INTERMEDIATE ALGEBRA

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THIRD EDITION

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PREFACE

In this edition we have maintained the use of clear, direct language that the student will find understandable and we have increased the rigor of the examples and explanations. The result is a blend of detailed topic development and general algebraic principles followed by procedural summaries, the combination of which allows the student a smoother transition between elementary algebra and college algebra.

Major Features Retained from the Second Edition

- Important concepts and algorithms are enclosed in **boxes** for easy identification and reference.
- Common algebraic errors are clearly identified in special **Words of Caution**. This helps to reduce the number of mistakes commonly made by the inexperienced student.
- Liberal use is made of **visual aids** such as number lines, shading, and other graphics.
- The **even-numbered exercises parallel the odd exercises** so that the assignment of either set provides complete coverage of the material.
- **Answers** to all **odd-numbered exercises**, as well as many selected solutions for these odd exercises, are provided in the back of this text.
- To prepare a student for taking a chapter test, both a **comprehensive summary** and a **review** are included at the end of each chapter. Complete solutions to all problems in these reviews are given in the answer section.

Major Changes from the Second Edition

- We have **increased the rigor of exercises** in Chapter One that are intended as a review of basic arithmetic.
- **Fractions** have been used more often in both examples and exercises so that the student is prepared to apply fraction skills to algebra.
- The **properties of real numbers** are organized into Sections 1–1 through 1–4 of Chapter One. These properties are referenced in later sections as they are applied to reinforce the link between conceptual algebra and applications.
- Many of the **problem sets** are expanded, including exercises that require problem solving, writing, and the use of concepts in order to be sure students understand why a procedure is done, not just how.
- Extensive **cumulative reviews** are added at the end of Chapters Three and Seven and a final review at the end of the book. **Critical Thinking** sections are also added, so that there is either a Cumulative Review or a Critical Thinking section at the end of each chapter.
- The **exponent rules** are revised slightly, so that the division rule is written with only one form. Exercises are revised to ensure that students can do the problems at each level of presentation.
- The **approach** to doing the mixture, solution, interest, coin, and similar problems is slightly revised, with other types of problems included in the problem sets.
- More emphasis is placed on **algebraic rules** used to work problems. These are emphasized in the explanations in the examples. Overall, the explanations are revised to use the properties of algebra more extensively.
- **Cramer's rule** is excluded as a method for solving higher-order systems; however, solving third-order determinants is retained.
- The addition-subtraction method for solving systems is revised to be only the **addition method**.

Ancillaries

- An **Instructor's Manual** contains four different tests for each chapter and two final examinations. The manual also contains solutions for all the even-numbered exercises in the textbook, as well as essays to help the instructor teach developmental mathematics students. Essays cover such topics as: writing in the mathematics classroom, running a lab, cooperative learning, and more.
- **EXP-Test**, a computerized test bank for IBM PCs and compatible hardware, contains all of the test questions in the Instructor's Manual and is available to adopters of the text.
- Fifteen **videotapes**, created by John Jobe of Oklahoma State University, review the most essential and difficult topics from the textbook.

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1 REVIEW OF ELEMENTARY TOPICS

Intermediate algebra is made up of two types of topics: (1) those that were introduced in beginning algebra and are expanded in this course, and (2) new topics not covered in beginning algebra. With most topics we begin with the ideas learned in beginning algebra and then develop these ideas further.

In Chapter One we review sets, the properties of real numbers, integral exponents, polynomials, and simplifying and evaluating algebraic expressions.

1–1 Sets

Ideas in all branches of mathematics—arithmetic, algebra, geometry, calculus, statistics, etc.—can be explained in terms of sets. For this reason you will find it helpful to have a basic understanding of sets.

Set

A **set** is a collection of objects or things.

Example 1 Sets

(a) The set of students attending college in the United States

(b) The set of *natural numbers*: 1, 2, 3, etc.

Element of a Set

The objects or things that make up a set are called its **elements** (or *members*). Sets are usually represented by listing their elements, separated by commas, within braces $\{\ \ \}$.

Example 2 Elements of sets

(a) Set $\{5, 7, 9\}$ has elements 5, 7, and 9.

