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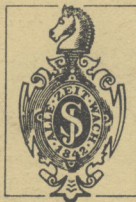
Volume 9

**Elastic and Charge Exchange Scattering
of Elementary Particles**

Supplement to Volume I/7 and Extension to High Energies

Subvolume b: Pion Nucleon Scattering

Part 2: Methods and Results of Phenomenological Analyses



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Editor in Chief: K.-H. Hellwege

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Ergänzung zu Band I/7 und Erweiterung für hohe Energien

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phänomenologischer Analysen

G. Höhler

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Units for cross sections

$$\frac{d\sigma}{dt} \left[\frac{\text{mb}}{\text{GeV}^2} \right] = 0.3894 \frac{d\sigma}{dt} [\text{GeV}^{-4}] = 1026.0 \frac{d\sigma}{dt} [\text{n.u.}] ;$$

$$\frac{d\sigma}{d\Omega} \left[\frac{\text{mb}}{\text{sr}} \right] = 0.3894 \frac{d\sigma}{d\Omega} [\text{GeV}^{-2}] = 19.99 \frac{d\sigma}{d\Omega} [\text{n.u.}] = \frac{q^2 [\text{GeV}^2]}{\pi} \frac{d\sigma}{dt} \left[\frac{\text{mb}}{\text{GeV}^2} \right]$$

$$1 \text{ mb} = 10^3 \mu\text{b} = 10^6 \text{ nb} = 10^{-27} \text{ cm}^2.$$

Masses and derived quantities

Pion: $\mu = \mu_{\pm} = 0.13957 \text{ GeV}$, $\mu_0 = 0.13496 \text{ GeV}$.

Nucleon: $m = m_p = 0.93828 \text{ GeV}$, $m_n = 0.93957 \text{ GeV}$.

$$1/\mu = 7.1649 \text{ GeV}^{-1}, \quad 1/\mu^2 = 51.336 \text{ GeV}^{-2}, \quad m/\mu = 6.7227, \quad m^2 = 45.195 \mu^2.$$

Compton wavelength: $\lambda_{\pi} = \hbar c/\mu c^2 = 1.4138 \text{ fm}$, $\lambda_{\pi}^2 = 19.989 \text{ mb}$, $\lambda_{\pi}/c = 4.716 \cdot 10^{-24} \text{ s}$.

We have always used the proton mass and the charged pion mass in kinematical calculations.

Pion-Nucleon coupling constant and other parameters

The pseudoscalar and pseudovector coupling constants g and f are related by $g^2/4\pi = (2m/\mu)^2 f^2$. Taking $f^2 = 0.079$, we have $g^2/4\pi = 14.28$, $g = 13.40$, $g^2/m = 26.70 \mu^{-1} = 191.3 \text{ GeV}^{-1}$, $g^2/2m^2 = 1.99 \mu^{-2}$.

Pion decay constant: $f_{\pi} = (0.945 \pm 0.001) \mu = 132 \text{ MeV}$. Axial vector coupling constant: $g_A = 1.26 \pm 0.01$.

Isospin relations

Amplitudes for the reactions $\pi^{\pm} p \rightarrow \pi^{\pm} p$ and $\pi^{-} p \rightarrow \pi^0 n$

$$A_+ = A^{3/2}, \quad A_- = A^+ - A^- = \frac{1}{3}(2A^N + A^{\Delta}).$$

$$A_- = \frac{1}{3}(2A^{1/2} + A^{3/2}) = A^+ + A^- = A^{\Delta},$$

$$A_0 = -\frac{\sqrt{2}}{3}(A^{1/2} - A^{3/2}) = -\sqrt{2}A^- = \frac{\sqrt{2}}{3}(A^N - A^{\Delta}) = \frac{1}{\sqrt{2}}(A_+ - A_-).$$

$I = 1/2$ amplitude

$$A^{1/2} = \frac{1}{2}(3A_- - A_+) = A^+ + 2A^-.$$

Isospin even and odd combinations

$$A^+ = \frac{1}{2}(A_- + A_+) = \frac{1}{3}(A^{1/2} + 2A^{3/2}); \quad A^- = \frac{1}{2}(A_- - A_+) = -\frac{1}{\sqrt{2}}A_0 = \frac{1}{3}(A^{1/2} - A^{3/2}).$$

Upper indices: 1/2 and 3/2 give the value of I . N and Δ belong to $I_u = 1/2$ and $3/2$, respectively.

Parameters of meson resonances

	η	ϱ	ω	S^*	ϕ	f	g
Mass [MeV]	548.8 ± 0.6	769 ± 3	782.6 ± 0.2	975 ± 4	1019.61 ± 0.07	1273 ± 5	1691 ± 5
Full width [MeV]	$0.83 \cdot 10^{-3}$ $\pm 0.12 \cdot 10^{-3}$	154 ± 5	9.9 ± 0.3	33 ± 6	4.21 ± 0.13	1.79 ± 20	200 ± 20
$I^G(J^P)C_n$	$0^+(0^-)+$	$1^+(1^-)-$	$0^-(1^-)-$	$0^+(0^+)+$	$0^-(1^-)-$	$0^+(2^+)+$	$1^+(3^-)-$

I = isospin, G = G -parity, J = spin, P = space parity, C_n = charge-conjugation parity for the neutral member of the multiplet (Review of Particle Properties: 82 R Roos).

