Symmetry Principles in Elementary Particle Physics

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PREFACE

The study of elementary particles and their interactions has brought to light symmetries and relationships which are nowadays objects of study in themselves. In this book we have attempted to present these symmetry principles and their associated conservation laws as a set of interrelated physical principles, explained in terms of the simplest appropriate mathematics. Mathematical excursions into the more abstract aspects of the subject have been avoided; thus in particular, we have not made explicit use of the formal apparatus of group theory. Similarly we have omitted all descriptions of the experimental methods by which quoted results have been obtained.

The level is thus intended to meet the needs of a graduate student working in particle physics, who wants an accurate but not too abstract explanation of the principles commonly quoted in the literature of his subject. It is to be hoped that many such readers would afterwards progress further with the aid of more advanced literature.

We have drawn heavily on material used by both of us for postgraduate lectures in Bristol, and we acknowledge the contribution which these postgraduate classes have made to our own powers of understanding and explanation.

Our thanks are due also to the many colleagues and friends who have over the years shed light on difficult topics through discussion, and to Miss Alma Dawes, Miss Margaret James, Miss Anna Love and Mrs Nancy Thorp who have typed our outpourings. We should also like to thank Dr J.W. Alcock for assistance with proof-reading and the editorial staff of the Cambridge University Press for their assistance at all stages.

W.M. Gibson
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CONTENTS

CHAPTER 4

Lorentz invariance

CHAPTER 5
Parity

CHAPTER 1

INTRODUCTION TO ELEMENTARY PARTICLES

1.1 Perspective

The general field to which this book contributes is known to some as the Physics of Elementary Particles, to others as High Energy Physics, and has recently been given the additional name of Sub-nuclear Physics. These titles are almost synonymous, emphasising respectively the search for basic components of which matter is made up, the need for high energies to probe the inner structure of matter, and the fact that the search leads us deeper than the atomic nucleus.

Many textbooks in this field have presented the elementary particles and their properties, leading from the regularity of these properties to the gradually uncovered theory of the fundamental interactions. We, however, take as our subject not the particles themselves but the symmetry principles and conservation laws by which their properties are governed. The understanding of these laws and principles has of course grown from the experimental study of the actual particles, and this fact must continually bring us back to the basis of observed fact, as we work through the essentially mathematical framework of the symmetry principles and conservation laws.

1.2 The particles

A purely empirical classification of the elementary particles may be made according to mass, with baryons having mass of the order of that of the proton, leptons having small or zero rest mass, and mesons intermediate mass.

It soon becomes clear, however, that properties other than mass can lead us to classification schemes of a more fundamental nature. First we have the question of spin and statistics: the important distinction here is between particles of half-integral spin which obey Fermi–Dirac statistics and particles of zero or integral spin which obey Bose–Einstein statistics. The former, known as fermions, can be created only as pairs with corresponding antiparticles, so that the total number of a given type of fermion is conserved (see §1.4). It is an observed fact, so far

Table 1.1. *Baryons and antibaryons*

		I_3	I	J	P	S	Y	B
N	p	$+\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	+	0	+1	+1
	n	$-\frac{1}{2}$						
	Λ^0	0	0	$\frac{1}{2}$	+	−1	0	+1
Σ	Σ^+	+1	1	$\frac{1}{2}$	+	−1	0	+1
	Σ^0	0						
	Σ^-	−1						
Ξ	Ξ^0	$+\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	+	−2	−1	+1
	Ξ^-	$-\frac{1}{2}$						
$\overline{N}$	$\bar{p}$	$-\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	−	0	−1	−1
	$\bar{n}$	$+\frac{1}{2}$						
	$\overline{\Lambda}^0$	0	0	$\frac{1}{2}$	−	+1	0	−1
$\overline{\Sigma}$	$\overline{\Sigma}^+$	+1	1	$\frac{1}{2}$	−	+1	0	−1
	$\overline{\Sigma}^0$	0						
	$\overline{\Sigma}^-$	−1						
$\overline{\Xi}$	$\overline{\Xi}^+$	$+\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	−	+2	+1	−1
	$\overline{\Xi}^0$	$-\frac{1}{2}$						

Table 1.2. *Mesons* ($B = 0$)

	I_3	I	G	J	P	C_n	$S = Y$
π^+	+1	1	−	0	−	+	0
π^0	0						
π^-	−1						
K^+	$+\frac{1}{2}$	$\frac{1}{2}$		0	−		1
K^0	$-\frac{1}{2}$						
$\overline{K}^0$	$+\frac{1}{2}$	$\frac{1}{2}$		0	−		−1
K^-	$-\frac{1}{2}$						
η	0	0	+	0	−	+	0
ρ^+	+1	1	+	1	−	−	0
ρ^0	0						
ρ^-	−1						
ω	0	0	−	1	−	−	0
ϕ	0	0	−	1	−	−	0

Table 1.3. *Leptons and antileptons* ($J = \frac{1}{2}$)

	L	Helicity
e^-	$+1$	$\pm\frac{1}{2}$
ν_e	$+1$	$-\frac{1}{2}$
e^+	-1	$\pm\frac{1}{2}$
$\bar{\nu}_e$	-1	$+\frac{1}{2}$
μ^-	$+1$	$\pm\frac{1}{2}$
ν_μ	$+1$	$-\frac{1}{2}$
μ^+	-1	$\pm\frac{1}{2}$
$\bar{\nu}_\mu$	-1	$+\frac{1}{2}$

unexplained, that all fermions have a non-zero baryon or lepton number, and are distinguishable from their antiparticles by the opposite values of these numbers. Bosons, on the other hand, can be created in numbers which are limited only indirectly by other conservation laws. For example high energy neutron–proton scattering can be accompanied by the creation of one, two, three or more pions. The baryons and the leptons are fermions, while all the strongly interacting mesons are bosons. The muon, originally classed as a meson on account of its mass, is classed as a lepton by virtue of its spin $\frac{1}{2}$ and (see later) its weakly interacting nature.

