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Strength of Materials

2nd Revised Edition

Surendra Singh

Strength of Materials

(For the Engineering Students of Degree, Diploma and AMIE Classes)

[In MKS Units]

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Second Revised Edition

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STRENGTH OF MATERIALS

By the same author

ENGINEERING MATERIALS
(2nd edition)

To

my dear mother

PERFACE TO THE SECOND EDITION

Subsidy from the National Book Trust for this second revised edition has enabled the book to be priced low enough to be within the easy reach of all students, including those votaries of *Saraswati* who have to brave the wrath of *Laxmi.*

Academic fraternity's welcome to the book is enthusing and I look back with satisfaction at the fulfilment of my hopes and expectations along with the needs of teachers and taughts.

Apart from tinkering here and other, a chapter on *Welded Joints* has been added and solved problems rearranged. Misprints that came to my notice have been eliminated.

Surely the student community shall find the revised edition more useful.

Suggestions for the improvement of the book shall be gratefully accepted.

SURENDRA SINGH

PREFACE TO THE FIRST EDITION

With hope and expectations I feel pleasure in placing this treatise in the hands of my readers—the students. I shall be guilty of misstatement if I were to claim any originality of matter. It is the same old stuff, only the presentation is mine. Having had a long association with students of various shades I had the occasion to know of what ails them—particularly those striving to learn the subject in preparing for competitive tests and examinations of professional bodies like AMIE (India) etc. where they lack the guidance of a teacher. To attain this, attempt has been made to avoid complicated procedures of solution even if it made the solution a little longer, cross references are frequently given and hints by way of foot-notes have been copiously provided for. Abundant use of diagrams has been made to explain and clarify knotty points.

A large number of well graded examination questions from various Universities and from other professional and competitive examinations have been solved. All important formulae used or derived in a particular chapter have been summarised at the end of that chapter to enable the students to refurbish their memories just before examination.

Syllabii for degree examinations in Engineering of various Universities and of AMIE (India) guided me in deciding the course contents of the text. May be, because of variations from university to university some aspect is still left wanting for a section of my readers. If brought to my notice this deficiency shall be made good at the time of revising the text.

I owe my gratitude to authors of numerous standard books on the subject, that I consulted; my friends whose opinions I sought and various universities and examining bodies for their examination questions that I used. Above all, it is the student community that deserves to be thanked for educating me about their difficulties and problems from which germinated the idea of my attempting this venture.

Sadhna Marg is undulating and littered with thorns of human treachery, ingratitude, deceit and cunningness which came my way in plenty and as such the first temptation was to dedicate the work to "Human Treachery." Defeatist as I am not, I spotted a bright streak of light in the vast dark sky, the light of innocent and selfless love—the pure love of a mother, which I received in abundance and I thus decided to dedicate this work to her.

How far my efforts have been successful is for my readers—the students—to judge. Even though all efforts have been made to avoid mistakes yet it would be difficult to claim perfection. Suggestions for improvement shall be gratefully acknowledged.

SURENDRA SINGH

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STRESSES AND STRAINS

1.1. GENERAL

External forces applied to a body have the tendency to deform the body which develops an internal resistance against the deforming forces. This resistance increases with the increase in deforming forces but only up to a certain limit, beyond which the deforming forces will cause the failure of that body. The ultimate internal resistance, to the external forces, offered by a body depends upon the type of deformation taking place and the nature of material of which the body is made.

In **strength of materials** the internal effects produced and the deformations of bodies caused by externally applied forces are studied. Whereas in **Engineering mechanics** the study is confined to relations between externally applied forces on rigid bodies, either at rest or in motion.

1.2. STRESS

External forces acting on a rigid body are termed as **loads.** All externally applied loads deform an elastic material. As the material undergoes deformation it sets up internal resistance to the deforming forces. The quantum of internal resisting forces correspondingly increases with the increase in externally applied loads only up to a certain limit beyond which any increase in applied loads will continue the process of deformation to the stage of failure. The deformation is known as **strain** and the resisting forces are called **stresses.** Stress per unit area is termed as **unit stress** and the total internal force within a single member is generally called **total stress.** Usually, however, the word "stress" is used alone in place of unit stress. Since within elastic limit (Art. 1.4) the resistance offered by a body is the same as the load applied so the **unit stress may be defined as load per unit area** and be mathematically expressed as

$$p=\frac{P}{A} \qquad \ldots (1.1)$$

where Stress intensity $=p$

Load applied $=P$

Area of *X*-section of the loaded section $=A$.

If the load or total force P be expressed in kg and the area A in cm^2 then the unit of stress shall be kg/cm^2.

There are only two basic stresses which are:

(*i*) The **normal stresses** which act normal to the stressed surfaces under consideration. These are either *tensile* (Art. 1.2.1) or *compressive* (Art. 1.2.2) stresses.

(*ii*) The **shearing stresses** which act parallel to the stressed surfaces under consideration (Art. 1.8) A member could be experiencing any of the basic stresses or combinations thereof.

For the present we plan to study the following three types of stresses—(*i*) Tensile stresses, (*ii*) compressive stresses, and (*iii*) shear stresses.

