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Invitation to Fixed-Parameter Algorithms

Rolf Niedermeier

Oxford Lecture Series in Mathematics and its Applications
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Invitation to Fixed-Parameter Algorithms
Rolf Niedermeier

This research-level text is an application-oriented introduction to the growing and highly topical area of the development and analysis of efficient fixed-parameter algorithms for optimally solving computationally hard combinatorial problems.

The book is divided into three parts: a broad introduction that provides the general philosophy and motivation; followed by coverage of algorithmic methods developed over the years in fixed-parameter algorithmics forming the core of the book; and a discussion of the essentials from parameterized hardness theory with a focus on W[1]-hardness which parallels NP-hardness, then stating some relations to polynomial-time approximation algorithms, and finishing up with a list of selected case studies to show the wide range of applicability of the presented methodology.

Aimed at graduate and research mathematicians, programmers, algorithm designers, and computer scientists, the book introduces the basic techniques and results and provides a fresh view on this highly innovative field of algorithmic research.

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PREFACE

This book grew out of my *habilitationsschrift* at the University of Tübingen in 2002. Currently, there is only one monograph dealing with the issue of fixed-parameter algorithms: Rod G. Downey and Michael R. Fellows' groundbreaking monograph *Parameterized Complexity* (1999). Since then there have been numerous new results in this field of exactly solving combinatorially hard problems. Moreover, Downey and Fellows' monograph focuses more on structural complexity theory issues than on concrete algorithm design and analysis. By way of contrast, the objective of this book is to focus on the algorithmic side of parameterized complexity, giving a fresh view of this highly innovative field of algorithmic research.

The book is divided into three parts:

1. a broad introduction that provides the general philosophy and motivation;
2. a part on algorithmic methods developed over the years in fixed-parameter algorithmics, forming the core of the book; and
3. a final section discussing the essentials of parameterized hardness theory, focusing first on *W[1]*-hardness, which parallels *NP*-hardness, then stating some relations to polynomial-time approximation algorithms, and finishing up with a list of selected case studies to show the wide range of applicability of the methodology presented.

The book is intended for advanced students in computer science and related fields as well as people generally working with algorithms for discrete problems. It has particular relevance when studying ways to cope with computational intractability as expressed by *NP*-hardness theory.

The reader is recommended to start with Part I, but Parts II and III do not need to be read in the given order. Thus, from Chapter 7 on (with a few exceptions) there are almost no restrictions concerning the chosen order. The material presented can be used to form a course exclusively dedicated to the topic of fixed-parameter algorithms as well as to provide supplementary material for an advanced algorithms class.

We believe that the concept of fixed-parameter tractability is fundamental for the algorithmics of computationally hard discrete problems. Due to the ubiquity of the proposed problem parameterization approach discussed here, fixed-parameter algorithms should be seen as basic knowledge for every algorithm designer. May this book help to spread this news.

ACKNOWLEDGEMENTS

To all who helped, particularly the unnamed ones!

The first hint that I should study parameterized complexity came from Klaus-Jörn Lange. He pointed me to strange things like "towers of complexity" or "parameterization of languages by the slice". Shortly afterwards, during my 1998 stay at Charles University, Prague, Jaroslav Nešetřil showed me a bunch of papers by Rod G. Downey and Michael R. Fellows. This prompted my first steps in researching fixed-parameter algorithms in enjoyable cooperation with Peter Rossmanith.

However, I am most grateful to my (partially former) Ph.D. students who share(d) their ideas and time with me. I list them in alphabetical order: Jochen Alber, Michael Dom, Jiong Guo, Jens Gramm, Falk Hüffner, and Sebastian Wernicke. They helped me a lot in countless ways and without them this book would not exist. In addition, I greatly profited from working with my graduate students Nadja Betzler, Britta Dorn, Frederic Dorn, Erhan Kenar, Hannes Moser, Amalinda Oertel, David Pricking, Daniel Raible, Marion Renner, Christian Rödelsperger, Ramona Schmid, Anke Truss, and Johannes Uhlmann. All of them have been infected with fixed-parameter algorithmics in the broadest sense.

Finally, special thanks go to Hans Bodlaender, Mike Fellows, Henning Fernau, Edward Hirsch, and Ton Kloks for collaborations from which I have profited greatly.

I thank Rod Downey for making the connection with Oxford University Press and the staff at Oxford University Press for a smooth and enjoyable cooperation.

I apologize for omitting further names here—there are too many to name them all and it would be too dangerous to forget one of them.

