

Linear Algebra
and its applications

FOURTH EDITION



David C. Lay

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David C. Lay

University of Maryland—College Park

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*To my wife, Lillian, and our children,
Christina, Deborah, and Melissa, whose
support, encouragement, and faithful
prayers made this book possible.*

About the Author

David C. Lay holds a B.A. from Aurora University (Illinois), and an M.A. and Ph.D. from the University of California at Los Angeles. Lay has been an educator and research mathematician since 1966, mostly at the University of Maryland, College Park. He has also served as a visiting professor at the University of Amsterdam, the Free University in Amsterdam, and the University of Kaiserslautern, Germany. He has published more than 30 research articles on functional analysis and linear algebra.

As a founding member of the NSF-sponsored Linear Algebra Curriculum Study Group, Lay has been a leader in the current movement to modernize the linear algebra curriculum. Lay is also a co-author of several mathematics texts, including *Introduction to Functional Analysis* with Angus E. Taylor, *Calculus and Its Applications*, with L. J. Goldstein and D. I. Schneider, and *Linear Algebra Gems—Assets for Undergraduate Mathematics*, with D. Carlson, C. R. Johnson, and A. D. Porter.

Professor Lay has received four university awards for teaching excellence, including, in 1996, the title of Distinguished Scholar–Teacher of the University of Maryland. In 1994, he was given one of the Mathematical Association of America’s Awards for Distinguished College or University Teaching of Mathematics. He has been elected by the university students to membership in Alpha Lambda Delta National Scholastic Honor Society and Golden Key National Honor Society. In 1989, Aurora University conferred on him the Outstanding Alumnus award. Lay is a member of the American Mathematical Society, the Canadian Mathematical Society, the International Linear Algebra Society, the Mathematical Association of America, Sigma Xi, and the Society for Industrial and Applied Mathematics. Since 1992, he has served several terms on the national board of the Association of Christians in the Mathematical Sciences.

Preface

The response of students and teachers to the first three editions of *Linear Algebra and Its Applications* has been most gratifying. This *Fourth Edition* provides substantial support both for teaching and for using technology in the course. As before, the text provides a modern elementary introduction to linear algebra and a broad selection of interesting applications. The material is accessible to students with the maturity that should come from successful completion of two semesters of college-level mathematics, usually calculus.

The main goal of the text is to help students master the basic concepts and skills they will use later in their careers. The topics here follow the recommendations of the Linear Algebra Curriculum Study Group, which were based on a careful investigation of the real needs of the students and a consensus among professionals in many disciplines that use linear algebra. Hopefully, this course will be one of the most useful and interesting mathematics classes taken by undergraduates.

WHAT'S NEW IN THIS EDITION

The main goal of this revision was to update the exercises and provide additional content, both in the book and online.

1. More than 25 percent of the exercises are new or updated, especially the computational exercises. The exercise sets remain one of the most important features of this book, and these new exercises follow the same high standard of the exercise sets of the past three editions. They are crafted in a way that retells the substance of each of the sections they follow, developing the students' confidence while challenging them to practice and generalize the new ideas they have just encountered.
2. Twenty-five percent of chapter openers are new. These introductory vignettes provide applications of linear algebra and the motivation for developing the mathematics that follows. The text returns to that application in a section toward the end of the chapter.
3. A New Chapter: Chapter 8, The Geometry of Vector Spaces, provides a fresh topic that my students have really enjoyed studying. Sections 1, 2, and 3 provide the basic geometric tools. Then Section 6 uses these ideas to study Bezier curves and surfaces, which are used in engineering and online computer graphics (in Adobe® Illustrator® and Macromedia® FreeHand®). These four sections can be covered in four or five 50-minute class periods.

A second course in linear algebra applications typically begins with a substantial review of key ideas from the first course. If part of Chapter 8 is in the first course, the second course could include a brief review of sections 1 to 3 and then a focus on the geometry in sections 4 and 5. That would lead naturally into the online chapters 9 and 10, which have been used with Chapter 8 at a number of schools for the past five years.

4. The *Study Guide*, which has always been an integral part of the book, has been updated to cover the new Chapter 8. As with past editions, the *Study Guide* incorporates

detailed solutions to every third odd-numbered exercise as well as solutions to every odd-numbered writing exercise for which the text only provides a hint.

5. Two new chapters are now available online, and can be used in a second course:

Chapter 9. Optimization

Chapter 10. Finite-State Markov Chains

An access code is required and is available to qualified adopters. For more information, visit www.pearsonhighered.com/irc or contact your Pearson representative.