Conversion of units

Most of the equations contain only the physical quantities mass, energy, momentum, length and time (no electrical or magnetic quantities). In this case, the equations can be written in terms of only one natural unit, preferably an energy. If the rest mass μ of a particle is chosen, the units are shown in the first line of the following table; if 1 GeV is used as basic unit, the corresponding units are shown in the second line.

Basic unit	Energy	Mass	Momentum	Length	Time
μc^2	μc^2	μ	μc	$\lambda = \frac{\hbar}{\mu c}$	$\frac{\lambda}{c} = \frac{\hbar}{\mu c^2}$
1 GeV	GeV	GeV/ c^2	GeV/ c	GeV $^{-1}$	GeV $^{-1}/c$

Both these systems of units go usually under the expression $\hbar=c=1$. In this volume the charged pion mass is used for μ (natural units, "n.u."). As an example the equation $E^2=(pc)^2+m^2c^4$ can be written as

$$(E/\mu c^2)^2 = (p/\mu c)^2 + (m/\mu)^2 \text{ or } (E/\text{GeV})^2 = (p/\text{GeV}/c)^2 + (mc^2/\text{GeV})^2$$

In the literature and also in this volume there is an inconsistency concerning the treatment of c in the dimension. In some cases, $c=1$ is understood (for instance resonance masses are usually given in GeV) but in others, c is written in the dimension (momenta are given in GeV/ c).

Length	fm	n.u.	GeV $^{-1}$	mb·GeV
1 fm = 10^{-13} cm =	—	0.7073	5.068	1.9733
1 n.u. = $1\mu^{-1}$ =	1.414	—	7.165	2.7898
1 GeV $^{-1}$ =	0.1973	0.1396	—	0.3894
1 mb·GeV =	0.5068	0.3585	2.568	—
Area	mb	n.u.	GeV $^{-2}$	mb $^{1/2}$ GeV $^{-1}$
1 mb = 10^{-27} cm 2 =	—	0.05003	2.568	1.6025
1 n.u. = $1\mu^{-2}$ =	19.99	—	51.34	32.03
1 GeV $^{-2}$ =	0.3894	0.01948	—	0.6240
1 mb $^{1/2}$ GeV $^{-1}$ =	0.6240	0.03122	1.6025	—

Units for amplitudes

Invariant amplitudes A, C and spin non-flip and flip amplitudes G, H :

$$A[\text{GeV}^{-1}] = \frac{A[\mu^{-1}]}{0.1396} = \frac{A[\text{fm}]}{0.1973} = \frac{A[\text{mb} \cdot \text{GeV}]}{0.3894} = \frac{A[\text{mb}^{1/2}]}{0.6240}$$

Invariant amplitude B and helicity amplitudes F_{++}, G_{++} :

$$B[\text{GeV}^{-2}] = \frac{B[\text{mb}]}{0.3894} = \frac{B[\text{mb}^{1/2} \text{GeV}^{-1}]}{0.6240} = 51.33 B[\mu^{-2}]$$

Vorwort

Band I/9, der Daten über die Elastische und Ladungsaustausch-Streuung von Elementarteilchen enthält, besteht aus zwei Teilbänden a, b, von denen I/9a bereits 1980 erschienen ist.

In Teilband I/9b sind die Daten über die π -N Streuung zusammengestellt. Da dieser Prozess über viele Jahrzehnte das besondere Interesse sowohl der Experimentalphysiker wie der Theoretiker fand, wurde es nötig, den Teilband noch einmal zu unterteilen. Teil I/9b1 (erschienen Ende 1981) enthält die experimentellen Daten. Die π -N Streuung ist einer der wenigen Prozesse, bei denen es gelingt, aus den Experimenten Streuamplituden abzuleiten. Die Zusammenstellung dieser Streuamplituden ist der wesentliche Inhalt dieses (letzten) Teiles I/9b2.

Da die Berechnung der Streuamplituden aus den experimentellen Daten nicht ganz einfach ist und gewisse Annahmen erfordert, hat sich um diesen Problemkreis eine reiche Literatur entwickelt. Im ersten Kapitel wird eine kritische Übersicht über die verschiedenen Methoden der Partialwellenanalyse und der Bestimmung der Streuamplituden gegeben.

Da die Streuamplituden Mittelwerte über die einzelnen Experimente darstellen, sind sie zuverlässiger als die ursprünglichen Daten. Es ist daher für manche Zwecke sinnvoll, die aus den Streuamplituden zurückberechneten meßbaren Größen zu verwenden. Diese Größen sind in einem weiteren Kapitel zusammengestellt. Weiterhin werden Anwendungen der Streuamplituden auf eine Reihe von speziellen Problemen diskutiert.

