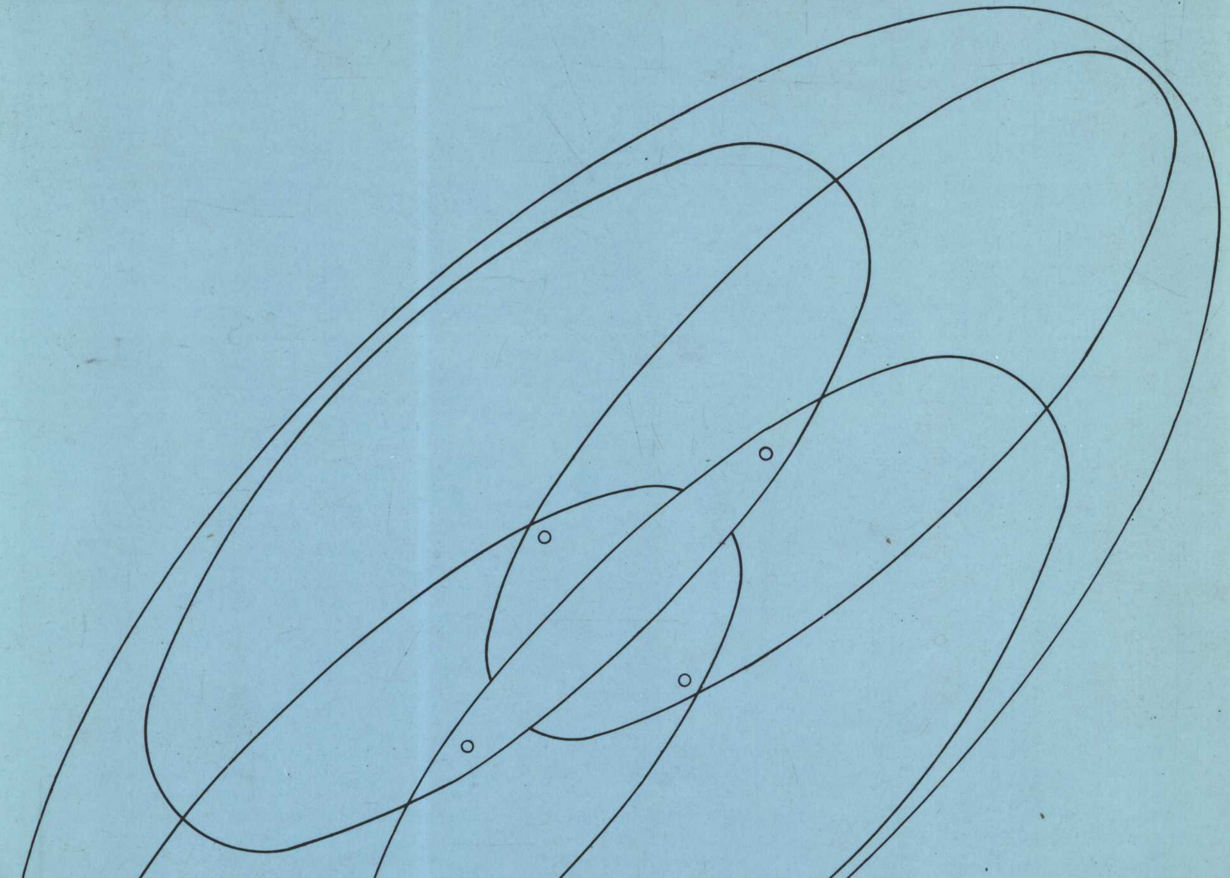
AN INTRODUCTION TO MECHANICS

KLEPPNER
KOLENKOW



INTERNATIONAL STUDENT EDITION

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USEFUL PHYSICAL AND ASTRONOMICAL DATA