As has been hinted above, a further useful classification of particles may be made according to the nature of their interactions. This leads us to group the baryons and mesons together as hadrons, strongly interacting particles (see §1.3), while the leptons remain apart as weakly interacting as well as light in mass.

The actual known particles are listed, according to the above principles, in tables 1.1 to 1.3. Quoted in these tables are the values of the quantum numbers which are discussed in §1.4.

By reason of its relation to the electron and the muon via the weak interaction, the neutrino (having zero rest-mass) is classed as a lepton, while the other object of zero rest-mass, the photon, has to be treated in a class of its own, as the quantum of the electromagnetic field.

1.3 Types of interaction

Different types of interaction between particles may be distinguished by their values of the coupling constant, a dimensionless number related to the strength of the interaction, and also to the typical value of cross-section for processes proceeding via this interaction.

There is an element of convention in the specification of the coupling constant for different types of interaction, but the general aim is to express the interaction energy as a fraction of the mass which is equivalent to the range of the interaction.

Strong interaction. For the strong interaction, the mutual energy of two particles separated by a distance r may be expressed as

$$E = \frac{g^2}{r} e^{-r/a}$$

where g is analogous to electric charge, and a is the range of the interaction, expressible as the Compton wavelength of a particle (actually the pion) of mass m given by

$$a = \frac{\hbar}{mc}$$

Thus the interaction energy when $r = a$ may be put as

$$E = \frac{g^2 mc}{\hbar}$$

whence

$$\frac{E}{mc^2} = \frac{g^2}{\hbar c}$$

This is the quantity generally used as the coupling constant for the strong interaction; it has value

$$\frac{g^2}{\hbar c} \sim 15$$

Electromagnetic interaction. The electromagnetic interaction has a strength characterised by the quantity $e^2/\hbar c$, which is known as the fine structure constant and has value 1/137. One may describe this quantity by an argument similar to that used above for the strong interaction; but since there is no unique range for a force obeying an inverse square law, one must say that $e^2/\hbar c$ is the interaction energy of two electronic charges separated by a general distance r, expressed as a fraction of the rest-energy of an object which would have Compton wavelength r.

Weak interaction. For the weak interaction we have to use the fact that decay rates lead us to a dimensional measure of interaction strength

$$G = 1.4 \times 10^{-49} \text{ erg cm}^3$$

The range is unknown, so to get a dimensionless number it is necessary to introduce a standard of length, such as the Compton wavelength of the pion or of the proton ($\hbar/m_p c$). This gives a coupling constant

$$\frac{G}{\hbar c}\left(\frac{m_p c}{\hbar}\right)^2$$

of order 10^{-5} (or 2×10^{-7} if one uses $\hbar/m_\pi c$). In fact the true range may be much smaller than $\hbar/m_p c$, in which case the interaction energy would be greater than the number 10^{-5} suggests.

Gravitational interaction. It is of interest to compare the three interactions which are important in elementary particle physics with a fourth, the gravitational interaction which is far too weak to have any significance in this field. If we use G' as the gravitational constant, and consider two electrons, we get a gravitational coupling constant

$$\frac{G' m^2}{\hbar c} \sim 10^{-45}$$

a number which amply demonstrates the difference in scale between gravitational effects on the one hand and electromagnetic or nuclear effects on the other.

1.4. Conservation laws

Many of the regularities observed in physics may be expressed as conservation laws, each of which states that the magnitude of some quantity is constant. The most familiar such laws are the laws of conservation of energy and momentum, which are universally valid, in quantum mechanics as in classical mechanics. Equally rigid are the laws of conservation of angular momentum and of electric charge. Conservation laws of this type are to be distinguished from those which apply in idealised systems to which real situations may or may not approximate. Laws of this latter type arise in the quantum-mechanical description of the interactions between elementary particles, and the principal ones will form a basis for our consideration of the symmetry principles.

To set up a few signposts to the topics under review, we may draw attention to the quantum numbers quoted for the individual particles in tables 1.1 to 1.3. The baryons, which can undergo transitions into each other, are given a baryon number $B = +1$, while their antiparticles are given a value $B = -1$. The fact that baryons, being fermions, can be created and annihilated only in particle–antiparticle pairs is then expressed by saying that the total baryon number B is conserved. This law appears to be in the universally valid class.

The other class of fermions, the leptons, appears to obey a similar but independent law of conservation of total number. For this purpose we assign lepton numbers $L = +1$ for e^+, μ^+, ν and $L = -1$ for $e^-, \mu^-, \bar{\nu}$. The conservation of lepton number expressed in this way appears to be as valid as the corresponding conservation of baryon number. It appears, further, that we may divide the leptons and antileptons into electronic ($e^+, \nu, e^-, \bar{\nu}$) and muonic ($\mu^+, \nu_\mu, \mu^-, \bar{\nu}_\mu$), the numbers of which are conserved separately. We could thus allocate electronic and muonic lepton numbers separately, and say that their totals were conserved separately in all known processes.

The intrinsic parity P of the particles may be linked with the parity $P = (-)^l$ associated with orbital angular momentum l in the relative motion of the particles, to calculate the total parity of a system. The law of conservation of parity, stating that total parity is conserved, is valid for processes which occur through the strong nuclear interaction, or through the electromagnetic interaction, but is violated in the weak interaction.

Also giving rise to conditionally obeyed conservation laws, to be discussed in later chapters of this book, are the quantum numbers listed as I (isospin), C (charge conjugation symmetry) and G (G-parity), together with S (strangeness) or Y (hypercharge).