Sehr nützlich sollte auch die Formelzusammenstellung im Anhang sein, insbesondere bei der Lektüre von Originalarbeiten mit ihren sehr uneinheitlichen Bezeichnungen.

Die kritische und umfassende Zusammenstellung der Ergebnisse und Methoden der π -N Streuung dürfte einmalig sein. Sie wurde nur möglich durch die große Kompetenz und Mühe des Autors, der diesen Problemen einen großen Teil seiner wissenschaftlichen Aktivität gewidmet hat. Alle bei der Fertigstellung des Bandes aufgetretenen technischen und administrativen Schwierigkeiten wurden in der üblichen souveränen Weise durch den Gesamtherausgeber, die Redaktion, besonders Herrn Dr. W. Polzin und Frau D. Dolle, und den Verlag überwunden. Ihnen allen gebührt besonderer Dank. Dieser Bandteil wurde, wie alle Bände des Landolt-Börnstein, ohne finanzielle Unterstützung von anderer Seite veröffentlicht.

Genf, Oktober 1982

Der Herausgeber

Preface

Volume I/9 containing data on Elastic and Charge Exchange Scattering of Elementary Particles, consists of two subvolumes a, b, the first of which, I/9a, is already available since 1980.

In subvolume I/9b the data of the π -N scattering have been compiled. Since this process has attracted the special interest of both experimentalists and theorists during many decades, it became necessary to subdivide this subvolume into two parts. Part I/9b1 (published late 1981) contains the experimental data. π -N scattering is one of the few processes for which it is possible to derive scattering amplitudes from the experiments. The compilation of these scattering amplitudes is the essential content of part I/9b2.

Since the derivation of the scattering amplitudes from the experimental data is not simple and requires certain assumptions, a comprehensive literature developed around this field of problems. Hence a critical review of the various methods of partial wave analysis and the determination of scattering amplitudes is presented in the first chapter.

Since the scattering amplitudes represent average values over individual experiments, they are more reliable than the original data. For certain purposes it seems useful therefore to utilize measurable quantities reconstructed from the scattering amplitudes. These quantities are compiled in another chapter. In addition, applications of the scattering amplitudes to a number of special problems are discussed.

The compilation of formulae in the Appendix should turn out to be very useful, in particular facilitating the reading of original papers with their rather non-uniform notation.

The critical and comprehensive review of the results and methods of π -N scattering should be regarded as unique. It became possible only by the remarkable competence and efforts of the author who has dedicated a large part of his scientific activity to these problems. All technical and administrative difficulties which arose during the production of this volume were mastered in the usual expert way by the editor in chief, the editorial office, especially Dr. W. Polzin and Frau D. Dolle, and the publisher. Special thanks are due to all of them. This part of the volume, like all other volumes of Landolt-Börnstein, has been published without financial support from any outside source.

Geneva, October 1982

The Editor

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Contents

Pion-nucleon scattering

Subvolume b2: Methods and results of phenomenological analyses

2 Methods and results of phenomenological analyses	1
2.0 Introduction	1
2.1 Methods of partial wave and amplitude analyses	4
2.1.1 General remarks on partial wave and amplitude analyses	4
2.1.2 Elastic phase shift analysis	5
2.1.3 Inelastic partial wave analysis – the continuum ambiguity	5
2.1.4 Classification of partial wave analyses	7
2.1.5 The SACLAY and CERN phase shift analyses	8
2.1.6 The analysis LONDON 73 (“CBC”)	9
2.1.7 The Karlsruhe-Helsinki analyses KH 78 and KH 80	10
2.1.8 The analysis LENINGRAD 78 and the SIN analyses	14
2.1.9 The Carnegie-Mellon-Berkeley analysis (CMU-LBL 79)	15
2.1.10 New energy-dependent analyses	18
2.1.11 BARRELETs method	20
2.1.12 The analysis HENDRY 80	23
2.1.13 π^-p scattering near the η -production threshold	24
2.1.14 The tail of high partial waves	25
2.1.15 Amplitude analyses without dispersion constraints	27
2.1.16 Amplitude analyses with dispersion constraints	31
2.1.17 Electromagnetic corrections	32
2.1.17.1 Introduction	32
2.1.17.2 Models for the electromagnetic corrections	32
2.1.17.3 Radiative corrections	34
2.1.17.4 Electromagnetic corrections to polarization data	34
2.1.17.5 The $\Delta^{++} - \Delta^0$ mass difference	34
2.2 Results for pion-nucleon scattering amplitudes	36
2.2.1 πN forward amplitudes from total cross sections and dispersion relations	36
2.2.1.1 Introduction	36
2.2.1.2 The input for the total cross section	36
2.2.1.3 Evaluation of the dispersion relation	39
2.2.1.4 Results for forward amplitudes	41
2.2.1.5 Remarks on the literature on pion-nucleon forward amplitudes	54
2.2.2 πN partial wave amplitudes	55
2.2.3 Invariant amplitudes	102
2.2.4 s -channel helicity amplitudes	128
2.3 Measurable quantities reconstructed from partial waves	148
2.3.1 Differential cross sections	148
2.3.2 Polarization and spin-rotation parameters	168
2.4 Further results derived from πN partial waves	198
2.4.1 Parameters of pion-nucleon resonances	198
2.4.1.1 The notion of a resonance	198
2.4.1.2 Parametrizations of resonances	202
2.4.1.3 Results for the parameters of nucleon resonances	204
2.4.1.4 Nucleon resonances in other reactions than $\pi N \rightarrow \pi N$	214