Gravitational constant, G	6.67×10^{-11} N·m^2/kg^2
Acceleration of gravity at earth's surface, g	$\approx \begin{cases} 9.8 \text{ m/s}^2 \\ 32 \text{ ft/s}^2 \end{cases}$
Speed of light, c	3.00×10^8 m/s^1
Mass of electron	9.11×10^{-31} kg
Mass of proton	1.67×10^{-27} kg
Charge of electron, e	1.60×10^{-19} C
Planck's constant, h	6.63×10^{-34} J·s
Mass of earth	5.98×10^{24} kg
Mean radius of earth	6.37×10^6 m
Radius of earth's orbit	1.49×10^{11} m
Period of earth's rotation	8.64×10^4 s
Period of earth's revolution	3.16×10^7 s
Mass of moon	7.34×10^{22} kg
Mean radius of moon	1.74×10^6 m
Radius of moon's orbit	3.84×10^8 m
Mass of sun	1.99×10^{30} kg

THE GREEK ALPHABET

α	A	alpha	ν	N	nu
β	B	beta	ξ	Ξ	xi
γ	Γ	gamma	ο	O	omicron
δ	Δ	delta	π	Π	pi
ϵ	E	epsilon	ρ	P	rho
ζ	Z	zeta	σ	Σ	sigma
i	I	iota	τ	T	tau
η	H	eta	υ	ϒ	upsilon
θ	Θ	theta	ϕ	Φ	phi
κ	K	kappa	χ	X	chi
λ	Λ	lambda	ψ	Ψ	psi
μ	M	mu	ω	Ω	omega

UNITS AND CONVERSION FACTORS

	SI	Cgs	ENGLISH
Time	second (s)	second	second
Length	meter (m)	centimeter (cm) 1 cm = 10^{-2} m	foot (ft) 1 ft = 0.305 m 1 in = 2.54 cm 1 mi = 5280 ft 1 mi = 1.61 km
Mass	kilogram (kg)	gram (g) 1 g = 10^{-3} kg	slug 1 slug = 14.6 kg 1 lb_m = 0.454 kg
Force	newton (N)	dyne (dyn) 1 dyn = 10^{-5} N	pound 1 lb = 4.45 N
Energy	joule (J)	erg 1 erg = 10^{-7} J	foot · pound (ft · lb) 1 ft · lb = 1.36 J
Power	watt (W)	erg/second 1 erg/s = 10^{-7} W	foot pound/second 1 ft·lb/s = 1.36 W 1 horsepower (hp) = 550 ft·lb/s 1 hp = 746 W

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1 Angstrom (Å) = 10^{-10} m
1 atomic mass unit (amu) = 1.66×10^{-27} kg
= 931 MeV
1 electronvolt (eV) = 1.60×10^{-19} J
1 mi/h = 0.447 m/s
1 lb/in^2 = 6.90×10^3 N/m^2

PREFIXES FOR POWERS OF TEN

10^{12}	tera	T
10^{9}	giga	G
10^{6}	mega	M
10^{3}	kilo	k
10^{-3}	milli	m
10^{-6}	micro	μ
10^{-9}	nano	n
10^{-12}	pico	p
10^{-15}	femto	f

AN
INTRODUCTION
TO
MECHANICS

To our parents

Beatrice and Otto
Katherine and John

PREFACE

There is good reason for the tradition that students of science and engineering start college physics with the study of mechanics: mechanics is the cornerstone of pure and applied science. The concept of energy, for example, is essential for the study of the evolution of the universe, the properties of elementary particles, and the mechanisms of biochemical reactions. The concept of energy is also essential to the design of a cardiac pacemaker and to the analysis of the limits of growth of industrial society. However, there are difficulties in presenting an introductory course in mechanics which is both exciting and intellectually rewarding. Mechanics is a mature science and a satisfying discussion of its principles is easily lost in a superficial treatment. At the other extreme, attempts to "enrich" the subject by emphasizing advanced topics can produce a false sophistication which emphasizes technique rather than understanding.

This text was developed from a first-year course which we taught for a number of years at the Massachusetts Institute of Technology and, earlier, at Harvard University. We have tried to present mechanics in an engaging form which offers a strong base for future work in pure and applied science. Our approach departs from tradition more in depth and style than in the choice of topics; nevertheless, it reflects a view of mechanics held by twentieth-century physicists.

