THE PHYSICS PROBLEM SOLVER

REGISTERED TRADEMARK

- An essential supplement to any class text.
- Includes every type of problem that might be assigned or given on exams.
- Students can learn and understand the subject thoroughly by reviewing the problems in the order given, and seeing the solutions.
- Designed to save students hours of time in finding solutions to problems.
- Problems are arranged in order of complexity, from elementary to advanced.
- Every problem worked out in step-by-step detail.
- Over 1,000 pages.
- Fully indexed for locating specific problems rapidly.

Staff of Research and Education Association

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REGISTERED TRADEMARK

Staff of Research and Education Association,

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THE PHYSICS PROBLEM SOLVER ®

Printed in the United States of America

Library of Congress Catalog Card Number 76-332

International Standard Book Number 0-87891-507-9

Revised Printing, 1988

WHAT THIS BOOK IS FOR

For as long as physics has been taught in schools, students have found this subject difficult to understand and learn. Despite the publication of hundreds of textbooks in this field, each one intended to provide an improvement over previous textbooks, students continue to remain perplexed, and the subject is often taken in class only to meet school/departmental requirements for a selected course of study.

In a study of the problem, REA found the following basic reasons underlying students' difficulties with physics taught in schools:

(a) No systematic rules of analysis have been developed which students may follow in a step-by-step manner to solve the usual problems encountered. This results from the fact that the numerous different conditions and principles which may be involved in a problem, lead to many possible different methods of solution. To prescribe a set of rules to be followed for each of the possible variations, would involve an enormous number of rules and steps to be searched through by students, and this task would perhaps be more burdensome than solving the problem directly with some accompanying trial and error to find the correct solution route.

(b) Textbooks currently available will usually explain a given principle in a few pages written by a professional who has an insight into the subject matter that is not shared by students. The explanations are often written in an abstract manner which leaves the students confused as to the application of the principle. The explanations given are not sufficiently detailed and extensive to make the student aware of the wide range of applications and different aspects of the principle being studied. The numerous possible variations of principles and their applications are usually not discussed, and it is left for the students to discover these for themselves while doing exercises. Accordingly, the average student is expected to rediscover that which has been long known and practiced, but not published or explained extensively.

(c) The examples usually following the explanation of a topic are too few in number and too simple to enable the student to obtain a thorough grasp of the principles involved. The explanations do not provide sufficient basis to enable a student to solve problems that may be subsequently assigned for homework or given on examinations.

The examples are presented in abbreviated form which leaves out much material between steps, and requires that students derive the omitted material themselves. As a result, students find the examples difficult to understand--contrary to the purpose of the examples.

Examples are, furthermore, often worded in a confusing manner. They do not state the problem and then present the solution. Instead, they pass through a general discussion, never revealing what is to be solved for.

Examples, also, do not always include diagrams/graphs, wherever appropriate, and students do not obtain the training to draw diagrams or graphs to simplify and organize their thinking.

(d) Students can learn the subject only by doing the exercises themselves and reviewing them in class, to obtain experience in applying the principles with their different ramifications.

In doing the exercises by themselves, students find that they are required to devote considerably more time to physics than to other subjects of comparable credits, because they are uncertain with regard to the selection and application of the theorems and principles involved. It is also often necessary for students to discover those "tricks" not revealed in their texts (or review books), that make it possible to solve problems easily. Students must usually resort to methods of trial-and-error to discover these "tricks," and as a result they find that they may sometimes spend several hours to solve a single problem.

(e) When reviewing the exercises in classrooms, instructors usually request students to take turns in writing solutions on the board and explaining them to the class. Students often find it difficult to explain in a manner that holds the interest of the class, and enables the remaining students to follow the material written on the board. The remaining students seated in the class are, furthermore, too occupied with copying the material from the board, to listen to the oral explanations and concentrate on the methods of solution.

This book is intended to aid students in physics to overcome the difficulties described, by supplying detailed illustrations of the solution methods which are usually not apparent to students. The solution methods are illustrated by problems selected from those that are most often assigned for class work and given on examinations. The problems are arranged in order of complexity to enable students to learn and understand a particular topic by reviewing the problems in sequence. The problems are illustrated with detailed step-by-step explanations, to save the students the large amount of time that is often needed to fill in the gaps that are usually found between steps of illustrations in textbooks or review/outline books.

The staff of REA considers physics a subject that is best learned by allowing students to view the methods of analysis and solution techniques themselves. This approach to learning the subject matter is similar to that practiced in various scientific laboratories, particularly in the medical fields.

