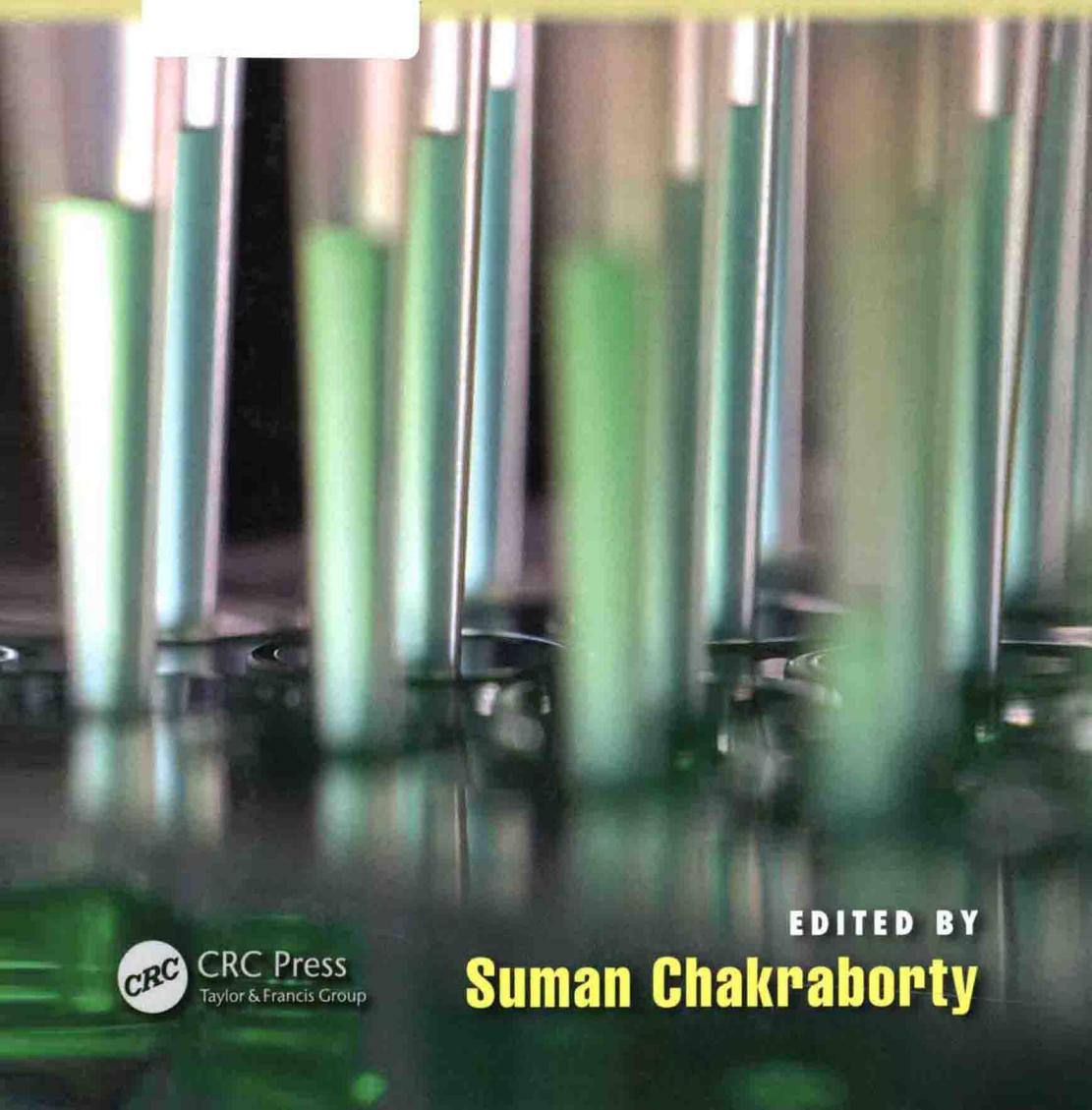
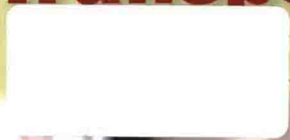