Our book is written primarily for students who come to the course knowing some calculus, enough to differentiate and integrate simple functions.[1] It has also been used successfully in courses requiring only concurrent registration in calculus. (For a course of this nature, Chapter 1 should be treated as a resource chapter, deferring the detailed discussion of vector kinematics for a time. Other suggestions are listed in To The Teacher.) Our experience has been that the principal source of difficulty for most students is in learning how to apply mathematics to physical problems, not with mathematical techniques as such. The elements of calculus can be mastered relatively easily, but the development of problem-solving ability requires careful guidance. We have provided numerous worked examples throughout the text to help supply this guidance. Some of the examples, particularly in the early chapters, are essentially pedagogical. Many examples, however, illustrate principles and techniques by application to problems of real physical interest.

The first chapter is a mathematical introduction, chiefly on vectors and kinematics. The concept of rate of change of a vector,

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[1] The background provided in "Quick Calculus" by Daniel Kleppner and Norman Ramsey, John Wiley & Sons, New York, 1965, is adequate.

probably the most difficult mathematical concept in the text, plays an important role throughout mechanics. Consequently, this topic is developed with care, both analytically and geometrically. The geometrical approach, in particular, later proves to be invaluable for visualizing the dynamics of angular momentum.

Chapter 2 discusses inertial systems, Newton's laws, and some common forces. Much of the discussion centers on applying Newton's laws, since analyzing even simple problems according to general principles can be a challenging task at first. Visualizing a complex system in terms of its essentials, selecting suitable inertial coordinates, and distinguishing between forces and accelerations are all acquired skills. The numerous illustrative examples in the text have been carefully chosen to help develop these skills.

Momentum and energy are developed in the following two chapters. Chapter 3, on momentum, applies Newton's laws to extended systems. Students frequently become confused when they try to apply momentum considerations to rockets and other systems involving flow of mass. Our approach is to apply a differential method to a system defined so that no mass crosses its boundary during the chosen time interval. This ensures that no contribution to the total momentum is overlooked. The chapter concludes with a discussion of momentum flux. Chapter 4, on energy, develops the work-energy theorem and its application to conservative and nonconservative forces. The conservation laws for momentum and energy are illustrated by a discussion of collision problems.

Chapter 5 deals with some mathematical aspects of conservative forces and potential energy; this material is not needed elsewhere in the text, but it will be of interest to students who want a mathematically complete treatment of the subject.

Students usually find it difficult to grasp the properties of angular momentum and rigid body motion, partly because rotational motion lies so far from their experience that they cannot rely on intuition. As a result, introductory texts often slight these topics, despite their importance. We have found that rotational motion can be made understandable by emphasizing physical reasoning rather than mathematical formalism, by appealing to geometric arguments, and by providing numerous worked examples. In Chapter 6 angular momentum is introduced, and the dynamics of fixed axis rotation is treated. Chapter 7 develops the important features of rigid body motion by applying vector arguments to systems dominated by spin angular momentum. An elementary treatment of general rigid body motion is presented in the last sections of Chapter 7 to show how Euler's equations can be developed from simple physical arguments. This more advanced material is optional however; we do not usually treat it in our own course.

Chapter 8, on noninertial coordinate systems, completes the development of the principles of newtonian mechanics. Up to this point in the text, inertial systems have been used exclusively in order to avoid confusion between forces and accelerations. Our discussion of noninertial systems emphasizes their value as computational tools and their implications for the foundations of mechanics.

Chapters 9 and 10 treat central force motion and the harmonic oscillator, respectively. Although no new physical concepts are involved, these chapters illustrate the application of the principles of mechanics to topics of general interest and importance in physics. Much of the algebraic complexity of the harmonic oscillator is avoided by focusing the discussion on energy, and by using simple approximations.

Chapters 11 through 14 present a discussion of the principles of special relativity and some of its applications. We attempt to emphasize the harmony between relativistic and classical thought, believing, for example, that it is more valuable to show how the classical conservation laws are unified in relativity than to dwell at length on the so-called "paradoxes." Our treatment is concise and minimizes algebraic complexities. Chapter 14 shows how ideas of symmetry play a fundamental role in the formulation of relativity. Although we have kept the beginning students in mind, the concepts here are more subtle than in the previous chapters. Chapter 14 can be omitted if desired; but by illustrating how symmetry bears on the principles of mechanics, it offers an exciting mode of thought and a powerful new tool.