6. PowerPoint® slides are now available for the 25 core sections of the text; also included are 75 figures from the text.

DISTINCTIVE FEATURES

Early Introduction of Key Concepts

Many fundamental ideas of linear algebra are introduced within the first seven lectures, in the concrete setting of \mathbb{R}^n , and then gradually examined from different points of view. Later generalizations of these concepts appear as natural extensions of familiar ideas, visualized through the geometric intuition developed in Chapter 1. A major achievement of this text is that the level of difficulty is fairly even throughout the course.

A Modern View of Matrix Multiplication

Good notation is crucial, and the text reflects the way scientists and engineers actually use linear algebra in practice. The definitions and proofs focus on the columns of a matrix rather than on the matrix entries. A central theme is to view a matrix–vector product Ax as a linear combination of the columns of A . This modern approach simplifies many arguments, and it ties vector space ideas into the study of linear systems.

Linear Transformations

Linear transformations form a “thread” that is woven into the fabric of the text. Their use enhances the geometric flavor of the text. In Chapter 1, for instance, linear transformations provide a dynamic and graphical view of matrix–vector multiplication.

Eigenvalues and Dynamical Systems

Eigenvalues appear fairly early in the text, in Chapters 5 and 7. Because this material is spread over several weeks, students have more time than usual to absorb and review these critical concepts. Eigenvalues are motivated by and applied to discrete and continuous dynamical systems, which appear in Sections 1.10, 4.8, and 4.9, and in five sections of Chapter 5. Some courses reach Chapter 5 after about five weeks by covering Sections 2.8 and 2.9 instead of Chapter 4. These two optional sections present all the vector space concepts from Chapter 4 needed for Chapter 5.

Orthogonality and Least-Squares Problems

These topics receive a more comprehensive treatment than is commonly found in beginning texts. The Linear Algebra Curriculum Study Group has emphasized the need for a substantial unit on orthogonality and least-squares problems, because orthogonality plays such an important role in computer calculations and numerical linear algebra and because inconsistent linear systems arise so often in practical work.

PEDAGOGICAL FEATURES

Applications

A broad selection of applications illustrates the power of linear algebra to explain fundamental principles and simplify calculations in engineering, computer science, mathematics, physics, biology, economics, and statistics. Some applications appear in separate sections; others are treated in examples and exercises. In addition, each chapter opens with an introductory vignette that sets the stage for some application of linear algebra and provides a motivation for developing the mathematics that follows. Later, the text returns to that application in a section near the end of the chapter.

A Strong Geometric Emphasis

Every major concept in the course is given a geometric interpretation, because many students learn better when they can visualize an idea. There are substantially more drawings here than usual, and some of the figures have never before appeared in a linear algebra text.

Examples

This text devotes a larger proportion of its expository material to examples than do most linear algebra texts. There are more examples than an instructor would ordinarily present in class. But because the examples are written carefully, with lots of detail, students can read them on their own.

Theorems and Proofs

Important results are stated as theorems. Other useful facts are displayed in tinted boxes, for easy reference. Most of the theorems have formal proofs, written with the beginning student in mind. In a few cases, the essential calculations of a proof are exhibited in a carefully chosen example. Some routine verifications are saved for exercises, when they will benefit students.

Practice Problems

A few carefully selected Practice Problems appear just before each exercise set. Complete solutions follow the exercise set. These problems either focus on potential trouble spots in the exercise set or provide a “warm-up” for the exercises, and the solutions often contain helpful hints or warnings about the homework.

Exercises

The abundant supply of exercises ranges from routine computations to conceptual questions that require more thought. A good number of innovative questions pinpoint conceptual difficulties that I have found on student papers over the years. Each exercise set is carefully arranged in the same general order as the text; homework assignments are readily available when only part of a section is discussed. A notable feature of the exercises is their numerical simplicity. Problems “unfold” quickly, so students spend little time on numerical calculations. The exercises concentrate on teaching understanding rather than mechanical calculations. The exercises in the *Fourth Edition* maintain the integrity of the exercises from the third edition, while providing fresh problems for students and instructors.

Exercises marked with the symbol [M] are designed to be worked with the aid of a “Matrix program” (a computer program, such as MATLAB[®], Maple[™], Mathematica[®], MathCad[®], or Derive[™], or a programmable calculator with matrix capabilities, such as those manufactured by Texas Instruments).

