

Reasoning from Evidence

Inductive

Logic

William Gustason

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INDUCTIVE LOGIC

William Gustason

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Preface

This book grew out of notes used for a one-semester undergraduate course in inductive logic. Its aim is to provide a general introduction to the principles and problems of inductive inference. Topics found in elementary, general-purpose logic texts are covered in greater depth, and there is extensive coverage of subjects typically not found in them at all. No previous training in logic is required; the elements of deductive logic (specifically sentential logic) necessary for studying some of the topics covered are introduced in the last section of the first chapter. Beyond that, a familiarity with some very basic high school algebra will be helpful in a few places, and even in those contexts assistance for readers without a technical background is often provided.

At present, few colleges and universities are offering one-semester courses on induction, but their number is slowly growing. Inductive logic surely is as worthy of a full-semester introductory treatment as deductive logic, and because it focuses on such topics as chance, decision making, and scientific method, many students find it more appealing than the sometimes dry formalities (at least from their viewpoint) of sentential and quantificational logic. Ideally, those faculty contemplating the addition of an inductive logic course to their curriculum will find this text to be one that suits their needs. It is also hoped that students and interested readers will find the issues discussed here stimulating and worthy of further investigation (many references for further reading are provided throughout and a bibliography is provided at the end of the last chapter).

Of course, there are other applications, too. This book can serve as a useful reference source for undergraduate and graduate-level philosophy courses in subjects such as the philosophy of science and epistemology. Those in other fields, such as statistics and decision theory, should find it a useful supplement as well. Naturally, the book would fail in its purpose if it could not be read with profit by an interested reader having no formal academic commitment.

It is widely agreed that inductive logic has made less progress in the twentieth century than deductive logic, so it is not surprising that there is less of a consensus on what are the “standard” topics for an introductory discussion. Moreover, controversial issues arise more frequently concerning even the most elementary material in the field. I have tried to select topics based, first, on what consensus there appears to be, and second, on what is important and can be readily absorbed by students with a minimal background in logic and philosophy. Included in the discussion of topics that historically have been much debated is coverage of widely differing points of view. On the problem of justification, for example, several well-known approaches are covered in the fifth and seventh chapters. However, considerations of space and continuity preclude such coverage for all topics on which controversy exists (for example, the Dutch Book approach to justifying the calculus). In such cases, though, the instructor should be able to easily supplement the discussion with his or her own material if desired.

The text also provides some flexibility in order of coverage. Chapters Two and Three (on basic forms of induction and probability, respectively) could be covered in reverse order; Chapter Six (on hypotheses and problems in confirmation) could then be taken up directly following coverage of the second chapter. Because the text probably offers more material than would be covered in a one-semester (or certainly in a one-quarter) course, all or parts of Chapters Four and Seven can be omitted if coverage of technical matters is not a high priority (the last section of Chapter Five could be ignored, too). However, if such coverage is desired, all or parts of Chapters Two and Six could be omitted.

The final chapter (on the basic theories of inductive probability) offers more challenging material than the preceding ones, and an understanding of the third, fourth, and fifth chapters is essential. Instructors should expect to spend more than the usual amount of class time covering it. However, it will be time well spent for those who want a more than cursory examination of the work of Carnap, Reichenbach, and Bayesian theorists.

Anyone who writes a book owes many debts. The publisher’s reviewers provided some valuable suggestions and corrections, as did my friends and colleagues, Rod Bertolet, Howard Kenreuther, Annie Pirruccello, and Ted Ulrich. Thanks go also to the many students who have used this material in one form or another in the past; their advice was especially helpful. Finally, I must thank Arthur Burks of the University of Michigan, from whom I first learned about induction and probability. That he has made many important contributions to these fields is well known; that he has been a guiding influence and inspiration to his many students is not.

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Inductive and Deductive Logic

Section 1. The Appraisal of Arguments

Logic is an area of philosophy, but it is also a science. Like mathematics it is a formal (or *a priori*) science whose problems can be dealt with in a study rather than a laboratory. It is sometimes described as the science of “reasoning” or “inference,” and sometimes as the study of the “laws of thought”; but although these characterizations can be initially helpful, they can also be misleading. Reasoning (inferring, thinking) is a mental process and as such is the province of the empirical sciences of psychology and physiology. Logic does not study mental or brain activity and hence is not concerned with the actual process of reasoning. But the beginning and end points of the process are of prime interest. That is, we reason *from* certain items of information *to* another such item. The former are the **premises** of the inference, the latter its **conclusion**. And the question to be asked is: Do the premises *warrant* or *justify* the conclusion? Do they provide adequate grounds for accepting it?