In using this book, students may review and study the illustrated problems at their own pace; they are not limited to the time allowed for explaining problems on the board in class.

When students want to look up a particular type of problem and solution, they can readily locate it in the book by referring to the index which has been extensively prepared. It is also possible to locate a particular type of problem by glancing at just the material within the boxed portions. To facilitate rapid scanning of the problems, each problem has a heavy border around it. Furthermore, each problem is identified with a number immediately above the problem at the right-hand margin.

To obtain maximum benefit from the book, students should familiarize themselves with the section, "How To Use This Book," located in the front pages.

To meet the objectives of this book, staff members of REA have selected problems usually encountered in assignments and examinations, and have solved each problem meticulously to illustrate the steps which are usually difficult for students to comprehend. Gratitude for their patient work in this area is due to Michael Abrams, Philip Druck, Metin Durgut, Steven Landovitz, Rae Mendelson, and Daniel Wyschogrod. Anthony Longhitano deserves special praise for his contributions and conscientious efforts.

Gratitude is also expressed to the many persons involved in the difficult task of typing the manuscript with its endless changes, and to the REA art staff who prepared the numerous detailed illustrations together with the layout and physical features of the book.

Finally, special thanks are due to Helen Kaufmann for her unique talents in rendering those difficult border-line decisions and in making constructive suggestions related to the design and organization of the book.

Max Fogiel, Ph.D.
Program Director

HOW TO USE THIS BOOK

This book can be an invaluable aid to students in physics as a supplement to their textbooks. The book is subdivided into 37 chapters, each dealing with a separate topic. The subject matter is developed beginning with fundamental concepts of vector quantities and extending through all major fields currently studied in physics. For students in science and engineering, advanced problems and solution techniques have been included.

TO LEARN AND UNDERSTAND A TOPIC THOROUGHLY

1. Refer to your class text and read the section pertaining to the topic. You should become acquainted with the principles discussed there. These principles, however, may not be clear to you at that time.

2. Then locate the topic you are looking for by referring to the "Table of Contents" in the front of this book, "The Physics Problem Solver."

3. Turn to the page where the topic begins and review the problems under each topic, in the order given. For each topic, the problems are arranged in order of complexity, from the simplest to the most difficult. Some problems may appear similar to others, but each problem has been selected to illustrate a different point or solution method.

To learn and understand a topic thorougly and retain its contents, it will be generally necessary for students to review the problems several times. Repeated review is essential in order to gain experience in recognizing the principles that should be applied, and in selecting the best solution technique.

TO FIND A PARTICULAR PROBLEM

To locate one or more problems related to a particular subject matter, refer to the index. In using the index, be certain to note that the numbers given there refer to problem numbers, not to page numbers. This arrangement of the index is intended to facilitate finding a problem more rapidly, since two or more problems may appear on a page.

If a particular type of problem cannot be found readily, it is recommended that the student refer to the "Table of Contents" in the front pages, and then turn to the chapter which is applicable to the problem being sought. By scanning or glancing at the material that is boxed, it will generally be possible to find problems related to the one being sought, without consuming considerable time. After the problems have been located, the solutions can be reviewed and studied in detail. For this purpose of locating problems rapidly, students should acquaint themselves with the organization of the books as found in the "Table of Contents."

In preparing for an exam, it is useful to find the topics to be covered on the exam in the "Table of Contents," and then review the problems under those topics several times. This should equip the student with what might be needed for the exam.

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Chapter No. **Page No.**

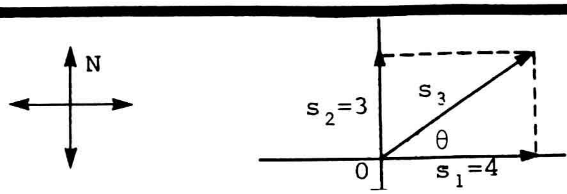
CHAPTER 1

VECTORS

VECTOR FUNDAMENTALS

• PROBLEM 1

Find the resultant of the vectors $\vec{S}_1$ and $\vec{S}_2$ specified in the figure.



Solution. From the Pythagorean theorem, $S_1^2 + S_2^2 = S_3^2$, or $4^2 + 3^2 = S_3^2$, and so we get $S_3 = 5$ units. The direction of S_3 may be specified by the angle θ which it makes with S_1.

$$\sin\theta = \frac{S_2}{S_3} = 0.60 \quad \text{gives } \theta = 37°.$$

Resultant $\vec{S}_3$ therefore represents a displacement of 5 units from 0 in the direction 37° north of east.

