parts of the country and from abroad. In the basic research he worked in the area of linear and nonlinear stability of layer stratification (heterogeneous fluids) and thermal stratification and double diffusive convection with and without the external constraints of Lorentz and/or Coriolis force. Keeping in view the needs of the country, he focused his attention on surface instability in composite layers, i.e., fluids bounded by porous layers with an application in materials science.

After his excellent service as Professor of Mathematics, Dean, Faculty of Sciences, Principal of the prestigious Central College and Co-ordinator of the UGC-DSA Programme, Bangalore University, Professor Rudraiah was called upon to take over as Vice-Chancellor of Gulbarga University in 1990. During his tenure of six years from 1990 to 1996, Professor Rudraiah established a number of job-oriented post-graduate programmes in different branches of Science, Humanities, Social Sciences and Management. In addition, he gave a major thrust to research by initiating and encouraging basic and applied research in the above fields. With his varied academic and administrative experience and capacity to make quick and incisive decisions he was directly responsible for the growth of the university to become one of the leading universities in a brief span of six years. Apart from being a successful academician and researcher and an administrator *par excellence*, Professor Rudraiah is also an institution builder. As President of the Indian Society of Theoretical and Applied Mechanics (ISTAM) for two terms and President of the Mathematical Section of the Indian Science Congress, he gave impetus to ISTAM to shape itself as an effective vehicle for interaction between basic and applied researchers in India. His research career culminated in the establishment of the National Research Institute for Applied Mathematics in Bangalore in 1996.

After his successful completion of two terms as Vice-Chancellor of Gulbarga University in 1996, Professor Rudraiah was awarded the position of a senior scientist by the Indian National Science Academy. He decided to work at the UGC-DSA centre in fluid mechanics, Department of Mathematics, Bangalore University.

Professor Rudraiah, as a member of the board of studies in mathematics of several universities both inside and outside the state and as chairman of the board of studies in mathematics of Bangalore University, played a significant role in mathematics education at undergraduate and postgraduate levels. He was chairman of the common syllabus committee to frame the common syllabi for the undergraduate courses in mathematics for all of the universities in Karnataka State.

Professor Rudraiah has received national and international honours in recognition of his outstanding achievements in the professional and related fields of activities. He was awarded the KIT fellowship of Japan. He is a recipient of the FICCI Award for his outstanding contribution in mathematics, including physical sciences. He is also a recipient of the Rajyotsava award in the field of education from the Government of Karnataka. He was awarded a National Lectureship by the UGC.

Professor Rudraiah is a Fellow of all three scientific academies in India, namely the Indian National Science Academy, the National Academy of Sciences and the Indian Academy of Sciences. He is also a Fellow of the Institute of Mathematics and its Applications, UK. He is a member of the editorial boards of many research journals, including the *International Journal of Porous Media*, USA.

Professor Rudraiah has been blessed with a charming wife, Manonmani Rudraiah. With her suave and gentle manners she has contributed a lot in her self-effacing way to Professor Rudraiah's success. They have one son who is an engineer and runs an industry.

For some of us, who were fortunate to have worked with him, it is a great pleasure to wish Professor Rudraiah and his family a long, healthy, happy and productive life and further accomplishments. His students, colleagues and friends all over the world join the members of the organising committee in paying tribute to him.

This Festschrift is a small token of appreciation and a tribute to the foresight, vision and creative imagination of Professor Rudraiah. The editors are indeed grateful to all the distinguished authors who readily agreed to send in their contributions, and to The Gordon and Breach Publishing Group for bringing out this volume. It is our hope that the papers in this volume will prove a useful source for research workers in the areas of fluid mechanics, heat transfer, stability of flows, MHD and waves.