Physics cannot be learned passively; there is absolutely no substitute for tackling challenging problems. Here is where students gain the sense of satisfaction and involvement produced by a genuine understanding of the principles of physics. The collection of problems in this book was developed over many years of classroom use. A few problems are straightforward and intended for drill; most emphasize basic principles and require serious thought and effort. We have tried to choose problems which make this effort worthwhile in the spirit of Piet Hein's aphorism

Problems worthy
  of attack
prove their worth
  by hitting back[1]

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[1] From *Grooks I*, by Piet Hein, copyrighted 1966, The M.I.T. Press.

It gives us pleasure to acknowledge the many contributions to this book from our colleagues and from our students. In particular, we thank Professors George B. Benedek and David E. Pritchard for a number of examples and problems. We should also like to thank Lynne Rieck and Mary Pat Fitzgerald for their cheerful fortitude in typing the manuscript.

Daniel Kleppner
Robert J. Kolenkow

TO THE TEACHER

The first eight chapters form a comprehensive introduction to classical mechanics and constitute the heart of a one-semester course. In a 12-week semester, we have generally covered the first 8 chapters and parts of Chapters 9 or 10. However, Chapter 5 and some of the advanced topics in Chapters 7 and 8 are usually omitted, although some students pursue them independently.

Chapters 11, 12, and 13 present a complete introduction to special relativity. Chapter 14, on transformation theory and four-vectors, provides deeper insight into the subject for interested students. We have used the chapters on relativity in a three-week short course and also as part of the second-term course in electricity and magnetism.

The problems at the end of each chapter are generally graded in difficulty. They are also cumulative; concepts and techniques from earlier chapters are repeatedly called upon in later sections of the book. The hope is that by the end of the course the student will have developed a good intuition for tackling new problems, that he will be able to make an intelligent estimate, for instance, about whether to start from the momentum approach or from the energy approach, and that he will know how to set off on a new tack if his first approach is unsuccessful. Many students report a deep sense of satisfaction from acquiring these skills.

Many of the problems require a symbolic rather than a numerical solution. This is not meant to minimize the importance of numerical work but to reinforce the habit of analyzing problems symbolically. Answers are given to some problems; in others, a numerical "answer clue" is provided to allow the student to check his symbolic result. Some of the problems are challenging and require serious thought and discussion. Since too many such problems at once can result in frustration, each assignment should have a mix of easier and harder problems.

Chapter 1 Although we would prefer to start a course in mechanics by discussing physics rather than mathematics, there are real advantages to devoting the first few lectures to the mathematics of motion. The concepts of kinematics are straightforward for the most part, and it is helpful to have them clearly in hand before tackling the much subtler problems presented by newtonian dynamics in Chapter 2. A departure from tradition in this chapter is the discussion of kinematics using polar coordinates. Many students find this topic troublesome at first, requiring serious effort. However, we feel that the effort will be amply rewarded. In the first place, by being able to use polar coordinates freely, the kinematics of rotational motion are much easier to understand;

the mystery of radial acceleration disappears. More important, this topic gives valuable insights into the nature of a time-varying vector, insights which not only simplify the dynamics of particle motion in Chapter 2 but which are invaluable to the discussion of momentum flux in Chapter 3, angular momentum in Chapters 6 and 7, and the use of noninertial coordinates in Chapter 8. Thus, the effort put into understanding the nature of time-varying vectors in Chapter 1 pays important dividends throughout the course.

If the course is intended for students who are concurrently beginning their study of calculus, we recommend that parts of Chapter 1 be deferred. Chapter 2 can be started after having covered only the first six sections of Chapter 1. Starting with Example 2.5, the kinematics of rotational motion are needed; at this point the ideas presented in Section 1.9 should be introduced. Section 1.7, on the integration of vectors, can be postponed until the class has become familiar with integrals. Occasional examples and problems involving integration will have to be omitted until that time. Section 1.8, on the geometric interpretation of vector differentiation, is essential preparation for Chapters 6 and 7 but need not be discussed earlier.

