Interdisciplinary Mathematical Sciences – Vol. 17

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Festschrift Masatoshi Fukushima

In Honor of Masatoshi Fukushima's Sanju

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Preface

Compiling a Festschrift and presenting it at a larger international conference is arguably the most personal way the academic community can express its appreciation to one of its members. More than five decades, Professor Masatoshi Fukushima has contributed to our discipline and is still surprising us with highly original contributions and deep insights. He has long been recognised as one of the towering figures in stochastic analysis who has shaped, and in fact initiated, the probabilistic part of the theory of Dirichlet spaces. His scholarship is much admired, as have his mentorship of the younger generation and his contributions to the international mathematical community.

The overwhelming and cordial responses we have received when inviting contributions for the Festschrift Masatoshi Fukushima to be published on the occasion of his Sanju is further testimony of Professor Fukushima's standing in our community. The Festschrift contains 26 articles, some being surveys, some being cutting-edge research contributions, as well as an update of Professor Fukushima's bibliography published a few years ago in "Selecta Masatoshi Fukushima". We would like to take the opportunity to thank again all colleagues for their contributions, and in particular for collaborating with us smoothly. We also want to thank those who helped us as referees, and special thanks go to K. Yamazaki, A. Arranz-Carreño and K. Evans for their tex-nical support. Further thanks go to Ms. Tan Rok Ting of World Scientific Publishing Co. Pte. Ltd. for her constant and truly generous cooperation and also to the publisher for the uncomplicated collaboration.

We consider it as a great honour to present to Professor Fukushima this Festschrift on behalf of his colleagues, students and friends, as a tribute to his mathematical achievements and as gratitude for his friendship.

Zhen-Qing Chen (Seattle)
Niels Jacob (Swansea)
Masayoshi Takeda (Sendai)
Toshihiro Uemura (Osaka)

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Part 1

Professor Fukushima's Work

Chapter 1

The mathematical work of Masatoshi Fukushima - An Essay

Zhen-Qing Chen, Niels Jacob, Masayoshi Takeda and Toshihiro Uemura

Over more than five decades Professor Fukushima has made remarkable contributions to mathematics, especially to stochastic analysis and its applications to science, in particular to mathematical physics. For many of his colleagues, students and friends it becomes an excellent opportunity to express their appreciation of himself and his work by celebrating his Sanju, the 80th birthday in the Japanese tradition, with an international conference and by publishing a Festschrift in his honour. Naturally such a Festschrift should contain a scientific account of Professor Fukushima's oeuvre. When making here an attempt, i.e. writing "un essai" in the classical meaning, to present such an account, we must be aware that we are not discussing a completed oeuvre. Professor Fukushima is continuing to make highly original research contributions with deep insights. Obviously it is pre-matured to describe or guess the lasting impact these latest contributions will have. However it is fair to say that some of his earlier works are meanwhile considered as to belong to the "classical canon" of stochastic analysis, and hence we will concentrate more on those contributions. But we will see how some of his latest research indeed connect with his earlier work in a natural way.

Professor Fukushima started his graduate studies in 1959 as a student of Kiyoshi Itô at Kyoto University. For anyone who is interested in the history of probability theory it is a temptation to write about the amazing changes in the field during the three decades between the publication of the works of P. Lévy [24], J. L. Doob [8], K. Itô [19] and [20], and S. Kakutani [22], and the publication of the monograph of K. Itô and H. P. McKean [21]. This period also includes the contributions of the E. B. Dynkin school, and we refer to the monographs [10] - [11], and for us quite important the work of W. Feller, in particular [12] and [13], as well as the contributions of G. A. Hunt [16] - [18]. (Clearly, this list is not exhausting.) However we have already some good, although brief, account of this period, especially when having in mind the developments in Japan, see D. W. Stroock and S. R. S. Varadhan [29], H. Föllmer [15], and in particular to M. Fukushima [F39] as well as [F49]. We should also mention the work of Kôsaku Yosida combining one-parameter operator semigroup theory with probability theory and potential theory on which S. Watanabe

[30] gave a readable account.

For Professor Fukushima, early in his career, the problem to understand Feller's boundary theory for one-dimensional diffusion processes in higher dimensions, say Brownian motion in bounded domains, took an important place, see [F1] and [F2], and in some sense this is a topic that still occupies him after 50 years, see for example [F44]. Already in these early papers the idea was to use Dirichlet spaces as introduced by A. Beurling and J. Deny [2] and [3]. Feller's work in one dimension could use the fact that (in a rough formulation) integrable functions with an integrable distributional derivative are absolutely continuous. Clearly this does not apply in higher dimensions, but the results of the paper by J. Deny and J.-L. Lions [7], see also J. L. Doob [9], provided a type of substitute, i.e. allowing to pass to quasi everywhere fine continuous versions. In [7] many technical tools originated from classical potential theory, by which we mean really the theory of harmonic functions and the Newton potential, and thus the method had a natural limitation. In the paper [F2] Professor Fukushima introduced already a significant change into the theory of Dirichlet spaces as initiated by A. Beurling and J. Deny: the concept of "Dirichlet spaces related to $L^2(X; m)$" was introduced, where X is a certain Hausdorff space with some finite measure m. Although this looks at first glance as technical or minor, today this is the standard definition of (symmetric) Dirichlet spaces (with some more general space X and measure m), once we add regularity, see the comments to [F3] below.