2.4.1.5	Models for nucleon resonances	216
2.4.1.6	Degeneracy of the complex pole positions	221
2.4.2	Meson-nucleon coupling constants	222
2.4.2.1	Determination of the pion-nucleon coupling constant f^2	222
2.4.2.2	Other meson-nucleon coupling constants	226
2.4.3	Zero trajectories	229
2.4.3.1	Introduction	229
2.4.3.2	General properties of zero trajectories	229
2.4.3.3	Zero trajectories of invariant amplitudes	231
2.4.3.4	Zero trajectories of transversity amplitudes	236
2.4.3.5	Zero trajectories of other amplitudes	237
2.4.4	The saturation of isospin bounds	238
2.4.4.1	Introduction	238
2.4.4.2	Isospin bounds for backward scattering	238
2.4.4.3	Isospin bounds for cross sections and polarizations	239
2.4.4.4	Isospin bounds at high energies	240
2.4.4.5	Further isospin bounds	242
2.4.5	Invariant amplitudes in the unphysical region and near threshold	242
2.4.5.1	Analytic continuation at fixed t	242
2.4.5.2	Analytic continuation along other paths	243
2.4.5.3	Errors of invariant amplitudes in the unphysical region	243
2.4.6	$\pi\pi N\bar{N}$ partial waves in the unphysical region	244
2.4.6.1	$\pi\pi N\bar{N}$ partial waves at $t \leq 0$	244
2.4.6.2	$\pi\pi$ -scattering phase shifts	265
2.4.6.3	Extrapolation of $\pi\pi N\bar{N}$ partial waves to $t > 0$	268
2.4.7	Low energy expansions	273
2.4.7.1	The subthreshold expansion at $v = t = 0$	273
2.4.7.2	The threshold expansion	276
2.4.7.3	Connection between the two expansions	282
2.5	Applications of πN and $\pi\pi N\bar{N}$ amplitudes	285
2.5.1	Low-energy models for the pion-nucleon system	285
2.5.1.1	Introduction	285
2.5.1.2	The ADLER-WEISBERGER relation	285
2.5.1.3	ADLER's consistency condition and the pion-nucleon sigma term	287
2.5.1.4	The GOLDBERGER-TREIMAN discrepancy	294
2.5.1.5	Current algebra models	294
2.5.1.6	Dispersion models	296
2.5.1.7	Models for applications in nuclear physics	297
2.5.2	Electromagnetic nucleon and pion form factors	301
2.5.2.1	Introduction	301
2.5.2.2	Dispersion theory of the nucleon isovector form factors	302
2.5.2.3	Dispersion analysis of nucleon form factor data	304
2.5.2.4	The pion form factor	308
2.5.3	Two-pion exchange in nucleon-nucleon scattering	310
2.5.3.1	Introduction	310
2.5.3.2	Two-pion-exchange in NN forward scattering	310
2.5.3.3	Quark models of nucleon-nucleon scattering	313
2.5.4	$NN \leftrightarrow \pi\pi$ amplitudes at $t > 4m^2$	313
2.6	Analysis of high energy data	314
2.6.1	Introduction	314
2.6.2	The Coulomb interference region	314
2.6.3	Diffraction scattering: fits to the data and amplitude analysis	321
2.6.3.1	The shape of the diffraction peak	322
2.6.3.2	The momentum dependence of C^+ and of the slope of $\text{Im}C^+$	326
2.6.3.3	The partial wave decomposition of the diffraction peak	328
2.6.3.4	The inelastic overlap function	330
2.6.3.5	A high energy approximation of the impact parameter formalism	331