(b) Set $\{a, d, f, h, k\}$ has elements a, d, f, h, and k.

A set may contain just a few elements, many elements, or no elements at all. A set is usually named by a capital letter such as A, N, or W. Letters used to name elements of sets are usually lowercase. The expression "$A = \{m, t, c\}$" is read "A is the set whose elements are m, t, and c."

Roster Method

A class roster is a list of the members of the class. When we represent a set by $\{3, 8, 9, 11\}$, we are representing the set by a **roster** (or *list*) of its members.

Modified Roster Method. Sometimes the number of elements in a set is so large that it is not convenient or even possible to list all the members. In such cases we modify the roster notation. For example, the set of natural numbers can be represented as follows:

$$\{1, 2, 3, \ldots\}$$

This is read "The set whose elements are 1, 2, 3, and so on." The three dots to the right of the number 3 indicate that the remaining numbers are to be found by counting in the same way we have begun—namely, by adding 1 to the preceding number to find the next number.

Equal Sets

Two sets are **equal** if they both have exactly the same members.

Example 3 Equal sets and unequal sets

(a) $\{1, 5, 7\} = \{5, 1, 7\}$. Notice that both sets have exactly the same elements, even though they are not listed in the same order.

(b) $\{1, 5, 5, 5\} = \{5, 1\}$. Notice that both sets have exactly the same elements. It is not necessary to write the same element more than once in the roster of a set.

(c) $\{7, 8, 11\} \neq \{7, 11\}$. These sets are not equal because they do not both have exactly the same elements.

Rule Method (Set-Builder)

A set can also be represented by giving a **rule** describing its members in such a way that we definitely know whether a particular element is in that set or is not in that set.

$$\{x \mid x \text{ is an even number}\}$$

is read "The set of **all x such that x is an even number.**"

The rule

Example 4 Changing from set-builder notation to roster notation
Write $\{x \mid x + 2 = 5\}$ in roster notation.

$$\{x \mid x + 2 = 5\} = \{3\}$$

Example 5 Changing from roster notation to set-builder notation
Write $\{0, 3, 6, 9\}$ in set-builder notation.

$$\{0, 3, 6, 9\} = \{x \mid x \text{ is a digit exactly divisible by } 3\}$$

Note: The letter used in set-builder notation does not affect the elements of the set. Therefore the set given in Example 5 could be written as $\{y \mid y$ is a digit exactly divisible by $3\}$. ■

Any letter

Symbol ∈

The expression $2 \in A$ is read "2 is an element of set A." If $A = \{2, 3, 4\}$, we can say: $2 \in A$, $3 \in A$, and $4 \in A$. If we wish to show that a number or object is *not* an element of a given set, we use the symbol $\notin$, which is read "is not an element of." If $A = \{2, 3, 4\}$, then $5 \notin A$, which is read "5 is not an element of set A." Notice that $\in$ looks like the first letter of the word *element*.

Example 6 The use of ∈ and ∉

If $F = \{x \mid x \text{ is a negative number}\}$, then $-7 \in F$, $-\frac{1}{2} \in F$, and $6 \notin F$.

Empty Set (Null Set)

Set $B = \{1, 5\}$ has two elements. Set $C = \{5\}$ has only one element. Set $D = \{\ \ \}$ is empty, since it has no elements. A set having no elements is called the **empty set** (or *null set*). We use the symbol $\{\ \ \}$ or $\varnothing$ to represent the empty set.

Note: The combination of symbols $\{\varnothing\}$ *does not* represent the empty set. Technically, $\{\varnothing\}$ represents a set that contains the empty set. ■

Example 7 The empty set

(a) The set of all people in your math class who are 10 ft tall $= \varnothing$.

(b) The set of all positive numbers less than $0 = \{\ \ \}$.

Universal Set

A **universal set** is a set that consists of all the elements being considered in a particular problem. The universal set is represented by the letter U.

Example 8 Universal sets

(a) Suppose we are going to consider only digits. Then $U = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$, since U contains *all* the digits.

(b) Suppose we are going to consider only negative numbers. Then $U = \{x \mid x$ is a negative number$\}$.