These questions make clear that reasoning can be done well or badly, according to whether the premises do or do not warrant the conclusion. So we can rephrase our question in two parts: Is the inference of the conclusion from the premises a correct (or “good”) one, and if so, on what basis do we judge it to be correct? One of the major tasks of logic is determining what good reasoning consists in and developing methods that will enable us to distinguish good inferences from bad ones. Logic is thus a normative science as well as a descriptive one; it not only studies the mechanisms underlying good reasoning, but in doing so it provides us with standards by which we *should* reason.

We express our thoughts—and hence the inferences in which they occur—in language. When we reason, we attempt to show that the conclusion is *true* given the premises, and truths are communicated through the use of sentences. Therefore premises and conclusions may

be represented by the sentences of a given language, but not all sentences may serve in this capacity. Interrogative sentences ("Is the door closed?") and imperative sentences ("Close the door!") are not used to say something true or false, but rather to express questions and commands, respectively. Our main concern is with *indicative* sentences ("The door is closed."), for it is chiefly through them that we make claims that are judged true or false, and hence that may serve as premises and conclusions.

However, the matter is a bit more complicated. We cannot simply ascribe truth and falsehood to indicative sentences *per se*. The very same sentence can be used on different occasions to express different items of information, some true and some false. The sentence, "Jones was in Chicago yesterday" might be true when said by someone on May 14 of Sam Jones but false when said on June 22 of Betty Jones. Conversely, different sentences may in different contexts be used to express the same item of information; for example, "Jones was in Chicago yesterday" and "Sam Jones was in Chicago on May 13, 1992." We must therefore distinguish between the sentence itself and the information it expresses in a particular context of its use (either spoken or written), and only the latter may be said to be true or false.

Let us use the term 'statement' to mean what a sentence expresses in a given context of use. It is then statements rather than the sentences used to express them that are properly the bearers of truth and falsehood, and hereafter when we write down a sentence and call it a "statement" we will mean that piece of (true-or-false) information the sentence was used to express on a given occasion. The premises and conclusions of our reasoning may be regarded as **statements** in this sense.

A particular instance of reasoning may thus be represented in language as a list of statements, and we now introduce a special term for such lists:

An **argument** is a set of statements, one of which is marked off as the conclusion of the argument, the rest serving as its premises.

The conclusion states the claim being argued whereas the premises provide the grounds being offered in support of the conclusion. Of course 'argument' is used in many ways in everyday speech; we speak of two people "having an argument" for example, and in mathematics we often talk of a certain quantity as an "argument" of a function, but hereafter, unless otherwise noted, we will use 'argument' only in the technical sense just mentioned.

Arguments are formulated in ordinary language in many ways. Usually the conclusion is signalled by a word such as ‘therefore,’ ‘hence,’ ‘so,’ ‘thus,’ and so forth, as in “The sky is clouding up, the temperature is falling, and the barometer is dropping rapidly; hence it will rain soon.” Sometimes the conclusion is stated first with the premises following a word like ‘since’ or ‘for,’ as in “Jones will be defeated in the next election, for he supports higher taxes and most voters are opposed to a tax increase.” And very often we do not explicitly state all the premises, presuming that those left out can easily be supplied by our listeners. If someone says, “Mrs. Lopez is a U.S. citizen over the age of 65, so she’s eligible for Medicare,” the missing premise is that all U.S. citizens over 65 are eligible for Medicare.

At its most elementary level, logic is concerned with the analysis and evaluation of arguments. Arguments come in many varieties, from the simple examples just mentioned to highly complex ones found in almost every human endeavor from theoretical physics to jury trials. A mathematical proof is a chain of arguments, the conclusion of one serving as a premise of a subsequent one. The logician’s task is to find methods for appraising arguments: for developing a basis for classifying arguments according to whether their premises warrant the conclusion (the “good” ones) or whether they do not (the “bad” or “fallacious” ones). We will now look at two types of appraisal; they divide our subject into its two basic subareas: deductive logic and inductive logic.