IIT KHARAGPUR RESEARCH MONOGRAPH SERIES

# Microfluidics and Microscale Transport Processes



CRC Press  
Taylor & Francis Group

EDITED BY

**Suman Chakraborty**

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# Microfluidics and Microscale Transport Processes

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EDITED BY  
Sunil Chakraborty



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## About the Series

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The Indian Institute of Technology (IIT) Kharagpur has been a forerunner in research publications and this monograph series is a natural culmination. Empowered with more than sixty years of cumulative experience, the faculty and alumni have collaborated to present the *IIT Kharagpur Research Monograph Series*.

Initiated during the diamond jubilee year of the institute, this series collates research and developments in various branches of science and engineering in a coherent manner. An ongoing endeavor, the series is expected to serve as a reference source of fundamental research as well as providing direction to young researchers. The presentations included in this series appear in a format that may serve as stand-alone texts or reference books.

The specific objective of this research monograph series is to encourage the outstanding faculty and esteemed alumni to spread and share knowledge and information to the global community for the betterment of mankind.

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## *The Institute*

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IIT Kharagpur is one of the pioneering technological institutes in India and was the first of its kind to be established immediately after that country's independence. The institute was founded on August 18, 1951, at Hijli, Kharagpur, West Bengal, India. IIT Kharagpur has the largest campus of all the IITs, with an area of 2,100 acres. At present, it has thirty-four departments, centers, and schools, and approximately 10,000 undergraduate, post-graduate, and research students. With a current strength of nearly 600, the number of faculty is expected to double within approximately five years. The faculty and alumni of IIT Kharagpur have global exposure in the fields of science and engineering. Their experience and contributions will be promoted through this monograph series.

More on IIT Kharagpur is available at [www.iitkgp.ac.in](http://www.iitkgp.ac.in).

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# *Preface*

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The advancements in micro- and nano-fabrication techniques, especially in the last few decades, have led research communities all over the world to invest unprecedented levels of attention in the science and technology of micro- and nano-scale devices and their related applications. Simultaneously, sheer scientific curiosity has led to an insatiable hunger to delve deeper into the microscopic realm in order to uncover new phenomena as well as to develop a deeper and more fundamental understanding of them. Beyond the traditional vanguards of this micro/nano brigade in the form of the considerably well-developed field of microelectronics (that has also pervaded the life of the general populace through an extensive range of products on the consumer electronics front), the existence of intriguing possibilities afforded by micro-/nanoscale transport processes has also come to be realized. The scope of such possibilities being comprehensively interdisciplinary in nature, it has captured the attention and imagination of scientists from diverse backgrounds. The applications of this inter-disciplinary field of inquiry traverse the broad spectrum from biotechnology and biomedical engineering to ink-jet printing and thermal management of electronic devices/systems.

Interestingly, although the cutting-edge nature of this new technology may make it appear to be limited to the abstruse confines of highly sophisticated research laboratories, there are few technological endeavors on the global front today that are of as high immediacy to the alleviation of human suffering (particularly under conditions of inequitable access to health and life opportunities). Such immediacy of societal relevance was recognized early on at different leading academic and research institutes worldwide, including IIT Kharagpur. These institutes today lead the world with a dedication to developing certain key aspects of the involved technological applications—their primary aim being to offer cost-effective solutions and alternatives to everyday health-care problems.