• PROBLEM 2

Three forces acting at a point are $\vec{F}_1 = 2\hat{i} - \hat{j} + 3\hat{k}$, $\vec{F}_2 = -\hat{i} + 3\hat{j} + 2\hat{k}$, and $\vec{F}_3 = -\hat{i} + 2\hat{j} - \hat{k}$. Find the directions and magnitudes of $\vec{F}_1 + \vec{F}_2 + \vec{F}_3$, $\vec{F}_1 - \vec{F}_2 + \vec{F}_3$, and $\vec{F}_1 + \vec{F}_2 - \vec{F}_3$.

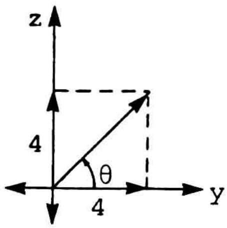


Fig. A

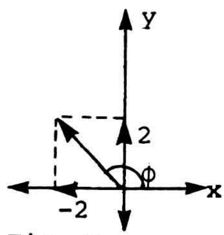


Fig. B

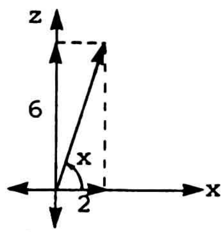


Fig. C

Solution: When vectors are added (or subtracted), their components in the directions of the unit vectors add (or subtract) algebraically. Thus since

$\vec{F}_1 = 2\hat{i} - \hat{j} + 3\hat{k}$, $\vec{F}_2 = -\hat{i} + 3\hat{j} + 2\hat{k}$, $\vec{F}_3 = -\hat{i} + 2\hat{j} - \hat{k}$,

then it follows that

$$\vec{F}_1 + \vec{F}_2 + \vec{F}_3 = (2 - 1 - 1)\hat{i} + (-1 + 3 + 2)\hat{j}$$

$$+ (3 + 2 - 1)k$$

$$= 0\hat{\imath} + 4\hat{\jmath} + 4\hat{k}.$$

Similarly,

$$\vec{F}_1 - \vec{F}_2 + \vec{F}_3 = \left[2 - (-1) - 1\right]\hat{\imath} + \left[-1 - (3) + 2\right]\hat{\jmath} + \left[3 - (2) - 1\right]\hat{k}$$

$$= 2\hat{\imath} - 2\hat{\jmath} + 0\hat{k}$$

and

$$\vec{F}_1 + \vec{F}_2 - \vec{F}_3 = \left[2 - 1 - (-1)\right]\hat{\imath} + \left[-1 + 3 - (2)\right]\hat{\jmath} + \left[3 + 2 - (-1)\right]\hat{k}$$

$$= 2\hat{\imath} + 0\hat{\jmath} + 6\hat{k}$$

The vector $\vec{F}_1 + \vec{F}_2 + \vec{F}_3$ thus has no component in the x-direction, one of 4 units in the y-direction, and one of 4 units in the z-direction. It therefore has a magnitude of $\sqrt{4^2 + 4^2}$ units = $4\sqrt{2}$ units = 5.66 units, and lies in the y- z plane, making an angle θ with the y - axis, as shown in figure (a), where $\tan\theta = 4/4 = 1$. Thus $\theta = 45°$.

Similarly, $\vec{F}_1 - \vec{F}_2 + \vec{F}_3$ has a magnitude of $2\sqrt{2}$ units = 2.82 units, and lies in the x - y plane, making an angle ϕ with the x-axis, as shown in figure (b), where $\tan\phi = +2/-2 = -1$. Thus $\phi = 315°$.

Also, $\vec{F}_1 + \vec{F}_2 - \vec{F}_3$ has a magnitude of $\sqrt{2^2 + 6^2}$ units = $2\sqrt{10}$ units = 6.32 units, and lies in the x - z plane at an angle χ to the x-axis, as shown in figure (c), where $\tan\chi = 6/2 = 3$. Thus $\chi = 71°34'$.

