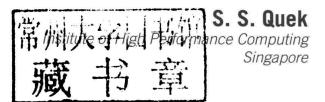


The Finite Element Method

A Practical Course

Second Edition

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Butterworth-Heinemann is an imprint of Elsevier The Boulevard, Langford Lane, Kidlington, Oxford, OX51GB, UK 225 Wyman Street, Waltham, MA 02451, USA

First edition 2013

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British Library Cataloguing in Publication Data

A catalog record for this book is available from the British Library.

Library of Congress Cataloging-in-Publication Data

A catalog record for this book is available from the Library of Congress.

ISBN: 978-0-08-098356-1

For information on all Butterworth-Heinemann publications visit our website at books.elsevier.com

Printed and bound by CPI Group (UK) Ltd, Croydon, CR0 4YY 13 14 15 16 17 10 9 8 7 6 5 4 3 2 1



To Zuona, Yun, Kun, Run, and my extended family for their love and support.

To all my fellow students for their company in studying this subject

G. R. Liu

To my wife, Lingzhi, and my children, Matthew and Michelle for their love and support.

To my mentor, Dr. Liu for his guidance

S. S. Quek

Biography

G.R. Liu

Dr Liu received his PhD from Tohoku University, Japan, in 1991. He was a Postdoctoral Fellow at Northwestern University, USA. He was a Professor at the Department of Mechanical Engineering, National University of Singapore. He founded the Centre for Advanced Computations in Engineering Science (ACES), National University of Singapore, and served as the Director of the ACES from 1998 to 2010. He founded the Association for Computational Mechanics (Singapore) (SACM) and served as the President of the SACM from 2002 to 2010. He is an executive council member of the International Association for Computational Mechanics, and the President of the Asia-



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S.S. Quek

Dr Quek received his PhD from the National University of Singapore. He joined the Institute of High Performance Computing (IHPC) under the Agency for Science, Technology, and Research (A*STAR), Singapore, and participated in the prestigious Visiting Investigator Program led by Professor David J. Srolovitz (currently at the University of Pennsylvania, USA). Through this program, Dr Quek established himself as a competent researcher in the field of computational mechanics and, in particular, computational modeling of dislocation plasticity. Dr Quek is currently a scientist at IHPC and also an Adjunct Assistant Professor at the Department of Mechanical Engineering,



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National University of Singapore. He has authored several technical publications including a book chapter and prestigious international journal papers. His other research interests include microstructure effects on the mechanics of materials, thin film mechanics, dislocation dynamics, nano-scale mechanics, and microstructure evolution.

Preface to the First Edition

For the past half a century, the Finite Element Method (FEM) has been developed into an indispensable technology in the modeling and simulation of engineering systems. In the development of an advanced engineering system, engineers have to go through a very rigorous process of modeling, simulation, visualization, analysis, designing, prototyping, testing, and finally, fabrication/construction. As such, techniques related to modeling and simulation play an increasingly important role in building advanced engineering systems quickly and effectively, and therefore the applications of the FEM have multiplied rapidly.

This book provides unified and detailed course material on the FEM for engineers and university students to solve linear problems in mechanical and civil engineering, with the main foci on structural mechanics and heat transfer. The aim of the book is to provide the necessary concepts, theories, and techniques of the FEM for readers, such that, at the end of this course, they are able to understand and use a commercial FEM package comfortably. Important fundamental and classical theories are introduced in a straightforward and easy to understand way. Modern, state-of-the-art treatment of engineering problems in designing and analysing structural and thermal systems, including microstructural systems, are also discussed. Useful key techniques in FEMs are described in depth, and case studies are provided to demonstrate the theory, methodology, techniques, and practical applications of the FEM (through the introduction of data input files of a typical FEM software package). Equipped with the concepts, theories, and modeling techniques described in this book, readers should be able to use a commercial FEM software package for solving engineering structural problems in a professional manner.

The general philosophy governing the book is to make all the topics insightful but simple, informative but concise, theoretical but applicable.