Section 2. Deductive Logic: Validity and Form _____

It may seem strange to begin a study of inductive logic by first looking at deductive logic, but there is a good reason for this: the fundamental concepts of the former are more readily grasped when contrasted with those of the latter. Thus far, we have described arguments simply as being good or bad, correct or incorrect. These vague and not very helpful terms will now be replaced by more precise ones, and in deductive logic the terms we need are ‘valid’ and ‘invalid.’ When an argument is assessed from the standpoint of deductive logic, these are the terms that express the evaluation.

We begin with a simple example. Your car’s fuel system is acting up, and from your account of the symptoms the mechanic tells you that either the carburetor or the fuel pump is at fault. An inspection of the former shows that everything is in order; the conclusion drawn is that the pump is the culprit.

We have here a simple argument that is undeniably a good—or valid—one

I. Either the carburetor or the fuel pump is at fault

The carburetor is not at fault

∴ The fuel pump is at fault

(Words for the conclusion will hereafter be replaced by the three-dot sign.)

In a valid argument like (I), the conclusion *follows from* the premises; in an invalid one, it does not. But what does ‘follow from’ mean here? Notice that if both premises are true, the conclusion cannot fail to be true as well. That is, the truth of the premises *guarantees* a true conclusion. The attempt to suppose that the premises are both true but the conclusion false lands us in an outright contradiction. In argument (I), if the second premise is true then the carburetor is not at fault, and if the conclusion is false then the fuel pump is not at fault; hence *neither* of them is at fault. But the first premise tells us that it is *either* one *or* the other! There is hence no way the conclusion can be false if both premises are true. More fully,

An argument is **valid** if and only if it is impossible for all of its premises to be true but its conclusion false.

Of course in many cases not all of the premises *are* true. Yet the argument can still be valid; it is important to realize that validity is a conditional notion: *if* we have true premises we thereby have a true conclusion as well. Each of the following arguments has a false premise, and in one of them the conclusion is false too. Both, however, are still valid; were the premises all true, the conclusion’s truth would be guaranteed:

If 3 is even, then 3 is a multiple of 2

3 is even

∴ 3 is a multiple of 2

Either 3 or 5 is odd

3 is not odd

∴ 5 is odd

To repeat, in many valid arguments not all premises are true, but what makes them valid is that *if* all premises were true the conclusion could not possibly be false. Valid arguments are, so to speak, “truth preserving.”

It is an important fact that we can argue validly from false premises as well as true ones. In the empirical sciences, as we shall see later, scientists typically formulate hypotheses to explain various phenomena, and many of them are eventually shown to be false. This is often

done by validly deducing from them a testable consequence. If experimental tests show the consequence to be false, then the hypothesis from which they were deduced is disconfirmed.

The upshot is that in evaluating an argument, there are at least two quite distinct questions to be asked:

Is the argument valid?

Are all of its premises in fact true?

The questions are equally important but only the first one falls within the province of logic. If the premises are about subatomic particles or the mating habits of polar bears or Napoleon's military prowess, then to determine their truth the persons to be consulted are (respectively) a physicist, zoologist, or historian, certainly not a logician. To establish definitively the truth of the conclusion, a "yes" answer to both questions is required, but a logician's expertise is relevant only to answering the first of them.

The following argument admits of a "yes" answer to the second question but not to the first:

- II.** If dogs are mammals, then they are vertebrates
 Dogs are vertebrates
 ∴ Dogs are mammals

All three statements are true, but the trouble is that the conclusion *does not follow from* the premises. If we rely on just the information provided by the premises, there is no reason for us to rule out dogs falling into the category of nonmammalian vertebrate. So although the conclusion is true along with the premises being true, its truth is not established by those premises—they do not guarantee its truth. Argument (II) is thus **invalid**—*it is possible for its conclusion to be false when all of its premises are true*.

So an invalid argument need not actually have true premises and a false conclusion; argument (II) shows that it is the possibility of such that makes it invalid. Of course, many invalid arguments do in fact have true premises and false conclusions, and they provide a means of showing that arguments like (II) are invalid as well. That is, if someone were to propose argument (II), we could show that person the error of his or her ways by comparing it to the following argument where the reasoning takes exactly the same form but has an obviously false conclusion accompanying true premises:

- III.** If whales are fish, then they are aquatic animals
 Whales are aquatic animals
 ∴ Whales are fish

The truth that dogs are mammals no more follows from the premises of (II) than the falsehood that whales are fish follows from the premises of (III).