2.6.3.6	pp-scattering amplitudes in the ISR range (see also p. 404)	333
2.6.3.7	Elastic scattering at large $ t $	335
2.6.3.8	The isospin even s -channel helicity flip amplitude	337
2.6.4	Models of diffraction scattering	339
2.6.4.1	The CHOU-YANG model	339
2.6.4.2	The CHENG-WU model	344
2.6.4.3	Geometrical scaling	345
2.6.4.4	The VAN HOVE model or the inelastic overlap function	347
2.6.4.5	Other s -channel models	348
2.6.4.6	Constituent models	349
2.6.4.7	General remarks on t -channel models	349
2.6.4.8	HARARI's dual absorptive model	350
2.6.4.9	Recent work on Regge pole phenomenology	351
2.6.4.10	LIPKINS two-component pomeron	353
2.6.4.11	Diffraction in electromagnetic processes	354
2.6.4.12	The transition to the wide-angle region	356
2.6.4.13	Theoretical papers on diffraction scattering	358
2.6.5	Near-forward charge-exchange scattering	359
2.6.5.1	The shape of the $\pi^- \rightarrow \pi^0 n$ near-forward cross section	359
2.6.5.2	Validity of the reggeized q -exchange model	363
2.6.5.3	Determination of $\pi^- p$ charge-exchange scattering amplitudes at high energies	364
2.6.5.4	The amplitudes of the reggeized q -exchange model	368
2.6.5.5	Other models	371
2.6.5.6	Superconvergence, finite energy sum rules and duality	373
2.6.5.7	Charge-exchange scattering at intermediate energies	375
2.6.6	Wide-angle scattering	376
2.6.6.1	Experimental results on wide-angle scattering at high energies	376
2.6.6.2	Hard scattering models	377
2.6.6.3	Geometrical models	380
2.6.6.4	Statistical models	381
2.6.7	Near-backward scattering	382
2.6.7.1	Experimental data for near-backward scattering at high energies	382
2.6.7.2	Dispersion analysis of backward scattering data	386
2.6.7.3	Isospin analysis of near-backward scattering data	388
2.6.7.4	Baryon-exchange Regge pole analyses of near-backward data	388
2.6.7.5	Line-reversed reactions	391
2.6.7.6	Dominance of peripheral partial waves in backward scattering	392
2.6.7.7	Expansion methods for the evaluation of dispersion integrals	394
2.6.8	Total elastic and charge-exchange cross sections	396
2.6.9	Survey index for the experimental data in the near-backward u -range	401
3	Bibliography	405
3.1	Books, reviews and lecture notes	405
3.2	Original papers and preprints	424
3.3	List of pion-nucleon scattering experiments	478

Appendix: Collection of pion-nucleon scattering formulas

1	Kinematics	480
1.1	Notation	480
1.2	Kinematics in the center of mass frame	481
1.2.1	Relations between energy and momentum variables	482
1.2.2	Relations which include the scattering angle or t	482
1.3	Relations between center of mass and laboratory frame variables	483
1.3.1	Relations between energy and momentum variables	484
1.3.2	Relations which include the scattering angle or t	484

1.3.3	Difference and differential formulas	485
1.3.4	Scattering at fixed laboratory scattering angle θ_π	485
1.4	Kinematics of the t -channel reaction $\pi\pi \rightarrow \pi\pi$	485
1.4.1	Physical region $t \geq 4m^2$	486
1.4.2	Unphysical region $t < 4m^2$	486
1.5	u -channel kinematics	486
1.6	The brick-wall frame	486
2	Scattering amplitudes	487
2.1	The spin structure	487
2.1.1	Spin no-flip and flip amplitudes $G(s, t)$ and $H(s, t)$	487
2.1.2	Transversity amplitudes $F(\pm)$	488
2.1.3	s -channel helicity amplitudes F_{++}, F_{+-}	489
2.1.4	s -channel laboratory frame amplitudes G_L and H_L	489
2.1.5	Invariant amplitudes A, B and combinations C, D	489
2.1.6	t -channel amplitudes	491
2.1.7	u -channel amplitudes	491
2.1.8	Relations between different amplitudes	492
2.1.9	High energy approximations	493
2.2	The isospin structure	494
2.2.1	Isospin operators and states	494
2.2.2	Matrix representations	495
2.2.3	Isospin invariance	496
2.2.4	Isospin relations for $\pi\pi \rightarrow \pi\pi$ scattering	498
2.3	Electromagnetic contributions	499
3	Expansions of scattering amplitudes	500
3.1	s -channel partial wave expansion	500
3.2	Expansion of transversity amplitudes	502
3.3	Impact parameter representations	503
3.3.1	The Fourier-Bessel transform	503
3.3.2	Two-dimensional Fourier transforms	505
3.3.3	Use of longitudinal and transverse momentum variables	506
3.4	t -channel partial wave expansion	507
3.4.1	$\pi\pi \rightarrow \pi\pi$ partial wave expansion in the range $t < 4\mu^2$	507
3.4.2	$\pi\pi \rightarrow \pi\pi$ partial wave expansion in the range $t > 4\mu^2$	509
3.5	Low energy expansions and threshold behaviour	509
3.5.1	Expansion at $v=t=0$	509
3.5.2	Expansions at the s -channel threshold	510
3.5.3	Behaviour near $t=4\mu^2$	511
3.5.4	Expansion at the t -channel threshold $t=4m^2$	512
4	Measurable quantities	514
4.1	Description of spin states	514
4.2	Differential cross section measurements	515
4.3	Polarization measurements	515
4.4	Relations between measurable quantities and amplitudes	516
4.4.1	Amplitudes G, H , and s -channel helicity amplitudes	516
4.4.2	Invariant amplitudes and t -channel helicity amplitudes	517
4.4.3	Legendre expansions	517
4.5	$\pi\pi \rightarrow \pi\pi$ and $\pi\pi \rightarrow \pi\pi$ cross sections	518
4.6	Electromagnetic contributions to cross sections and polarizations	518
5	Unitarity	519
5.1	Unitarity for $\pi\pi$ scattering amplitudes	519
5.1.1	Forward scattering	520
5.1.2	Elastic unitarity	520
5.1.3	Unitarity for partial waves and impact parameter transforms	522
5.2	Unitarity for $\pi\pi\pi$ -amplitudes	522
5.3	Unitarity bounds	523
5.4	POMERANCHUK-type theorems	525