The book unifies topics on mechanics for solids and structures, energy principles, weighted residual approach, the FEM, and techniques of modeling and computation, as well as the use of commercial software packages. The FEM was originally invented for solving mechanics problems in solids and structures. It is thus appropriate to learn the FEM via problems involving the mechanics of solids. Mechanics for solid structures comprises a vast subject by itself, which needs volumes of books to describe it thoroughly. This book will devote one chapter to briefly cover the mechanics of solids and structures by presenting the important basic principles. It focuses on the derivation of key governing equations for 3D solids. Drawings are used to illustrate all the field variables in solids, as well as the relationships between them. Equations for various types of solids and structure components, such as 2D solids, trusses, beams, and plates, are then deduced from the general equations for 3D solids. It has been found from our teaching practices that this method of delivering the basics of the mechanics of solid structures is very effective. The introduction of the general 3D equations before examining the other structural components actually gives students a firm fundamental background, from which the other equations can be easily derived and naturally understood. Understanding is then enforced by studying the examples and case studies that are solved using the FEM in other chapters. Our practice of teaching in the past few years has shown that most students managed to understand the fundamental basics of mechanics without too much difficulty, and many of them do not even possess an engineering background.

We have also observed that, over the past few years of handling industrial projects, many engineers are asked to use commercial FEM software packages to simulate engineering systems. Many do not have proper knowledge of the FEM, and are willing to learn via self-study. They thus need a book that describes the FEM in their language, and not with the use of overly technical symbols and terminology.

Without such a book, many would end up using the software packages blindly like a "black box." This book therefore aims to throw light into the "black box" so that users can see clearly what is going on inside by relating things that are done in the software with the theoretical concepts in the FEM. Detailed description and references are provided in case studies to show how the FEM's formulation and techniques are implemented in the software package.

We believe that being informative need not necessarily mean being exhaustive. There are a large number of techniques developed during the last five decades in the area of FEM, and it is not practically possible to be exhaustive. Our past experience has found that very few of the vast number of FEM techniques are still frequently used. This book does not want to be an encyclopedia (as that is already available), but intends to be informative enough for the really useful modeling techniques that are and will be alive for years to come. Useful techniques are also often very interesting, and by describing the key features of these lively techniques, this book is written to instill an appreciation of them for solving practical problems. It is with this appreciation that we hope readers will be enticed even more to the FEM.

Theories can be well accepted and appreciated if their applications can be demonstrated explicitly. The case studies used in the book also serve the purpose of demonstrating the finite element theories. They include a number of applications of the FEM for the modeling and simulation of microstructures and microsystems. Most of the case studies are idealized practical problems to clearly bring forward the concepts of the FEM, and will be presented in a manner that makes it easier for readers to follow. Following through these case studies, ideally in front of a workstation, helps the reader to easily understand the important concepts, procedures, and theories.

A picture tells a thousand words. Numerous drawings and charts are used to describe important concepts and theories. This is very important and will definitely be welcomed by readers, especially those from non-engineering backgrounds.

The book provides practical techniques for using commercial software packages, (e.g., ABAQUS and ANSYS). The case studies and examples calculated using one software package could easily be repeated using any other commercial software package. Commonly encountered problems in modeling and simulation using commercial software packages are discussed, and rules-of-thumb and guidelines are also provided to solve these problems effectively in professional ways.

Note that the focus of this book is on developing a good understanding of the fundamentals and principles of linear FE analysis. We have chosen ABAQUS for most of the examples as it can easily handle linear analyses. However, with further reading, readers could also extend the use of ABAQUS for projects involving non-linear FE analyses too. A chapter-by-chapter description of the book is given below.

Chapter 1: Highlights the role and importance of the FEM in computational modeling and simulation required in the design process for engineering systems. The general aspects of computational modeling and simulation of physical problems in engineering are discussed. Procedures for the establishment of mathematical and computational models of physical problems are outlined. Issues related to geometrical simplification, domain discretization, numerical computation, and visualization that are required in using the FEM are discussed.

Chapter 2: Describes the basics of mechanics for solids and structures. Important field variables of solid mechanics are introduced, and the key dynamic equations of these variables are derived. Mechanics for 2D and 3D solids, trusses, beams, frames, and plates are covered in a concise and easy to understand manner. Readers with a mechanics background may skip this chapter.