The papers [F3] and [F4] are Professor Fukushima's early seminal contribution in which he laid the foundation of what is nowadays understood as the modern theory of Dirichlet forms. Most of this work was done while he was a post doctoral fellow at the University of Illinois, Urbana - Champaign, working in the group of J. L. Doob. In [F3] parts of the quadratic form based potential theory of symmetric Dirichlet forms are studied, and in particular "regular representations" are discussed. The basic problem was to find for a given L^2 - related Dirichlet space an isometric version on a locally compact metric space that contains "sufficiently many" regular (continuous) elements. Now the ground was prepared for [F4], the construction of a strong Markov process associated to a regular Dirichlet space. The ideas picked up from the Deny - Lions paper [7], and employed already in [F2], were combined with the regular representation result. Using the natural capacity in a regular Dirichlet space it is possible to obtain quasi-continuous refinements of elements and this makes it eventually possible to construct the corresponding Hunt process (up to an invariant negligible set). This result was immediately recognised as a breakthrough and Professor Fukushima was invited to give a talk at the ICM in Helsinki 1978.

The construction of the Hunt process associated with a Dirichlet form we find now in the monograph [F43] and its predecessors [F10] and [F23] follows the modification by M. L. Silverstein which appeared in [27], not the paper [F4]. Professor Fukushima always points to the important contributions of Martin Silverstein to the theory of Dirichlet forms [27] and the boundary theory of symmetric Markov processes [28], as did Martin Silverstein appreciated his work. Martin Silverstein was among the first to recognize the importance of the regular representation of Dirichlet forms considered by Professor Fukushima, see [27], Section 2. Maybe in times without citation index, impact factors, etc., it was much easier to add to academic standards also civilized standards in human relations, respect for the work of colleagues, and a certain intellectual honesty.

The construction of a Hunt process being associated with a regular Dirichlet form was already a big step forward, but immediately several question arose, for example:

1. Itô's stochastic calculus is an analysis on path level. How can we derive a corresponding theory for symmetric Markov processes which may not be semi-martingales?
2. The processes are constructed up to a set of zero capacity. Is it possible to remove such exceptional set?
3. Can this approach to stochastic processes help to treat problems in science, especially mathematical physics?
4. Is it eventually possible to extend Feller's boundary theory to higher dimensions?

Itô calculus changed its face under the influence of P. A. Meyer and his school. The basic objects for a "reasonable" stochastic calculus were identified as semi-martingales, but the Hunt processes constructed with the help of Dirichlet forms are in general not semi-martingales. The breakthrough was the paper [F8] where the "Fukushima decomposition" was introduced. Using versions of elements in the Dirichlet space, additive functionals were introduced with the help of the associated Hunt process and it was proved that every such additive functional admits a decomposition into a martingale and an additive functional of zero energy. Then a stochastic calculus for additive functionals was established. Meanwhile the Fukushima decomposition and the related stochastic calculus is a core element of stochastic analysis with many applications in the studies of diffusions in infinite dimensions or on fractals, and in the study of Markov processes in random environment, just to mention some areas.

One of the hardest problem is to construct "nice" versions of the process associated with a Dirichlet form. The best solution would be to get rid of any exceptional set. Several papers of Professor Fukushima's, including [F13, F16, F21], are devoted to this problem. Arguably the most influential one is that jointly with H. Kaneko [F16]. Here (r,p)-capacities, originally introduced by P. Malliavin [26], were taken up with the aim to get version up to a set of (r,p)-capacity zero. These capacities are monotone in r and in p , and it may happen that for r or p large the only set of

(r, p)-capacity zero is the empty set. H. Kaneko [23] has used this to study diffusion processes in an L^p-setting.

Many of the non-Japanese colleagues of Professor Fukushima may not know about the book [F9] in Japanese co-authored with the physicist K. Ishii titled "Natural Phenomena and Stochastic Processes" first published in 1980. Influenced by the work of K. Ishii, Professor Fukushima turned to the study of the spectra of random Schrödinger operators and related topics, see [F6] written with H. Nagai and S. Nakao, [F7] with S. Nakao, as well as [F18] with S. Nakao and M. Takeda. Subsequently he continued to publish on mathematical physics and was in closed contact with colleagues in Bielefeld, including S. Albeverio, W. Karwowski and L. Streit, see for example [F11]. His interest and work had much impact on the later work of S. Kotani.

Within our essay we can not discuss all of Professor Fukushima's contributions, so we mention only briefly his work on ergodic theorems [F5, F12], on problems related to large deviation, for example [F15] with M. Takeda, on stochastic analysis on fractals [F20, F22, F27] in collaboration with T. Shima and M. Takeda, on Dirichlet forms related to complex analysis of several variables [F17] and [F14, F19] with M. Okada, or on BV functions, capacitary inequalities and the "geometry" of sets [F28] - [F32], including joint works with M. Hino or T. Uemura, or finally, already more related to the boundary theory of Markov processes, the study of reflecting diffusions with M. Tomisaki [F24] - [F26].

However we want and we have to return to the beginning of his career and the attempt to extend Feller's theory. Time changes of Markov processes are closely related to their boundary theory. Motivated by the Douglas integral, which was the main tool of J. Douglas to give a solution to the Plateau problem and that also characterizes the trace of reflecting Brownian motion in the unit disk on its boundary, Professor Fukushima with P. He and J. Ying in [F33] studied time changes of symmetric diffusions and the role played by the Feller measure. They showed that the jumping and killing measures for the time-changed process are bounded below by the Feller measure and the supplementary Feller measure, respectively. A complete solution to this problem is obtained in a joint work of Professor Fukushima with Z.-Q. Chen and J. Ying [F35], where not only the jumping and killing measures of the time-changed process are identified, but also the complete characterization of the Beurling - Deny decomposition, see also the extension in [F36].

The boundary theory for one-dimensional diffusions is well understood thanks to the fundamental works of W. Feller, K. Itô, and H. P. McKean. Much less is known for the boundary theory of multi-dimensional diffusions and of Markov processes with discontinuous sample paths. In his work with H. Tanaka [F34] Professor Fukushima studied the one-point extension of absorbing symmetric diffusions using excursion