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I.1. COMPUTER EXTENDED SERIES SOLUTION AND ANALYTIC CONTINUATION OF NONLINEAR BOUNDARY VALUE PROBLEMS

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1. INTRODUCTION

The problems associated with similarity solutions of fluid dynamics invariably result into regular perturbation problems. The successive approximation of the solution accounts for the slow variations. To unveil the analytic structure of the solution function it is necessary to calculate sufficiently large number of terms in the perturbation scheme. This long series also ensures in extending its region of validity and also yields more accurate solution. The sign pattern of the coefficients of the series give the direction of the singularity restricting the convergence of the series. Domb–Sykes plot helps in finding radius of convergence of the series and it enables in identifying the nature of singularity. There are number of techniques which can be used in achieving the above mentioned desirable features of the series. Review articles (Van Dyke, 1974, 1987) give listing of the prominent characters of this technique and they have made it popular in fluid mechanics. Bujurke and his collabo- rators (1992–1995) have successfully used this technique in analyzing number of quite complex problems. These are listed in Refs. 3–10. For the purpose of detail illustration one of the problems studied by (Ishizawa, 1966) in respect of similarity solutions of two-dimensional and axisymmetric unsteady squeezing of viscous flow between two parallel plates is considered here. Ishizawa obtains multifold series solution as well as pure numerical solution.

2. FORMULATION OF THE PROBLEM

Consider two plates at $y=\pm a(1-\alpha t)^{1/2}$ where a is the position at time $t=0$ as shown in Figure 1. When α is positive the two plates are squeezed until they touch at $t=1/\alpha$. When α is negative the two plates are separated. We shall assume the length (l, in the two-dimensional case) or the diameter (D, in the axisymmetric case) of the plates to be much larger than the gap width $2y$ at any time such that the end effects could be neglected. For continuity considerations the lateral velocity must be directly proportional to the distance from the centre. Let u, v, w be velocity components in the x, y and z directions respectively. For two-dimensional flow, we introduce the following transformations (Wang, 1976):

$$u=(\alpha x/(2(1-\alpha t)))f'(\eta), \tag{2.1}$$

$$w=(-a\alpha/(2(1-\alpha t)^{1/2}))f(\eta), \tag{2.2}$$

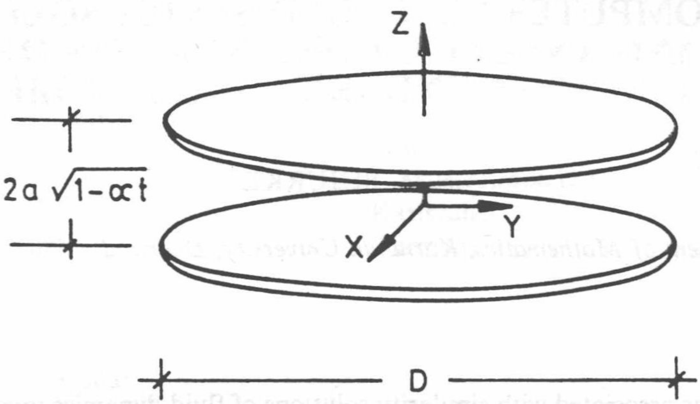

Figure 1 Schematic diagram of the problem.

and

$$\eta = z/(a(1-\alpha t)^{1/2}). \tag{2.3}$$

The two-dimensional Navier–Stokes equations (Batchelor, 1967) then reduce to an ordinary nonlinear differential equation

$$\eta f''' + 3f'' + f'f'' - ff''' = (1/S)f'''' \tag{2.4}$$

where $S = \alpha a^2/2\nu$ is the nondimensional parameter which represents the relative importance between unsteady and viscous forces (which can be termed as local Reynolds number). The boundary conditions are that on the plates the lateral velocities are zero and the normal velocity is equal to the velocity of the plate, i.e.

$$f(0) = f''(0) = f'(1) = 0, \qquad f(1) = 1. \tag{2.5}$$

Similarly, for axisymmetric flow we use the transforms

$$u = (\alpha x/(4(1-\alpha t)))f'(\eta), \tag{2.6}$$

$$v = (\alpha y/(4(1-\alpha t)))f'(\eta), \tag{2.7}$$

$$w = (-a\alpha/(2(1-\alpha t)^{1/2}))f(\eta) \tag{2.8}$$

and axisymmetric Navier–Stokes equations (Batchelor, 1967) reduce to

$$\eta f''' + 3f'' - ff''' = (1/S)f'''' \tag{2.9}$$

with the boundary conditions (2.5).

Therefore, the problem reduces to the solution of nonlinear ordinary differential equation

$$\eta f''' + 3f'' + \beta f'f'' - ff''' = (1/S)f'''', \tag{2.10}$$