As part of the year-long diamond jubilee celebrations of IIT Kharagpur, a plan was formulated to produce a series of monographs on the focal areas of this research. Among the key areas identified were microfluidics and microscale transport processes. This volume is a culmination of those ideas. In spite of the nature of the genesis, the scope of this monograph has been deliberately kept broad and very general. The aim has not been to showcase the research achievements of the microfluidics research group of IIT Kharagpur in isolation. Rather, in each chapter and in each subsection, the global context and the state-of-the-art level reached by various research groups that motivated such research achievements are first amply discussed and highlighted. In this respect, therefore, the current monograph is a rather comprehensive compendium of key indicators to the developments that have



taken place in the last decade or so in some of the most active research topics in microscale transport processes. Furthermore, each of the authors have tried to find an optimal balance between a discussion of concrete applications and a development of fundamental understanding pertinent to the research topic at hand. It is believed that this will make this volume a useful reference/accompaniment to standard texts for pedagogical purposes, particularly at the senior undergraduate or beginning graduate level. Established researchers or our colleagues from the industry will find this book to be an important resource for the latest developments in fluidic technology.

This monograph discusses a wide range of issues in the subdomains of capillary transport, fluidic resistance, electrokinetics, substrate modification, rotational microfluidics, and the applications of the phenomena of these subdomains in diverse situations ranging from non-biological (for instance, micro-heat pipes) to biological ones (such as DNA hybridization and cellular biomicrofluidics). In addition, the monograph addresses a generic problem in nanoparticle transport in colloidal suspensions. Finally, it also includes a chapter on Lattice-Boltzmann methods for phase-changing problems that represents a generic particle-based approach that may be useful in addressing many microfluidic problems of interdisciplinary relevance.

The editor wholeheartedly thanks all the contributing authors who ensured that important milestones (and deadlines!) were safely passed. The editor also finds himself in a delightfully unprecedented situation (compared to his previous editorial experiences) where each author is either a close colleague or friend, or his own student. He can only express his immense joy at having closely worked all these years with each and every one of them, and it is this work that has ultimately taken the form of the various chapters in this volume. The relentless efforts of Dr. Gagandeep Singh from CRC Press, right from the very inception of this monograph idea, through the proposal stage, and ultimately to the production stage, ensured a project as large as this never got derailed from the overall plan of completion. In many of the works that form a large part of the discussion in various chapters, the Microfluidics and Microscale Transport Processes Laboratory of IIT Kharagpur has been immensely fortunate to receive continuous support and tremendous encouragement from the government of India through its various sponsoring agencies (DST, DBT, DIT), as well as other nongovernment sponsoring agencies including Intel, General Motors, Tata Steel, and Delphi. The editor specially thanks the Indo-US Science and Technology Forum for its unstinted support. To the people at the helm of affairs at IIT Kharagpur, no amount of thanks will ever be enough. The editor also acknowledges the immense contributions of all his past and current research students toward the development of the laboratory to which he belongs, and to the fruition of many ideas. A special word of thanks is due to the editor's current doctoral student Jeevanjyoti Chakraborty for his active support in the various stages of the publication process of this monograph. Without his committed efforts, it would have been virtually impossible for the editor to adhere to the various deadlines

without sacrificing the academic rigor and scientific richness in the technical contents.

Finally, the editor gratefully acknowledges the continuing mental support of his parents and his wife, and thanks them for ungrudgingly tolerating the unearthly hours that his work often demanded throughout all these years. And to that curious little bundle of joy, his son, the editor has literally no words to explain how he gives an added meaning, and a hitherto unknown zest and richness to his own life and work!

**Suman Chakraborty**

*Mechanical Engineering Department, IIT Kharagpur*

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The MathWorks, Inc.

3 Apple Hill Drive

Natick, MA 01760-2098 USA

Tel: 508 647 7000

Fax: 508-647-7001

E-mail: [info@mathworks.com](mailto:info@mathworks.com)

Web: [www.mathworks.com](http://www.mathworks.com)

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## *Editor*

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**Dr. Suman Chakraborty** is currently a professor with joint affiliation in the Mechanical Engineering Department and the Advanced Technology Development Centre at the Indian Institute of Technology Kharagpur. He has research interests in the areas of microfluidics and micro- or nanoscale transport processes, including theoretical, computational, and experimental modeling, encompassing the underlying fundamentals as well as the bio-medical, biotechnological, chip cooling, and energy-related applications. He has been elected as a fellow of the Indian National Academy of Science (FNASc) and a fellow of the Indian National Academy of Engineering (FNAE). He is a recipient of the Indo-US Research Fellowship, the Scopus Young Scientist Award for his research in scientific/technical journals, and young scientist/young engineer awards from various national academies of science and engineering. He has also been an Alexander von Humboldt Fellow and a visiting professor at Stanford University. His writings have appeared in more than 190 international journal publications.