• PROBLEM 3

We consider the vector

$$\vec{A} = 3\hat{x} + \hat{y} + 2\hat{z}$$

(a) Find the length of $\vec{A}$.
(b) What is the length of the projection of $\vec{A}$ on the xy plane?
(c) Construct a vector in the xy plane and perpendicular to $\vec{A}$.
(d) Construct the unit vector $\hat{B}$.
(e) Find the scalar product with $\vec{A}$ of the vector $\vec{C} = 2\hat{x}$.
(f) Find the form of $\vec{A}$ and $\vec{C}$ in a reference frame obtained from the old reference frame by a rotation of $\pi/2$ clockwise looking along the positive z axis.
(g) Find the scalar product $\vec{A} \cdot \vec{C}$ in the primed coordinate system.
(h) Find the vector product $\vec{A} \times \vec{C}$.
(i) Form the vector $\vec{A} - \vec{C}$.

Fig. 1

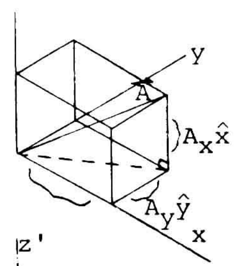


Fig. 2

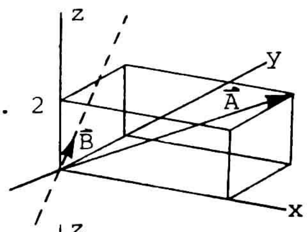


Fig. 3

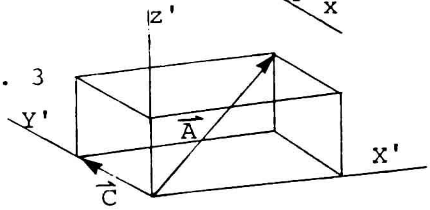
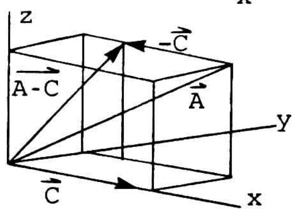


Fig. 4



The primed reference frame x', y', z', is generated from the unprimed system x, y, z, by a rotation of $+\pi/2$ about the z axis.

<u>Solution:</u> (a) When a vector is given in the form $A_x\hat{x} + A_y\hat{y} + A_z\hat{z}$, its length is given by $\sqrt{A_x^2 + A_y^2 + A_z^2}$. This can be seen from diagram 1. Vector $\vec{A}$ has components in the x, y and z directions. The x and y components form the legs of a right triangle. By the Pythagorean theorem the length of the hypotenuse of this triangle is $\sqrt{A_x^2 + A_y^2}$. But this line segment whose length is $\sqrt{A_x^2 + A_y^2}$ is one leg in a right triangle whose other leg is $A_z\hat{z}$ and whose hypotenuse is vector $\vec{A}$. Applying the Pythagorean theorem again, we find that the length of $\vec{A}$ is $\sqrt{A_x^2 + A_y^2 + A_z^2}$. Substituting our values we have $\sqrt{3^2 + 1^2 + 2^2} = \sqrt{14}$.

(b) We refer again to diagram 1. The projection of $\vec{A}$ on the xy plane is simply the dotted line which is the vector $A_x\hat{x} + A_y\hat{y}$. Its length is $\sqrt{A_x^2 + A_y^2}$ by the Pythagorean theorem. In our problem, the length is $\sqrt{3^2 + 1^2} = \sqrt{10}$.

(c) Construct a vector in the xy plane and perpendicular to A. We want a vector of the form

$$B = B_x\hat{x} + B_y\hat{y}$$

with the property $A \cdot B = 0$ (since $\vec{A} \cdot \vec{B} = |\vec{A}||\vec{B}| \cos \phi$ where ϕ is the angle between $\vec{A}$ and $\vec{B}$). Hence

$$(3\hat{x} + \hat{y} + 2\hat{z}) \cdot (B_x\hat{x} + B_y\hat{y}) = 0.$$

On taking the scalar product we find

$$3B_x + B_y = 0,$$

or

$$\frac{B_y}{B_x} = -3.$$

The length of the vector B is not determined by the specification of the problem. We have therefore determined just the slope of vector B, not its magnitude. See diagram 2.

(d) The unit vector B is the vector in the B direction but with the magnitude 1. It lies in the xy plane, and its slope (B_y/B_x) is equal to - 3. Therefore, $\hat{B}$ must satisfy the following two equations:

$$\hat{B}_x^2 + \hat{B}_y^2 = 1$$

$$\frac{\hat{B}_y}{\hat{B}_x} = -3$$

Solving simultaneously we have: $\hat{B}_x^2 + (-3\hat{B}_x)^2 = 1$ or $\hat{B}_x = 1/\sqrt{10}$ and $\hat{B}_y = -3/\sqrt{10}$.