CONTENTS

Part I

Foundations

A fixed-parameter algorithm is one that provides an *optimal* solution to a discrete combinatorial problem. As a rule, such a problem is *NP*-hard and that is why one must accept exponential running times for fixed-parameter algorithms. The fundamental idea is to restrict the corresponding, seemingly unavoidable, "combinatorial explosion" that causes the exponential growth in the running time of certain problem-specific parameters. It is hoped then that these parameters (in the concrete application behind the problem under consideration) might take only relatively "small" values, so that the exponential growth becomes affordable; that is, the fixed-parameter algorithm *efficiently* solves the given "parameterized problem".

As an example of "parameterization", consider the problem of placing as few queens as possible to attack all the squares on a chessboard. There is a way to place only five (which is optimal) queens on an 8×8 chessboard to do this. Here, a natural parameter is the size k of the solution set we search for, that is, the set of queens to be placed. Hence for 8×8 chessboards $k = 5$. What about general $n \times n$ chessboards? Can we find a minimum solution efficiently?

A "more serious" example is the following. Assume that one wants to establish transmission towers; the towers will be located on inhabited buildings, and each such building must be reachable by at least one transmission tower. In addition, assume that if a tower in location u can reach location v, then also one at v can reach u. Then, given all pairs that can reach each other, how many transmitters are needed to cover all the buildings? Again, a natural parameter to consider is the number of transmitters needed. Thus the task is to find a small number of transmission tower locations such that all buildings can be reached.

Both examples are instantiations of an *NP*-hard graph problem called DOMINATING SET:

> **Input:** An undirected graph $G = (V, E)$ and a nonnegative integer k.
> **Task:** Find a subset of vertices $S \subseteq V$ with k or fewer vertices such that each vertex in V is contained in S or has at least one neighbor in S.

An optimal solution to DOMINATING SET can be found in $O(n^{k+1})$ steps by simply trying all size-k subsets of the vertex set V of size n. According to parameterized complexity theory, there is little hope of doing significantly better than this. Fortunately, however, for restricted classes of graphs we can do better. For instance, for *planar* graphs (that is, graphs that can be drawn in the plane without edge crossings) DOMINATING SET can be solved in $O(8^k \cdot n)$ time. Note that DOMINATING SET remains *NP*-hard when restricted to planar graphs. Another algorithm even finds a solution in $O(c^{\sqrt{k}} \cdot n)$ time for some (larger) constant c. This is what we understand by fixed-parameter algorithms—the superpolynomial factor in the running time depends exclusively on the parameter k. Finally, again in case of planar graphs, there are simple data reduction rules that—in polynomial time—can shrink an original input graph with n vertices into a new one with only $O(k)$ vertices such that the search for an optimal solution can be done within the size $O(k)$ instance. All these results lead to the fundamental conclusion that the combinatorial explosion can be confined to the parameter k only, the central goal to be achieved by fixed-parameter algorithms. Generally speaking, a fixed-parameter algorithm solves a problem with an input instance of size n and a parameter k in

$$f(k) \cdot n^{O(1)}$$

time for some computable function f depending solely on k. That is, for every fixed parameter value it yields a solution in polynomial time and the degree of the polynomial is independent from k.

Fixed-parameter algorithms have been scattered around the literature for decades. As a method of algorithm design and analysis, parameterized complexity was systematized by Rod G. Downey and Michael R. Fellows and some of their co-authors during the 1990s. In particular, they developed a theory of parameterized computational complexity, which is a strong mathematical tool for guiding fixed-parameter algorithm design. In this book, we make use of parameterized computational complexity theory to the extent that is necessary to learn about the design and analysis of algorithms. More structural complexity-theoretic aspects are neglected in this work. Fixed-parameter algorithms are introduced as a valuable alternative to complement other algorithmic approaches for attacking hard combinatorial problems, such as approximation or heuristic algorithms.

Fixed-parameter algorithms adhere to a very natural concept when trying to solve hard combinatorial problems. In the following we give a concise description of the very basic ideas and objectives behind this work and parameterized complexity analysis. The focus of Part I is on encouraging the reader to adopt a parameterized view of the study of computationally hard problems. Besides simple motivating examples and the presentation of the elementary concepts needed throughout the book, the breadth of the parameterized complexity approach is illustrated by means of an extensive discussion of the *NP*-complete graph problem VERTEX COVER. Having dealt with this perhaps most popular parameterized problem, we finally move on and finish with a general discussion on the "art" of parameterizing problems.