Chapter 3: Introduces the general finite element procedure. Concepts of strong and weak forms of a system equations and the construction of shape functions for interpolation of field variables are described. The properties of the shape functions are also discussed with an emphasis on the sufficient

requirement of shape functions for establishing finite element equations. Hamilton's principle is introduced and applied to establish the general forms of the finite element equations. Methods to solve the finite element equation are discussed for static, eigenvalue analysis, as well as transient analysis.

Chapter 4: Details the procedure used to obtain finite element matrices for truss structures. The procedures to obtain shape functions, the strain matrix, local and global coordinate systems, and the assembly of global finite element system equations are described. Very straightforward examples are used to demonstrate a complete and detailed finite element procedure to compute displacements and stresses in truss structures. The reproduction of features and the convergence of the FEM as a reliable numerical tool are revealed through these examples.

Chapter 5: Deals with finite element matrices for beam structures. The procedures carried out to obtain shape functions and the strain matrix are described. Elements for thin beam elements are developed. Examples are presented to demonstrate application of the finite element procedure in a beam microstructure, using both ABAQUS and ANSYS.

Chapter 6: Shows the procedure for formulating the finite element matrices for frame structures, by combining the matrices for truss and beam elements. Details on obtaining the transformation matrix and the transformation of matrices between the local and global coordinate systems are described. An example is given to demonstrate the use of frame elements to solve practical engineering problems.

Chapter 7: Formulates the finite element matrices for 2D solids. Matrices for linear triangular elements, bilinear rectangular and quadrilateral elements are derived in detail. Area and natural coordinates are also introduced in the process. Iso-parametric formulation and higher order elements are also described. An example of analysing a micro device is used to study the accuracy and convergence of triangular and quadrilateral elements.

Chapter 8: Deals with finite element matrices for plates and shells. Matrices for rectangular plate elements based on the more practical Reissner–Mindlin plate theory are derived in detail. Shell elements are formulated simply by combining the plate elements and 2D solid plane stress elements. Examples of analyzing a micro device using ABAQUS are presented.

Chapter 9: Finite element matrices for 3D solids are developed. Tetrahedron elements and hexahedron elements are formulated in detail. Volume coordinates are introduced in the process. Formulation of higher order elements is also outlined. An example of using 3D elements for modeling a nano-scaled heterostructure system is presented.

Chapter 10: Special purpose elements and recent advanced methods are introduced and briefly discussed. Crack tip elements for use in many fracture mechanics problems are derived. Infinite elements formulated by mapping and a technique of using structure damping to simulate an infinite domain are both introduced. The finite strip method and the strip element method are also discussed. In addition, the meshfree methods and the smoothed finite element methods are discussed.

Chapter 11: Modeling techniques for the stress analyses of solids and structures are discussed. Use of symmetry, multipoint constraints, mesh compatibility, the modeling of offsets, supports, joints, and the imposition of multipoint constraints are all covered. Examples are included to demonstrate use of the modeling techniques.

Chapter 12: The FEM procedure for solving partial differential equations based on the weighted residual methods is presented. In particular, heat transfer problems in 1D and 2D are formulated. Issues in solving heat transfer problems are discussed. Examples are presented to demonstrate the use of ABAQUS for solving heat transfer problems.

Chapter 13: The basics of using commercial packages are discussed. Both ABAQUS and ANSYS are outlined so as to help a beginner to get a head start on using an FEM software package. We use

ABAQUS to introduce a typical FEM data input file, and use ANSYS for the introduction of creating an FEM model using a graphical user interface (GUI). An example is presented to provide a step-by-step outline of the procedure of creating an ABAQUS input file. Important information required by most FEM software packages is highlighted. A similar model is also created, executed, and post-processed using ANSYS with GUI. This chapter is meant for readers doing self-study on using an FEM package and ABAQUS and ANSYS are used to provide a basic idea of how to begin. For university courses, it is most effective to conduct hands-on sessions in computer rooms, together with students, when teaching about commercial software packages.