Notice that there can be different universal sets.

Finite Set

If in counting the elements of a set the counting comes to an end, the set is called a **finite set**.

Example 9 Finite sets

(a) $A = \{5, 9, 10, 13\}$

(b) $\varnothing = \{\ \ \}$

Infinite Set

If in counting the elements of a set the counting *never* comes to an end, the set is called an **infinite set**.

Example 10 Infinite sets

(a) $N = \{1, 2, 3, \ldots\}$ The natural numbers

(b) $J = \{x \mid x \text{ is a negative number}\}$

Subsets

A set A is called a **subset** of set B if every member of A is also a member of B. "A is a subset of B" is written in "$A \subseteq B$" or "$B \supseteq A$."

Note: The symbol $\subseteq$ is used to indicate that one *set* is a *subset* of another set. The symbol $\in$ is used to indicate that a particular *element* is a *member* of a particular set. ■

Example 11 Subsets

(a) $A = \{3, 5\}$ is a subset of $B = \{3, 5, 7\}$ because every member of A is also a member of B. Therefore $A \subseteq B$.

(b) $F = \{10, 7, 5\}$ is a subset of $G = \{5, 7, 10\}$ because every member of F is also a member of G. Every set is a subset of itself.

(c) $D = \{4, 7\}$ is *not* a subset of $E = \{7, 8, 5\}$ because $4 \in D$, but $4 \notin E$. Therefore $D \nsubseteq E$, which is read "D is not a subset of E."

(d) The empty set is a subset of every set. $\{\ \}$ is a subset of A because there is no member of $\{\ \}$ that is not also a member of A.

Example 12 List all the subsets of the set $\{a, b, c\}$.

$\{\ \}$	The subset having *no* elements	The empty set is a subset of every set
$\{a\}, \{b\}, \{c\}$	The subsets having *one* element	
$\{a, b\}, \{a, c\}, \{b, c\}$	The subsets having *two* elements	
$\{a, b, c\}$	The subset having *three* elements	Every set is a subset of itself

Venn Diagrams

A useful tool for helping you understand set concepts is the **Venn diagram**. A simple Venn diagram is shown in Figure 1–1.

All elements of the universal set U are considered to lie in the rectangle labeled U. All elements of set A are considered to lie in the circle labeled A. Since all elements of A also lie in the rectangle U, $A \subseteq U$.

Union of Sets

The **union of sets** A and B, written $A \cup B$, is the set that contains all the elements of A as well as all the elements of B. For an element to be in $A \cup B$, it must be in set A or set B.

In each of the Venn diagrams in Figures 1–2 and 1–3 the union is represented by the shaded area. In Figure 1–2, $A = \{b, c, g\}$, $F = \{b, c, d, e\}$, and $A \cup F = \{b, c, d, e, g\}$. In Figure 1–3, $A = \{b, c, g\}$, $B = \{h, i, j, k\}$, and $A \cup B = \{b, c, g, h, i, j, k\}$.