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## ***Contributors***

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**Debapriya Chakraborty**

Mechanical Engineering  
Department  
Indian Institute of Technology  
Kharagpur  
Kharagpur, India

**Jeevanjyoti Chakraborty**

Advanced Technology Development  
Centre  
Indian Institute of Technology  
Kharagpur  
Kharagpur, India

**Dipankar Chatterjee**

Simulation & Modeling Laboratory  
CSIR-Central Mechanical  
Engineering Research Institute  
Durgapur, India

**Siddhartha Das**

Department of Mechanical  
Engineering  
University of Alberta  
Edmonton, Canada

**Tamal Das**

Department of New Materials and  
Biosystems  
Max Planck Institute for Intelligent  
Systems  
Stuttgart, Germany

**Sunando DasGupta**

Chemical Engineering Department  
Indian Institute of Technology  
Kharagpur  
Kharagpur, India

**Ranabir Dey**

Mechanical Engineering  
Department  
Indian Institute of Technology  
Kharagpur  
Kharagpur, India

**Suvankar Ganguly**

Research & Development Division  
Tata Steel Limited  
Jamshedpur, India

**Lawrence Kulinsky**

Department of Mechanical &  
Aerospace Engineering  
University of California, Irvine  
Irvine, California

**Tapas K. Maiti**

Department of Biotechnology  
Indian Institute of Technology  
Kharagpur  
Kharagpur, India

**Marc Madou**

Department of Mechanical &  
Aerospace Engineering,  
Department of Biomedical  
Engineering, and Department  
of Chemical Engineering and  
Materials Science  
University of California, Irvine  
Irvine, California  
and  
Honorary Visiting Professor  
Indian Institute of Technology  
Kharagpur  
Kharagpur, India

**Rabibrata Mukherjee**

Chemical Engineering Department  
Indian Institute of Technology  
Kharagpur  
Kharagpur, India

**Balram Suman**

Energy Technology Company  
Chevron Corporation  
Houston, Texas

**Salar Soroori**

Department of Biomedical  
Engineering  
University of California, Irvine  
Irvine, California

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# Capillary Transport in Microchannels

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Debapriya Chakraborty and Suman Chakraborty

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## 1.1 Introduction

The word *capillary* derives its origin from a Latin word *capillaris*, meaning to resemble a strand of hair. It is commonly used to refer to a very small tube as small as a hair. As the size of a system is reduced, its surface area to volume ratio increases. Capillarity or capillary action refers to the ability of transport of fluids into these capillaries with or without the aid of an external driving force. Interestingly, unlike macroscale transport, spontaneous transport of the fluids is possible by exploiting the surface forces in these reduced scales. If we consider the length scale as  $L$ , the surface force scales as  $L^2$ , and the volumetric force scales as  $L^3$ , for a microdevice with characteristic length scale  $L \sim O(10^{-6}\text{m})$ , this ratio turns

out to be of the order of  $10^6$ , resulting in the dominance of surface effects over volumetric effects. Effects of many of the surface forces, which are not otherwise felt very prominently over macroscopic scales, may thus play decisive roles toward regulating the functionalities of microfluidic devices.

The phenomenon of capillarity governs many natural systems as well as functionalities of micro- and nanoscale devices. Transport of water/minerals from the roots to the branches is solely driven by the capillary action. Wicking is another form of capillarity, where the fluid is transported through the pores. Common examples of wicking are the soaking or absorbing of fluids on paper towels and lighting of oil lamps/candles. Capillary flows are exploited in thin layer chromatography, where dissolved solutes are transported with the solvent at various speeds. Different solutes get separated from the solvent mixtures depending upon their affinity for the solvent or the absorbent coating.