Most of the materials in the book are selected from lecture notes prepared for classes conducted by the first author since 1995 for both under- and post-graduate students. Those lecture notes were written using materials from many excellent, existing books on the FEM (as the number of references used are too many to be cited explicitly, they are listed in the Reference section of the book, while regrettably noting that some may have been unintentionally left out), and evolved over years of lecturing at the National University of Singapore, the Institute of Mechanics at the Chinese Academy of Science, Hunan University (China), Taiyuan University of Technology (China) and University of Cincinnati (USA). The materials have indeed been presented to readers/audiences of various backgrounds. The authors wish to express their sincere appreciation to the authors of all the existing FEM books, which some of these materials may have been referenced from (they are listed in the Reference section without explicitly citing them). FEM has been well developed and documented in detail in various existing books. In view of this, the authors have tried their best to limit the information in this book to the necessary minimum required to make it useful for those applying FEM in practice. Readers seeking more advanced theoretical material are advised to refer to books such as those by Zienkiewicz and Taylor (2000). The authors would also like to thank the students (both past and current) for their company in the study of the subject of FEM over the years, which has helped to develop this course into what it is today.

Preparing lectures for FEM courses is a very time-consuming task, as many drawings and pictures are required to explain all these theories, concepts, and techniques clearly. A set of colorful PowerPoint slides for the materials in the book has therefore been produced by the authors for lecturers to use. These slides can be found at the following website: www.textbooks.elsevier.com. It is aimed at reducing the amount of time taken in preparing lectures using this textbook. All the slides are grouped according to the chapters. The lecturer has the full freedom to cut and add slides according to the level of the class and the hours available for teaching the subject, or to simply use them as provided. Our suggestions on the use of material for undergraduate and graduate semester courses (typically ~40 lecture hours) are as follows.

Undergraduate Courses	Graduate Courses (FEM entry level)
Chapter 1 (With detailed coverage)	Chapter 1 (Skimming through)
Chapter 2 (With detailed coverage, may skip Section 2.6)	Chapter 2 (Skimming throughdetailed discussion on Section 2.6)
Chapter 3 (Detail with slower pace, may skip the proofs in Section 3.4.4)	Chapter 3 (May skip the proofs in Section 3.4.4)
Chapters 4-7 (Detail with slower pace)	Chapters 4–12
Chapter 9 (Informative)	Chapter 13 (Preferably done in a computer lab with
Chapter 11 (Selective)	hand-on sessions)
Chapter 13 (Preferably done in a computer lab with hand-on sessions)	

It is advisable that an undergraduate FEM course may focus more on the basics for both mechanics and FEM procedure, so that the students can have a solid knowledge for understanding and using the FEM. This is important, especially for undergraduate courses during the lower years. A graduate FEM course may focus on the FEM theory and formulation, so that the students can have a solid foundation for advanced use of FEM and even develop their new techniques for their graduate projects.

Finally, it is the authors' belief that people can rarely become an expert by reading a book. They may become an expert by hard work and persistent creative practices. In addition, the broad subject of the FEM is evolving continuously. This book aims to bring the reader into a 'workshop' with clear introductions to the basic tools there. It is up to the reader to perfect these tools and use them to innovate, in terms of using the FEM, or even to develop novel computational techniques.

G.R. Liu and S.S. Quek

Preface for the Second Edition

Since the first edition of this book was published in 2003, it has been used by many universities all over the world as a textbook and/or reference book. It has also been translated into other languages. We have subsequently received much encouragement, important comments, and suggestions. We would like to take this opportunity to thank all the users and readers for the use of this book and the constructive comments and suggestions that they have provided to us. Based on this feedback, the second edition contains the following major changes:

- 1. Editorial revision has been thoroughly carried out for the entire book.
- 2. Chapter 2 has been enriched for better and easier review on mechanics of solids and structures.
- 3. The number of exercises (review questions) has been significantly increased (more than double).
- 4. Coverage of axial symmetric problems has been added to Chapter 7.
- **5.** Applications of ABAQUS have been updated.
- **6.** Brief descriptions on the use of ANSYS have been added.
- Briefings on advanced meshfree methods and the smoothed finite element methods have been added.

For instructors using this book for a course, ancillary materials, such as PowerPoint lecture slides and a solutions manual (for instructors only), are available by registering at www.textbooks.elsevier. com.

G.R. Liu and S.S. Quek

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