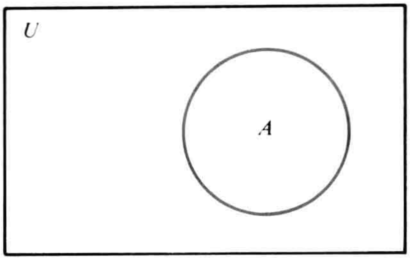

Figure 1–1 Venn Diagram

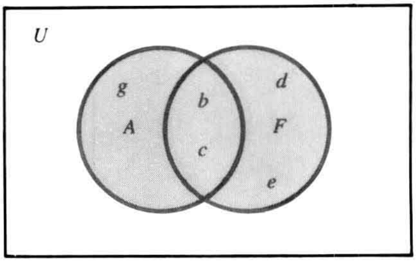

Figure 1–2 $A \cup F$

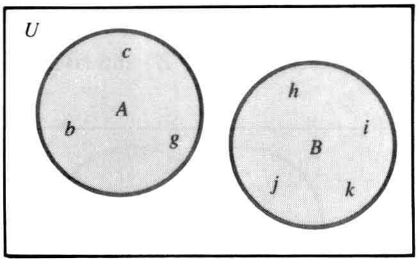

Figure 1–3 $A \cup B$

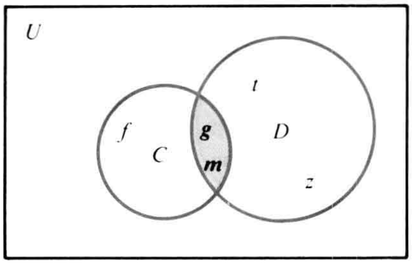

Figure 1–4 $C \cap D$

Intersection of Sets

The **intersection of sets** C and D, written $C \cap D$, is the set that contains only elements in *both* C and D. For an element to be in $C \cap D$, it must be in set C *and* set D.

Consider $C = \{g, f, m\}$ and $D = \{g, m, t, z\}$. Then $C \cap D = \{g, m\}$ because g and m are the only elements in both C and D. In the Venn diagram in Figure 1–4 the shaded area represents $C \cap D$ because that area lies in both circles.

Disjoint Sets

Disjoint sets are sets whose intersection is the empty set. Figure 1–3 is an example of a Venn diagram showing disjoint sets A and B.

Example 13 Disjoint sets

(a) If $A = \{b, c, g\}$ and $B = \{1, 2, 5, 7\}$, then $A \cap B = \varnothing$. Therefore A and B are disjoint sets.

(b) If $R = \{5, 7, 9\}$ and $T = \{9, 10, 12\}$, then $R \cap T = \{9\} \neq \varnothing$. Therefore R and T are *not* disjoint sets; they are intersecting sets.

Venn diagrams can be used to represent combinations consisting of both unions and intersections of sets.

Example 14 Use the Venn diagram with three intersecting circles in Figure 1–5 to represent $(A \cup B) \cap (B \cup C)$.

Step 1. Shade the portion representing $A \cup B$.

Step 2. Shade the portion representing $B \cup C$.

Step 3. The intersection consists of the overlapping areas in Figure 1–5.

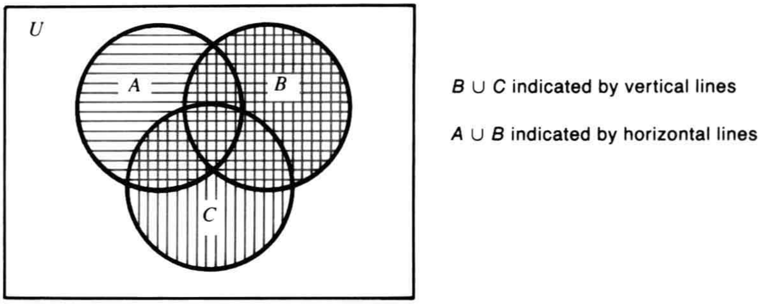

Figure 1–5 $(A \cup B)$ **and** $(B \cup C)$

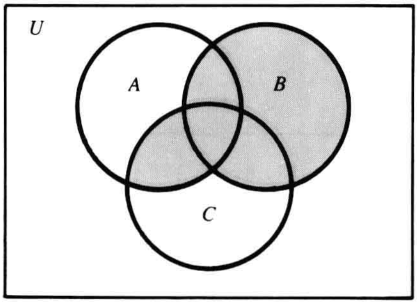

Figure 1–6 $(A \cup B) \cap (B \cup C)$

The intersection is shown in Figure 1–6.

We can use Venn diagram representations to verify whether combinations that seem different actually represent the same area.

Example 15 Use the Venn diagram with three intersecting circles in Figure 1–7 to represent $B \cup (A \cap C)$ and compare the diagram with Figure 1–6.

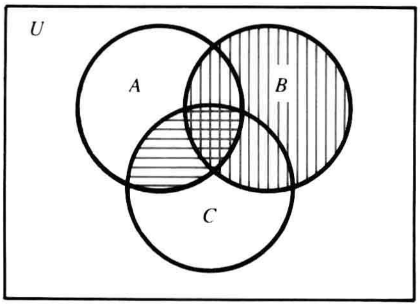

Figure 1–7 B **and** $A \cap C$