Capillary filling may be aided through several external driving forces, although it is primarily a surface tension-driven flow, wherein the actuation and control of fluid transport through a manipulation of the surface tension forces. The manipulation may be hydrodynamic, thermal, chemical, electrical, or optical in nature. Since surface tension forces scale with the linear dimensions, these become progressively more dominant with reduction of system length scale from macro to micro and further to nano. In a liquid-gas system, for example, molecules in the bulk of the liquid are pulled equally in all directions by the neighboring liquid molecules, resulting in no net force. At the interface, however, the molecules experience a net attractive force from other molecules inside the liquid, since these are not attracted as intensely by the molecules in the gaseous phase located on the other side. This is due to a denser molecular packing in the liquid phase than in the gas phase and a consequently stronger intermolecular force of attraction offered by the liquid molecules. To maintain interfacial equilibrium, molecules at the interface rearrange themselves to diminish the surface area (in order to minimize the surface energy), and a meniscus is developed in the form of a surface resembling a stretched elastic membrane. The pressure difference on either side of the meniscus leads to development of net normal force (pressure difference times the projected surface area). This normal force acting on the meniscus is balanced by the surface tension force in equilibrium, leading to a curved meniscus. Curvature of a meniscus essentially implicates a pressure jump across the same, which can act as a forcing parameter. In several cases, gradients of surface tension, as induced by gradients of temperature, concentration, or electrical voltage, may be utilized to realize differentials in a net driving force to manipulate microscale flows.

During the capillary filling of a microchannel, the fluid enters into a microchannel by the effects of surface tension. Further advancement of the capillary front is often observed with an additional driving influence



of the external driving effects. The fluid motion is opposed by the viscous resistances, as determined by the different flow regimes instantaneously prevailing within the liquid in the capillary. A critical assessment of the underlying consequences would effectively demand a comprehensive analysis of the complicated interplay between various forces dictating the interface evolution. The effects of surface wetting condition also play a crucial role in altering the contact line dynamics, bearing particular non-trivial interactions with the topological features of the solid boundaries. In an effort to describe these dynamics, the theoretical analysis presented in the next section is to introduce the dynamics of capillary driven flows by invoking fundamental fluid dynamic considerations for different external driving mechanisms studied by us, and at the same time restricting the analysis to well-suited semi-analytical frameworks without resorting to more complicated full-scale numerical analysis. However, we have also shown that the theoretical results obtained from a lumped analysis (or reduced order modeling) and even more detailed full-scale numerical simulations agree quite satisfactorily with the experimentally obtained capillary filling rates because of a consistent representation of the essential physics.

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## 1.2 Foundation of the Reduced Order Model

The dynamics of the capillary front may be described using a full solution of the multiphase equations as will be described later. Such solutions are computationally expensive, although they provide a detailed description of the involved shape of the meniscus. Alternatively, it is also possible to represent the governing equations to reduced order, which is also called reduced order model or lumped system analysis, where simple semi-analytical solutions are sought after, which describe the dynamics to a considerable extent. We start the analysis to develop reduced order model with the generalized governing equation (Navier's equation) for the balance of stresses in  $i$ th direction:

$$\frac{D}{Dt}(\rho u_i) = -(\nabla P)_i + (\nabla \cdot \tau)_i + \rho b_i \quad (1.1)$$

where  $D/Dt$  represents the total derivative of the variable,  $\rho$  is the density of the fluid,  $u_i$  represents the velocity in  $i$ th direction,  $\tau$  is the vector with components  $\tau_{ij}$  representing the stress, and  $\rho b_i$  represents the volumetric body force in  $i$ th direction. For a microchannel of height  $h_0$  and width  $w$ , the axial displacement of the centroid of the capillary meniscus (from the inlet of the microchannel),  $l$ , we integrate Equation 1.1 over the liquid domain