True/False Questions

To encourage students to read all of the text and to think critically, I have developed 300 simple true/false questions that appear in 33 sections of the text, just after the computational problems. They can be answered directly from the text, and they prepare students for the conceptual problems that follow. Students appreciate these questions—after they get used to the importance of reading the text carefully. Based on class testing and discussions with students, I decided not to put the answers in the text. (The *Study Guide* tells the students where to find the answers to the odd-numbered questions.) An additional 150 true/false questions (mostly at the ends of chapters) test understanding of the material. The text does provide simple T/F answers to most of these questions, but it omits the justifications for the answers (which usually require some thought).

Writing Exercises

An ability to write coherent mathematical statements in English is essential for all students of linear algebra, not just those who may go to graduate school in mathematics. The text includes many exercises for which a written justification is part of the answer. Conceptual exercises that require a short proof usually contain hints that help a student get started. For all odd-numbered writing exercises, either a solution is included at the back of the text or a hint is provided and the solution is given in the *Study Guide*, described below.

Computational Topics

The text stresses the impact of the computer on both the development and practice of linear algebra in science and engineering. Frequent Numerical Notes draw attention to issues in computing and distinguish between theoretical concepts, such as matrix inversion, and computer implementations, such as LU factorizations.

WEB SUPPORT

This Web site at www.pearsonhighered.com/lay contains support material for the textbook. For students, the Web site contains **review sheets** and **practice exams** (with solutions) that cover the main topics in the text. They come directly from courses I have taught in past years. Each review sheet identifies key definitions, theorems, and skills from a specified portion of the text.

Applications by Chapters

The Web site also contains seven Case Studies, which expand topics introduced at the beginning of each chapter, adding real-world data and opportunities for further exploration. In addition, more than 20 Application Projects either extend topics in the text or introduce new applications, such as cubic splines, airline flight routes, dominance matrices in sports competition, and error-correcting codes. Some mathematical applications are integration techniques, polynomial root location, conic sections, quadric surfaces, and extrema for functions of two variables. Numerical linear algebra topics, such as condition numbers, matrix factorizations, and the QR method for finding eigenvalues, are also included. Woven into each discussion are exercises that may involve large data sets (and thus require technology for their solution).

Getting Started with Technology


If your course includes some work with MATLAB, Maple, Mathematica, or TI calculators, you can read one of the projects on the Web site for an introduction to the

technology. In addition, the *Study Guide* provides introductory material for first-time users.

Data Files


Hundreds of files contain data for about 900 numerical exercises in the text, Case Studies, and Application Projects. The data are available at www.pearsonhighered.com/lay in a variety of formats—for MATLAB, Maple, Mathematica, and the TI-83+/86/89 graphic calculators. By allowing students to access matrices and vectors for a particular problem with only a few keystrokes, the data files eliminate data entry errors and save time on homework.

MATLAB Projects

These exploratory projects invite students to discover basic mathematical and numerical issues in linear algebra. Written by Rick Smith, they were developed to accompany a computational linear algebra course at the University of Florida, which has used *Linear Algebra and Its Applications* for many years. The projects are referenced by an icon  at appropriate points in the text. About half of the projects explore fundamental concepts such as the column space, diagonalization, and orthogonal projections; several projects focus on numerical issues such as flops, iterative methods, and the SVD; and a few projects explore applications such as Lagrange interpolation and Markov chains.

SUPPLEMENTS

Study Guide

A printed version of the *Study Guide* is available at low cost. I wrote this *Guide* to be an integral part of the course. An icon  in the text directs students to special subsections of the *Guide* that suggest how to master key concepts of the course. The *Guide* supplies a detailed solution to every third odd-numbered exercise, which allows students to check their work. A complete explanation is provided whenever an odd-numbered writing exercise has only a “Hint” in the answers. Frequent “Warnings” identify common errors and show how to prevent them. MATLAB boxes introduce commands as they are needed. Appendixes in the *Study Guide* provide comparable information about Maple, Mathematica, and TI graphing calculators (ISBN: 0-321-38883-6).