More needs to be said about how the word *impossible* is being used in our definition of validity. In one common sense, it is impossible for a human being to lift a 4,000-pound object or for an unsupported object near the earth not to fall to the ground or for litmus paper not to turn red when dipped in acid. This is called *physical* (or sometimes *causal*) impossibility, but in saying that it is impossible to have true premises accompanying a false conclusion, we are using 'impossible' in an even stronger sense. We would indeed be astounded if, for example, a baseball did not fall to the ground when released from my hand, but we can still entertain the supposition of such an event occurring, no matter how farfetched or miraculous it would be. On the other hand, we cannot suppose there to be an object that is both round and not round at the same time; no such miraculous situation can be entertained here, for roundness and nonroundness *logically* exclude one another whereas being-released-from-my-hand and not-falling-to-the-ground do not. Although we can consider the possibility of an unsupported object not falling, there is no "possibility" of an object being round and not round simultaneously. The latter is a *logical* impossibility, and to revert to argument (I), the supposition that its conclusion is false when its premises are true is impossible in this sense. To suppose the fuel pump is not at fault flatly contradicts the premises, just as 'this is not round' contradicts 'this is round.' In valid inference it is logically impossible for a false conclusion and true premises to occur together.¹

As a result the following argument is invalid as it stands:

- IV. Bert Jones is a 70-year-old human being
 Bert Jones's car weighs 3,500 pounds
 ∴ Bert Jones cannot lift his car

The supposition that the conclusion is false does not logically contradict the premises. Of course, if we added a further, well-confirmed premise—namely, that no 70-year-old human being can lift a 3,500 pound automobile—we would *then* have a valid argument.

The following argument has exactly the same form as argument (I):

- V. Either Alice missed her train or else the train was late
 Alice did not miss her train
 ∴ The train was late

Both arguments of course are valid, but what is important here is that they are valid *because* of the form they share; their content or subject

matter has no bearing on their assessment as valid inferences. Validity depends only on the meanings of purely logical terms like 'either-or' and 'not'; those of the other constituent expressions can be ignored. Using the letters ' p ' and ' q ' to represent any pair of statements whatever, the common form of (I) and (V) is expressed

- a. either p or q
 not- p
 $\therefore q$

Any argument of this form is valid. If this point is not already clear, we need only ask: Can there be an argument of form (a) with a false conclusion and true premises? The conclusion being false means that q is false. Now can we find a statement p such that both premises are true? No, because

- i. if we pick a statement for p that is true then the second premise is false
- ii. if we pick a statement for p that is false then the first premise is false (a disjunction or "either-or" statement is false whenever both component statements are false).

So in no case are there statements p and q that will give us true premises and false conclusion, and to show this, we did not need to specify particular statements for p and q . The point holds quite generally: *arguments are valid solely in virtue of their form.*

Now let us return to arguments (II) and (III). We saw that both are invalid and that they also share a common form:

- b. if p then q
 q
 $\therefore p$

Unlike form (a), we can obtain arguments with true premises and false conclusions from form (b) by finding appropriate statements for p and q . Argument (III) is a perfect example, where 'Whales are fish' replaces p and 'Whales are aquatic animals' replaces q . Not all arguments produced from form (b) will be like this; argument (II), which fallaciously inferred the truth that dogs are mammals, illustrates that. But the fact that there are any at all with true premises and false conclusion means that (b) is an *invalid argument form*; if we reason according to it, we can be led to false conclusions even if we begin with true premises. Such forms illuminate the sense of 'possible' used in our definition of invalidity: of all the specific arguments derivable from

such a form, some will lead us from truths to a falsehood. In a *valid argument form* like (a), this can never happen no matter what specific statements are substituted for p and q .

Corresponding to every argument form is a *rule of inference*. A rule is by the nature of the case general; it must be applicable to a number of similar instances. For argument form (a) we have the following rule:

From: a statement of the form *either p or q* and one of the form
 not- p
To infer: the statement q

This rule “governs” instances like arguments (I) and (V). Since arguments are valid solely in virtue of their form, the notion of validity should be regarded as applying primarily to rules of inference. The specific arguments derivable by substitution from the associated argument form are valid in virtue of being governed by the rule. Our primary definition of validity is thus:

A rule of inference is **valid** if and only if no argument derivable from its associated argument form has all true premises and a false conclusion. Otherwise, it is **invalid**.