6	Symmetries and analytic properties of invariant amplitudes	525
6.1	Behaviour of invariant amplitudes under symmetry operations	525
6.2	The MANDELSTAM representation	527
6.2.1	The double dispersion relation	527
6.2.2	Single-dispersion relations and absorptive parts	529
6.2.3	Further properties of absorptive parts and invariant amplitudes	530
6.2.4	Convergence of partial wave expansions	531
6.3	Fixed- t dispersion relations	533
6.3.1	General properties	533
6.3.2	Subtractions	535
6.3.3	Evaluation of the fixed- t dispersion relation	536
6.3.4	The expansion method	536
6.4	Forward dispersion relations	537
6.4.1	The basic formulas	537
6.4.2	Modified dispersion relations	538
6.4.3	The phase representation	538
6.4.4	Parametrizations for forward amplitudes at high energies	539
6.4.5	Derivative analyticity relations	540
6.4.6	Analytic continuation into the second sheet	540
6.5	The dispersion relation for $\partial C^+/\partial t$ at $t=0$	542
6.6	Fixed- u and fixed- s dispersion relations	542
6.7	Hyperbolic dispersion relations	543
6.7.1	General properties	543
6.7.2	Interior dispersion relations	543
6.8	Fixed center-of-mass angle dispersion relations	545
6.9	Backward dispersion relations	548
6.10	Fixed- v dispersion relations	548
7	Analytic properties of partial wave amplitudes	549
7.1	s -channel partial wave dispersion relations	549
7.1.1	The singularities of the partial wave amplitudes	549
7.1.2	The generalized πN potential	550
7.1.3	Unitary sum rules	551
7.1.4	Behaviour of $f_{t\pm}(s)$ at $s=0$	551
7.1.5	Partial wave dispersion relations in the W -plane	551
7.2	s -channel partial wave relations	552
7.2.1	Partial wave relations derived from fixed- t dispersion relations	552
7.2.2	Partial wave relations derived from hyperbolic dispersion relations	554
7.3	Analytic continuation of s -channel partial wave amplitudes into the second sheet	554
7.4	The FROISSART-GRIBOV representation for $\pi\pi NN$ wave amplitudes	556
7.4.1	The projection formulas	556
7.4.2	The nucleon pole term contributions	557
7.4.3	The FROISSART-GRIBOV projection at $t=4\mu^2$	558
7.5	t -channel partial wave dispersion relations	558
7.6	The CHEW-MANDELSTAM model for $\pi\pi$ -partial wave amplitudes	559
8	Low energy models	561
8.1	Nucleon and resonance exchanges	561
8.1.1	Nucleon exchange	561
8.1.2	Isobar exchange	562
8.1.3	Rho-exchange	565
8.2	The current algebra model	566
8.3	The dispersion model	568
8.3.1	Isospin odd amplitudes	568
8.3.2	Isospin even amplitudes	569
9	Regge models	569
9.1	Exchange of reggeized mesons in the t -channel	569
9.1.1	Analytic continuation of $\pi\pi NN$ partial waves in J	570

9.1.2 The Sommerfeld-Watson transform	570
9.1.3 The Regge pole formulas	571
9.2 Exchange of reggeized nucleons and nucleon resonances	573
9.2.1 Analytic continuation of πN partial waves in J	573
9.2.2 Exchange of Regge poles in the s -channel	574
9.2.3 Exchange of Regge poles in the u -channel	575
9.2.4 Line reversal	577
10 Isospin bounds	578
10.1 Equalities and bounds for a complete set of data	578
10.2 Bounds for differential cross sections and polarizations	579
10.3 Saturation of isospin bounds	580
11 Electromagnetic nucleon and pion form factors	580
11.1 The nucleon form factors	580
11.2 The pion form factor	584
11.3 Analytic continuation of electromagnetic form factors into the second sheet	587
12 References	588
Subject index	594
Table of the kinematical variables, see backside cover	