The vector B is then:

$$\hat{B} = (1/\sqrt{10})\hat{x} - (3/\sqrt{10})\hat{y}$$

(e) Converting the vectors into coordinate form and computing the dot product (scalar product):

$$(3\hat{x} + \hat{y} + 2\hat{z}) \cdot (2\hat{x} + 0\hat{y} + 0\hat{z}) =$$

$$6 + 0 + 0 = 6$$

(f) Find the form of $\vec{A}$ and $\vec{C}$ in a reference frame obtained from the old reference frame by a rotation of $\pi/2$ clockwise looking along the positive z axis. The new unit vectors $\hat{x}$, $\hat{y}$, $\hat{z}$ are related to the old $\hat{x}$, $\hat{y}$, $\hat{z}$ by (see fig. 3)

$$\hat{x}' = \hat{y}; \quad \hat{y}' = -\hat{x}; \quad \hat{z}' = \hat{z}.$$

Where $\hat{x}$ appeared we now have $-\hat{y}'$; where $\hat{y}$ appeared, we now have $\hat{x}'$, so that

$$A = \hat{x} - 3\hat{y}' + 2\hat{z}'; \qquad C = -2\hat{y}'.$$

(g) Using the results of part (f), we convert the vectors $\vec{A}$ and $\vec{C}$ into coordinate form in the primed coordinate system, giving us the following dot product:

$$\vec{A} \cdot \vec{C} = (\hat{x}' - 3\hat{y}' + 2\hat{z}') \cdot (0\hat{x}' - 2\hat{y}' + 0\hat{z}') =$$

$$0 + 6 + 0 = 6$$

This is exactly the result obtained in the unprimed system.

(h) Find the vector product $\vec{A} \times \vec{C}$. In the unprimed system $\vec{A} \times \vec{C}$ is defined as

$$\begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ 3 & 1 & 2 \\ 2 & 0 & 0 \end{vmatrix} = 4\hat{y} - 2\hat{z}.$$

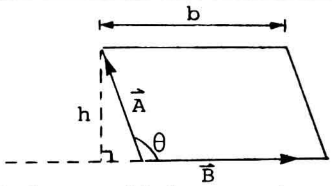
(i) Form the vector $\vec{A} - \vec{C}$. We have

$$\vec{A} - \vec{C} = (3 - 2)\hat{x} + \hat{y} + 2\hat{z} = \hat{x} + \hat{y} + 2\hat{z}.$$

● **PROBLEM 4**

Show that the area of a parallelogram, whose sides are formed by the vectors $\vec{A}$ and $\vec{B}$ (see figure) is given by

$$\text{Area} = |\vec{A} \times \vec{B}|.$$



Solution: The area of the parallelogram shown in the figure is

$$\text{Area} = bh$$

But $h = |\vec{A}| \sin\theta$ and $b = |\vec{B}|$

$$\text{Area} = |\vec{A}||\vec{B}| \sin\theta \qquad (1)$$

The left side of (1) is the magnitude of $\vec{A} \times \vec{B}$, hence

$$\text{Area} = |\vec{A} \times \vec{B}| = |\vec{A}||\vec{B}| \sin\theta$$

If we are interested in obtaining a vector area, we may write

$$\text{Area} = \vec{A} \times \vec{B}$$

where the direction of the area is the direction of $\vec{A} \times \vec{B}$. Such vector areas are useful in defining certain surface integrals used in physics.

DISPLACEMENT VECTORS

● **PROBLEM 5**

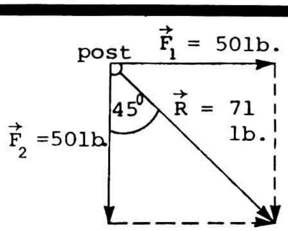
Two hikers set off in an eastward direction. Hiker 1 travels 3 km while hiker 2 travels 6 times the distance covered by hiker 1. What is the displacement of hiker 2?

Solution: From the information given the displacement vector is directed east. The magnitude of the displacement vector for hiker 2 is 6 times the magnitude of the displacement vector for hiker 1. Therefore, its magnitude is

$$6 \times (3 \text{ km}) = 18 \text{ km}$$

● **PROBLEM 6**

Two wires are attached to a corner fence post with the wires making an angle of 90° with each other. If each wire pulls on the post with a force of 50 pounds, what is the resultant force acting on the post? See Figure.



Solution: As shown in the figure, we complete the parallelogram. If we measure R and scale it, we find it is