Instructor’s Edition

For the convenience of instructors, this special edition includes brief answers to all exercises. A *Note to the Instructor* at the beginning of the text provides a commentary on the design and organization of the text, to help instructors plan their courses. It also describes other support available for instructors. (ISBN: 0-321-38518-7)

Instructor’s Technology Manuals

Each manual provides detailed guidance for integrating a specific software package or graphic calculator throughout the course, written by faculty who have already used the technology with this text. The following manuals are available to qualified instructors through the Pearson Instructor Resource Center, www.pearsonhighered.com/irc: MATLAB (ISBN: 0-321-53365-8), Maple (ISBN: 0-321-75605-3), Mathematica (ISBN: 0-321-38885-2), and the TI-83+/86/89 (ISBN: 0-321-38887-9).

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I am indeed grateful to many groups of people who have helped me over the years with various aspects of this book.

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A Note to Students

This course is potentially the most interesting and worthwhile undergraduate mathematics course you will complete. In fact, some students have written or spoken to me after graduation and said that they still use this text occasionally as a reference in their careers at major corporations and engineering graduate schools. The following remarks offer some practical advice and information to help you master the material and enjoy the course.

In linear algebra, the *concepts* are as important as the *computations*. The simple numerical exercises that begin each exercise set only help you check your understanding of basic procedures. Later in your career, computers will do the calculations, but you will have to choose the calculations, know how to interpret the results, and then explain the results to other people. For this reason, many exercises in the text ask you to explain or justify your calculations. A written explanation is often required as part of the answer. For odd-numbered exercises, you will find either the desired explanation or at least a good hint. You must avoid the temptation to look at such answers before you have tried to write out the solution yourself. Otherwise, you are likely to think you understand something when in fact you do not.

To master the concepts of linear algebra, you will have to read and reread the text carefully. New terms are in boldface type, sometimes enclosed in a definition box. A glossary of terms is included at the end of the text. Important facts are stated as theorems or are enclosed in tinted boxes, for easy reference. I encourage you to read the first five pages of the Preface to learn more about the structure of this text. This will give you a framework for understanding how the course may proceed.


In a practical sense, linear algebra is a language. You must learn this language the same way you would a foreign language—with daily work. Material presented in one section is not easily understood unless you have thoroughly studied the text and worked the exercises for the preceding sections. Keeping up with the course will save you lots of time and distress!

Numerical Notes

I hope you read the Numerical Notes in the text, even if you are not using a computer or graphic calculator with the text. In real life, most applications of linear algebra involve numerical computations that are subject to some numerical error, even though that error may be extremely small. The Numerical Notes will warn you of potential difficulties in using linear algebra later in your career, and if you study the notes now, you are more likely to remember them later.

If you enjoy reading the Numerical Notes, you may want to take a course later in numerical linear algebra. Because of the high demand for increased computing power, computer scientists and mathematicians work in numerical linear algebra to develop faster and more reliable algorithms for computations, and electrical engineers design faster and smaller computers to run the algorithms. This is an exciting field, and your first course in linear algebra will help you prepare for it.

Study Guide

To help you succeed in this course, I suggest that you purchase the *Study Guide* (www.mypearsonstore.com; 0-321-38883-6). Not only will it help you learn linear algebra, it also will show you how to study mathematics. At strategic points in your textbook, an icon  will direct you to special subsections in the *Study Guide* entitled “Mastering Linear Algebra Concepts.” There you will find suggestions for constructing effective review sheets of key concepts. The act of preparing the sheets is one of the secrets to success in the course, because you will construct *links between ideas*. These links are the “glue” that enables you to build a solid foundation for learning and *remembering* the main concepts in the course.

The *Study Guide* contains a detailed solution to every third odd-numbered exercise, plus solutions to all odd-numbered writing exercises for which only a hint is given in the Answers section of this book. The *Guide* is separate from the text because you must learn to write solutions by yourself, without much help. (I know from years of experience that easy access to solutions in the back of the text slows the mathematical development of most students.) The *Guide* also provides warnings of common errors and helpful hints that call attention to key exercises and potential exam questions.

If you have access to technology—MATLAB, Maple, Mathematica, or a TI graphing calculator—you can save many hours of homework time. The *Study Guide* is your “lab manual” that explains how to use each of these matrix utilities. It introduces new commands when they are needed. You can download from the website www.pearsonhighered.com/lay the data for more than 850 exercises in the text. (With a few keystrokes, you can display any numerical homework problem on your screen.) Special matrix commands will perform the computations for you!

What you do in your first few weeks of studying this course will set your pattern for the term and determine how well you finish the course. Please read “How to Study Linear Algebra” in the *Study Guide* as soon as possible. My students have found the strategies there very helpful, and I hope you will, too.