Subvolume b1: Tables of data

0 Introduction	1
1 Tables of data	2
1.1 General remarks	2
1.2 Survey indices	3
1.2.1 List of experiments	3
1.2.2 Survey index for differential cross sections and polarizations	10
1.3 Total cross sections	29
1.3.1 Total π^+p cross sections	29
1.3.2 Total π^-p cross sections	30
1.4 Differential cross sections	31
1.4.1 Differential cross sections for π^+p elastic scattering	31
1.4.2 Differential cross sections for π^-p elastic scattering	148
1.4.3 Differential cross sections for π^-p charge-exchange scattering	277
1.5 Polarization parameters	323
1.5.1 Polarization parameters for π^+p elastic scattering	323
1.5.2 Polarization parameters for π^-p elastic scattering	364
1.5.3 Polarization parameters for π^-p charge-exchange scattering	375
1.6 Spin-rotation parameters R and A	391
1.6.1 R parameter for π^-p elastic scattering	391
1.6.2 R parameter for π^+p elastic scattering	391
1.6.3 A parameter for π^-p elastic scattering	391
1.7 Literature on pion-nucleon scattering experiments	392
1.8 Other compilations of pion-nucleon scattering data and reports on related topics	400

2 Methods and results of phenomenological analyses

2.0 Introduction

Volumes I/7 and I/9b1 contain tables of the experimental data on elastic and charge-exchange pion-nucleon scattering. The present volume deals with the determination of scattering amplitudes from these data and applications within the framework of phenomenological theories.

The methods for partial wave and amplitude analysis are treated in Sect. 2.1 and results for various scattering amplitudes and for measurable quantities reconstructed from them are presented in Sects. 2.2 and 2.3, respectively. These tables give a condensed version of the experimental information.

Scattering amplitudes can be determined from data only if one uses in addition a theoretical input. Besides general theoretical principles (Lorentz invariance, T , C , and P -invariance, unitarity, fixed- t analyticity) one employs Mandelstam analyticity, which has the status of a working hypothesis, and isospin invariance, which is thought to be a good approximation for the strong interaction amplitudes.

Thus one could argue that the theoretical input leads to a model dependence of the result, which is due for instance to the necessary assumptions on the high energy behaviour and on the smoothing procedure in applications of the analyticity constraint. The magnitude of the resulting uncertainty can be estimated by repeating the analysis with reasonable modifications of these assumptions. It turns out to be much less serious than the model dependence in those cases, for which the analysis is based on a parametrization derived from a specific phenomenological model.

For a comparison with theoretical predictions, it is not sufficient to have long tables of four complex-valued amplitudes, which depend on two variables. One needs in addition a description of the amplitudes in certain kinematical regions in terms of a relatively small number of parameters, which have a direct physical interpretation and can be compared with the values calculated from models.

The most important parameters of the pion-nucleon system are those of the poles and zeros of the scattering amplitudes. The pole parameters are listed and discussed in Sects. 2.4.1 and 2.4.2. They are the "experimental" quantities, which are used in tests of various quark models of the nucleon. The properties of the zero trajectories (Sect. 2.4.3) show a remarkable similarity to the prediction from Veneziano-type dual models.

Section 2.4.4 deals with the saturation of isospin bounds. Kinematical regions in which these saturations occur are of interest in connection with the search for small violations of isospin invariance.

Analytic continuations of scattering amplitudes into unphysical regions have to be performed for several reasons, for instance for the determination of $\pi\pi NN$ amplitudes, for the test of predictions derived from Current Algebra and PCAC and for the study of zero trajectories. A brief treatment of $\pi\pi$ -scattering at low energies is included, because $\pi\pi$ -phase shifts are needed as part of the input (Sects. 2.4.5 and 2.4.6). The result is given not only in the form of tables (Tables 2.2.3.2 and 2.4.6.1) but also by the coefficients of the subthreshold expansion (Sect. 2.4.7). The coefficients of the effective range expansion at threshold are in practice also determined by a continuation, because it is not possible to obtain accurate values of the strong interaction amplitudes from πN scattering experiments at very low energies (Sect. 2.4.7).

Three applications of the continued amplitudes are treated in Sect. 2.5:

i) Many authors have proposed models for the πN system at low energies, which incorporate the predictions derived from Current Algebra and PCAC. These models have to be compared with amplitudes in the threshold and subthreshold regions (Sect. 2.5.1). One of the reasons why these models are studied is the possibility to calculate off-shell extrapolations of πN scattering amplitudes, which are needed in nuclear physics.

Another problem of current interest is a test of the prediction for the πN sigma term from quantum chromodynamics.

ii) Important properties of the electromagnetic nucleon form factors can be determined only if one applies the dispersion approach and includes the $I=J=1$ $\pi\pi NN$ partial waves of Sect. 2.4.6 as part of the input (Sect. 2.5.2). Since the pion form factor is also needed, we have added a summary on recent experimental results and investigations of its dispersion relation.

iii) Originally, simple one-boson-exchange models were used for the description of two-pion-exchange in nucleon-nucleon scattering. The results for the $\pi\pi NN$ partial waves are the basic input for a quantitative treatment (Sect. 2.5.3).

