

Advanced
Mathematics
and Mechanics
Applications Using

MATLAB[®]

Third Edition

Howard B. Wilson

University of Alabama

Louis H. Turcotte

Rose-Hulman Institute of Technology

David Halpern

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Library of Congress Cataloging-in-Publication Data

Wilson, H.B.

Advanced mathematics and mechanics applications using MATLAB / Howard B. Wilson, Louis H. Turcotte, David Halpern.—3rd ed.

p. cm.

ISBN 1-58488-262-X

1. MATLAB. 2. Engineering mathematics—Data processing. 3. Mechanics, Applied—Data processing. I. Turcotte, Louis H. II. Halpern, David. III. Title.

TA345 . W55 2002

620'.00151—dc21

2002071267

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International Standard Book Number 1-58488-262-X

Library of Congress Card Number 2002071267

Printed in the United States of America 1 2 3 4 5 6 7 8 9 0

Printed on acid-free paper

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International Standard Book Number 1-58488-262-X

Library of Congress Card Number 2002071267

Printed in the United States of America 1 2 3 4 5 6 7 8 9 0

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For my dear wife, Emma.

Howard B. Wilson

For my loving wife, Evelyn, our departed cat, Patches, and my parents.

Louis H. Turcotte

Preface

This book uses MATLAB[®] to analyze various applications in mathematics and mechanics. The authors hope to encourage engineers and scientists to consider this modern programming environment as an excellent alternative to languages such as FORTRAN or C++. MATLAB¹ embodies an interactive environment with a high level programming language supporting both numerical and graphical commands for two- and three-dimensional data analysis and presentation. The wealth of intrinsic mathematical commands to handle matrix algebra, Fourier series, differential equations, and complex-valued functions makes simple calculator operations of many tasks previously requiring subroutine libraries with cumbersome argument lists.

We analyze problems, drawn from our teaching and research interests, emphasizing linear and nonlinear differential equation methods. Linear partial differential equations and linear matrix differential equations are analyzed using eigenfunctions and series solutions. Several types of physical problems are considered. Among these are heat conduction, harmonic response of strings, membranes, beams, and trusses, geometrical properties of areas and volumes, flexure and buckling of indeterminate beams, elastostatic stress analysis, and multi-dimensional optimization.

Numerical integration of matrix differential equations is used in several examples illustrating the utility of such methods as well as essential aspects of numerical approximation. Attention is restricted to the Runge-Kutta method which is adequate to handle most situations. Space limitation led us to omit some interesting MATLAB features concerning predictor-corrector methods, stiff systems, and event locations.

This book is not an introductory numerical analysis text. It is most useful as a reference or a supplementary text in computationally oriented courses emphasizing applications. The authors have previously solved many of the examples in FORTRAN. Our MATLAB solutions consume over three hundred pages (over twelve thousand lines). Although few books published recently present this much code, comparable FORTRAN versions would probably be significantly longer. In fact, the conciseness of MATLAB was a primary motivation for writing the book.

The programs contain many comments and are intended for study as separate entities without an additional reference. Consequently, some deliberate redundancy

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exists between program comments and text discussions. We also list programs in a style we feel will be helpful to most readers. The source listings show line numbers adjacent to the MATLAB code. MATLAB code does not use line numbers or permit *goto* statements. We have numbered the lines to aid discussions of particular program segments. To conserve space, we often place multiple MATLAB statements on the same line when this does not interrupt the logical flow.

All of the programs presented are designed to operate under the 6.x version of MATLAB and Microsoft Windows. Both the text and graphics windows should be simultaneously visible. A windowed environment is essential for using capabilities like animation and interactive manipulation of three dimensional figures. The source code for all of the programs in the book is available from the CRC Press website at <http://www.crcpress.com>. The program collection is organized using an independent subdirectory for each of the thirteen chapters.

This third edition incorporates much new material on time dependent solutions of linear partial differential equations. Animation is used whenever seeing the solution evolve in time is helpful. Animation illustrates quite well phenomena like wave propagation in strings and membranes. The interactive zoom and rotation features in MATLAB are also valuable tools for interpreting graphical output.

Most programs in the book are academic examples, but some problem solutions are useful as stand-alone analysis tools. Examples include geometrical property calculation, differentiation or integration of splines, Gauss integration of arbitrary order, and frequency analysis of trusses and membranes.

A chapter on eigenvalue problems presents applications in stress analysis, elastic stability, and linear system dynamics. A chapter on analytic functions shows the efficiency of MATLAB for applying complex valued functions and the Fast Fourier Transform (FFT) to harmonic and biharmonic functions. Finally, the book concludes with a chapter applying multidimensional search to several nonlinear programming problems.

We emphasize that this book is primarily for those concerned with physical applications. A thorough grasp of Euclidean geometry, Newtonian mechanics, and some mathematics beyond calculus is essential to understand most of the topics. Finally, the authors enjoy interacting with students, teachers, and researchers applying advanced mathematics to real world problems. The availability of economical computer hardware and the friendly software interface in MATLAB makes computing increasingly attractive to the entire technical community. If we manage to cultivate interest in MATLAB among engineers who only spend part of their time using computers, our primary goal will have been achieved.

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