(However, we shall continue to employ the earlier definition too.) At its most basic level, then, deductive logic is concerned with developing methods that will enable us to distinguish valid from invalid rules of inference.

Exercises

1. Construct two arguments whose premises and conclusion are all false, one of which is valid and the other invalid.

2. Show that the following argument forms are invalid by finding statements for p and for q that yield true premises and a false conclusion.

a. either p or q
 $\therefore p$

b. if p then q
 not- p
 \therefore not- q

3. Give an informal proof or explanation of why the following forms are valid.

a. p
 \therefore either p or q

b. if p then q
 not- q
 \therefore not- p

Section 3. Inductive Logic: Evidence and Probability

Arguments (II) and (III) of the previous section were invalid, and once seen to be invalid they are of little further interest. However argument (IV)—about Bert Jones lifting his car—was in *some* sense a “good” one even though invalid. Many arguments are like this; although invalid by the standards of deductive logic, they are still worthwhile and important. Here are some further cases:

- I. 95 percent of all native Italians are Roman Catholics.
Angela is a native Italian.
∴ Angela is a Roman Catholic.

- II. Smith was killed in his home by a .38 caliber revolver
belonging to Jones.
Jones badly needed money to pay off a large gambling debt.
Jones has hated Smith for years.
Jones was having an affair with Smith’s wife, who would
collect on Smith’s life insurance policy in case of death.
Two reliable witnesses saw Jones leave Smith’s house about
10 minutes after the estimated time of death.
Jones’s fingerprints were found on the murder weapon.
Smith’s wife testifies that she conspired with Jones to
murder her husband.
∴ Jones murdered Smith.

- III. [Shortly after the invention of the microscope,
microorganisms were discovered in “fermentable” liquids
such as meat broth and sugared yeast water. Some
theorists claimed they arose through “spontaneous
generation,” but Louis Pasteur hypothesized that they
entered the liquids from the air using as their vehicles
airborne dust particles that came in contact with the
liquids.]

Premise: Earlier experiments conducted by Pasteur had shown, first, that the dust particles carry with them microorganisms and, second, that sterilized fermentable liquids exposed only to purified air do not develop microorganisms. Pasteur then had special flasks made with long, very thin goosenecks. When meat broth was poured in and sterilized, the moisture that collected in the necks allowed air to pass through but blocked the dust particles. Broth was also placed in standard, wide-necked flasks

and sterilized. In a short time, the liquids in these flasks developed microorganisms but those in the thin-necked flasks never did. In the former case, everything in the air entered the flask and microorganisms were found; in the latter, everything in the air *except* the dust entered and microorganisms were *not* found. Only when the thin-necked flasks were shaken vigorously enough to dislodge the dust that had collected in the neck were microorganisms detected in the broth.

Conclusion: Microorganisms enter fermentable liquids from the air by means of airborne dust particles.

All three arguments are invalid: Angela might be one of the few Italians who is not Roman Catholic; Jones conceivably is the victim of an elaborate and cunning frame-up; and it is barely possible that some component of the air other than the dust was blocked in the goose-necks or that some slip-up in the laboratory skewed the results of the experiment (in fact, of course, Pasteur's experiment was repeated many times with the same results).

But although it is *possible* in all three instances to have a false conclusion when the premises are true, it is *highly improbable* that this is the case. The premises provide strong evidence for the conclusion, and it is thus very likely the conclusion is true given the truth of the premises. In argument (II) the evidence contained in the premises makes the conclusion so probable that a jury would perhaps consider its truth to be "beyond reasonable doubt."

In deductive logic arguments are appraised as valid or invalid. Invalid arguments like the preceding three are still good ones when appraised in terms of how much *evidence* the premises provide for the conclusion. This kind of appraisal is the province of inductive logic, and we can now define the inductive counterpart of the term 'valid':

An argument is **inductively strong** if and only if, first, it is invalid, but second, the evidence supplied by its premises makes it highly improbable that its conclusion is false when all premises are true.

In valid inferences, true premises guarantee a true conclusion; in inductively strong ones, true premises make it highly probable that the conclusion is true. Moreover, whereas arguments are valid by virtue of their form, arguments are inductively strong by virtue of the strength of the evidence they contain.

Validity is an all-or-nothing affair: an argument cannot be "partly valid" or "two-thirds valid." Inductive strength, however, is a matter of degree according to *how much* evidence is provided by the premises