Since the physical range $t > 4m^2$ occurs in dispersion analyses of $\pi\pi NN$ -amplitudes, we have added in Sect. 2.5.4 a short summary of the $NN \leftrightarrow \pi\pi$ experiments available in this region, mentioning the first attempts to determine the partial wave amplitudes.

Section 2.6 is devoted to the analysis of πN scattering data at high energies. It can be expected that, in the coming years, a considerable part of the experimental and theoretical activity in pion-nucleon physics will concen-

2.0 Introduction

trate in this field. Therefore, I have summarized the most popular models, $\pi\pi$ -scattering is included because of the close similarity of the diffraction phenomena.

The last section contains a list of references for books and reviews (Sect. 3.1), original papers and recent pre-prints (Sect. 3.2), and papers on pion-nucleon scattering experiments (Sect. 3.3, see also Table 1.2.1 in Volume I/9b1).

A collection of pion-nucleon scattering formulas has been added in an Appendix, because it is my experience that one can lose considerable time on more or less trivial but tedious calculations, if it is necessary to work with formulas taken from various books, reviews and original papers. The reason is that the authors employ different conventions and notations, which are not always explained in full detail, or at a place where the definition is not easily found. Furthermore, the number of misprints is far from negligible.

To my knowledge, there exists no single book which covers all topics treated in the Appendix. Of course, printing and other errors cannot be avoided, but the present volume has a useful redundancy: it contains many numerical tables and figures, which have been calculated from the formulas of the Appendix. Furthermore, a shorter version of the collection of formulas has been used in our group for many years.

A reader who is interested in the derivation of a formula can find the reference in Sect. A.12.

The present volume has been written for physicists who are familiar with the main experimental facts and at least with the elementary parts of the theoretical formalism, which is described in several excellent textbooks and monographs. It will be useful for experimentalists who are studying possible interpretations of their results or planning new experiments, and also for theorists who want to test phenomenological models or rigorous predictions.

Part of the contents will be of interest to physicists who are working in other fields, where πN or $\pi N\bar{N}$ -amplitudes are needed, for instance in nuclear physics or in investigations of pionic atoms or of nucleon form factors.

The chapters on pion-nucleon scattering in the existing textbooks and monographs were written about 10 years ago or even earlier. Therefore, this book will be valuable for graduate students who want to become acquainted with the more recent developments.

Volume I/9b1 includes the results of all experimental investigations on pion-nucleon scattering which have been performed since the publication of Volume I/7. Because of the very large number of phenomenological and theoretical papers in this field, only a small fraction could be taken into account in Volume I/9b2. My choice was to give a fairly complete treatment of the phenomenological analysis of the experimental data, which leads to results for the scattering amplitudes and for various parameters of the pion-nucleon system. Special attention has been paid to applications of dispersion relations as constraints and for analytic continuations.

Since the work done in the fifties and sixties has been described in several excellent books and in many reviews and summer school lectures, I have concentrated on the progress made during the last decade.

It would have been unsatisfactory to present the amplitudes and the parameters without explaining why such a great effort has been made for their determination. Therefore, I have also discussed the present status of various models which have had some success in explaining properties of the pion-nucleon system, and a few applications of πN and $\pi N\bar{N}$ -amplitudes in other fields. The choice of topics reflects my own interests and limitations.

One of my aims was to present a "guide to the literature" in the following sense. In Sects. 2 and A.12, I have not only mentioned the publications actually used in writing this book but also many others, which would have to be discussed in a more detailed review. In particular, I have added references of the most recent papers, even if I have not been able to give a comment, because my manuscript had already been submitted. Furthermore, Sect. 3.1 contains a fairly complete list of books and reviews, which are related to the topics treated in this volume.

A well-known difficulty of books of this type is that parts will be outdated after a period which is appreciably smaller than the time interval between two editions. One can extend the space of time during which the book fulfills its purpose and is actually used by physicists working in this field by producing an informal "Supplement", which contains updated versions of the main tables and lists, corrections of misprints and short remarks on errors or omissions, which have been brought to the attention of the author. Updated versions of the tables will be prepared in any case for the continuation of the work of our group and we are willing to distribute copies on request.

In the fifties and sixties, pion-nucleon scattering was one of the main topics at international conferences on high energy physics. Later, many physicists left this field and started to work on other problems which seemed to be more promising.

Nowadays, interest is increasing again because of first attempts to calculate parameters of the pion-nucleon system using methods based on quantum chromodynamics, which is believed to have a good chance to be the correct theory of strong interactions. At present, the best evidence in favour of this theory comes from reactions other than hadron-hadron scattering, e.g. electron-positron interactions. However, as soon as the serious difficulties in deriving quantitative predictions for the interactions between hadrons have been overcome, pion-nucleon scattering amplitudes will play an important role for a critical test. They are known with a higher

accuracy than other strong scattering amplitudes and furthermore, they contain the experimental information on the excited states of the nucleon. A calculation of these states is one of the fundamental problems in a theory of strong interactions.

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G. H.