Chapter 2 The material in Chapter 2 often represents the student's first serious attempt to apply abstract principles to concrete situations. Newton's laws of motion are not self-evident; most people unconsciously follow aristotelian thought. We find that after an initial period of uncertainty, students become accustomed to analyzing problems according to principles rather than vague intuition. A common source of difficulty at first is to confuse force and acceleration. We therefore emphasize the use of inertial systems and recommend strongly that noninertial coordinate systems be reserved until Chapter 8, where their correct use is discussed. In particular, the use of centrifugal force in the early chapters can lead to endless confusion between inertial and noninertial systems and, in any case, it is not adequate for the analysis of motion in rotating coordinate systems.

Chapters 3 and 4 There are many different ways to derive the rocket equations. However, rocket problems are not the only ones in which there is a mass flow, so that it is important to adopt a method which is easily generalized. It is also desirable that the method be in harmony with the laws of conservation of momentum or, to put it more crudely, that there is no swindle involved. The differential approach used in Section 3.5 was developed to meet these requirements. The approach may not be elegant, but it is straightforward and quite general.

In Chapter 4, we attempt to emphasize the general nature of the work-energy theorem and the difference between conservative and nonconservative forces. Although the line integral is introduced and explained, only simple line integrals need to be evaluated, and general computational techniques should not be given undue attention.

Chapter 5 This chapter completes the discussion of energy and provides a useful introduction to potential theory and vector calculus. However, it is relatively advanced and will appeal only to students with an appetite for mathematics. The results are not needed elsewhere in the text, and we recommend leaving this chapter for optional use, or as a special topic.

Chapters 6 and 7 Most students find that angular momentum is the most difficult physical concept in elementary mechanics. The major conceptual hurdle is visualizing the vector properties of angular momentum. We therefore emphasize the vector nature of angular momentum repeatedly throughout these chapters. In particular, many features of rigid body motion can be understood intuitively by relying on the understanding of time-varying vectors developed in earlier chapters. It is more profitable to emphasize the qualitative features of rigid body motion than formal aspects such as the tensor of inertia. If desired, these qualitative arguments can be pressed quite far, as in the analysis of gyroscopic nutation in Note 7.2. The elementary discussion of Euler's equations in Section 7.7 is intended as optional reading only. Although Chapters 6 and 7 require hard work, many students develop a physical insight into angular momentum and rigid body motion which is seldom gained at the introductory level and which is often obscured by mathematics in advanced courses.

Chapter 8 The subject of noninertial systems offers a natural springboard to such speculative and interesting topics as transformation theory and the principle of equivalence. From a more practical point of view, the use of noninertial systems is an important technique for solving many physical problems.

Chapters 9 and 10 In these chapters the principles developed earlier are applied to two important problems, central force motion and the harmonic oscillator. Although both topics are generally treated rather formally, we have tried to simplify the mathematical development. The discussion of central force motion relies heavily on the conservation laws and on energy diagrams. The treatment of the harmonic oscillator sidesteps much of the usual algebraic complexity by focusing on the lightly damped oscillator. Applications and examples play an important role in both chapters.

Chapters 11 to 14 Special relativity offers an exciting change of pace to a course in mechanics. Our approach attempts to emphasize the connection of relativity with classical thought. We have used the Michelson-Morley experiment to motivate the discussion. Although the prominence of this experiment in Einstein's thought has been much exaggerated, this approach has the advantage of grounding the discussion on a real experiment.

We have tried to focus on the ideas of events and their transformations without emphasizing computational aids such as diagrammatic methods. This approach allows us to deemphasize many of the so-called paradoxes.

For many students, the real mystery of relativity lies not in the postulates or transformation laws but in why transformation principles should suddenly become the fundamental concept for generating new physical laws. This touches on the deepest and most provocative aspects of Einstein's thought. Chapter 14, on four-vectors, provides an introduction to transformation theory which unifies and summarizes the preceding development. The chapter is intended to be optional.

Daniel Kleppner
Robert J. Kolenkow

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