1.2.1 Tensile stress. Consider a straight bar of uniform *X*-section

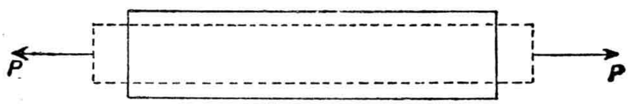

Fig. 1.1.

(Fig. 1.1) subjected to a pair of collinear forces acting in opposite directions and coinciding with the axis of the bar. If the forces are directed away from the bar then the bar tends to increase in length under the action of applied forces and the stresses developed in the bar are **tensile**. Tensile stresses are generally denoted by p_t.

1.2.2. Compressive stress. In the case discussed above if the forces are directed towards the bar (Fig. 1.2) then the bar tends to

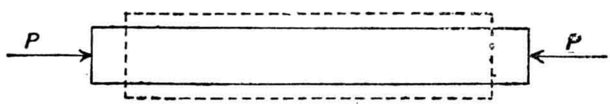

Fig. 1.2.

shorten in length under the action of the applied forces. The stresses developed in the bar are **compressive** and are generally denoted by p_c.

For shear stress refer to Art. 1.8.

1.3. STRAINS

Strain is a measure of the deformation produced in a member by the load. Direct stresses, tensile or compressive, produce change in length in the direction of the stress. If δl be the change in length l of a member caused by certain stresses then the strain

$$e = \frac{\delta l}{l} \qquad \ldots (1.2)$$

Strain (*for bodies subjected to normal tensile or compressive stresses*) **may be defined as change in length per unit length.** Tensile stresses increase the lengths whereas compressive stresses decrease the lengths as such tensile strains shall be taken as positive and compressive strains as negative. Since **strain is a ratio of two lengths, it has no units.**

1.4. STRESS-STRAIN CURVES FOR TENSION

Behaviour of materials subjected to tension is studied by plotting curves of stresses and corresponding strains observed by gradually increasing axially applied load to the point of failure of the specimen. Such curves, for different materials, differ in shape. Curves for ductile materials (*that elongate appreciably before failure*) and for brittle materials (*that show very small elongations before failure*) are discussed in the following articles.

1.4.1 Stress-strain curves for ductile materials. Mild steel is the most commonly used ductile material. A specimen mild steel in tension loaded with gradually increasing load shows initially the strains that are proportional to the stresses. Beyond a certain point p (Fig. 1.3), known as **limit of proportionality**, the stress-strain curve does not remain linear. The specimen, if stressed beyond e, known as **elastic limit**, does not return back to its original position when the load is removed. Property of materials to recover their original positions on removal of loads is termed as **elasticity**. If the specimen is loaded beyond e then on unloading the specimen a certain amount of strain called the **permanent set** is retained by the specimen. With further increase in load strain goes on increasing

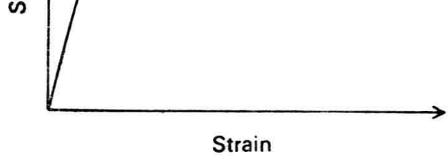

Fig. 1.3.

along ey up to y. Immediately beyond the point y there is an increase in strain even though there is no increase in stress. The stress corresponding to the point y is called the **yield stress**. At the yield stress the material begins to flow. At u the stress is the maximum and is known as **ultimate stress**. Beyond *u* the bar elongates even with decrease in stress and finally fails at a stage corresponding to point *f*.

The ratio of maximum load, that the specimen is capable of sustaining, to its original area of cross-section is termed as ultimate stress of the material.

After *u* the specimen is greatly reduced in cross-section area. At *f*, the point of failure, the reduced area is the least and this phenomenon is known as *necking*.

1.4.2. Stress-strain curve for brittle materials. Structural

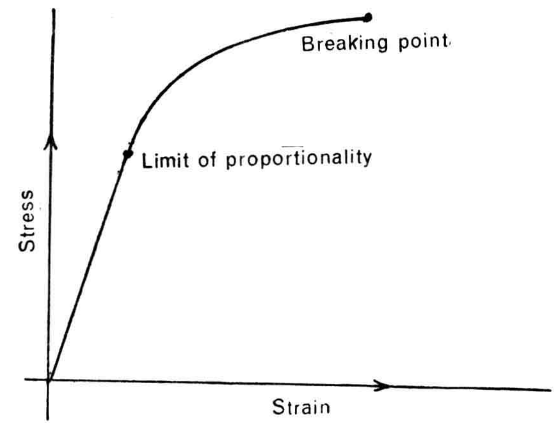

Fig. 1.4.

steel is the only material that exhibits a marked yield point. Most of the other materials show a gradual change from linear to the nonlinear range. Brittle materials have a very low proportional point and do not show the yield point (Fig. 1.4 shows a typical stress-strain curve for Cast Iron). In such cases where the yield point is not clearly shown, it is taken as the point of some definite amount of permanent strain. Usually specified strain is from 0.0005 to 0.0035 (0.05% to 0.35%). The point where the stress-strain curve is cut by a straight line parallel to its initial straight portion drawn at a

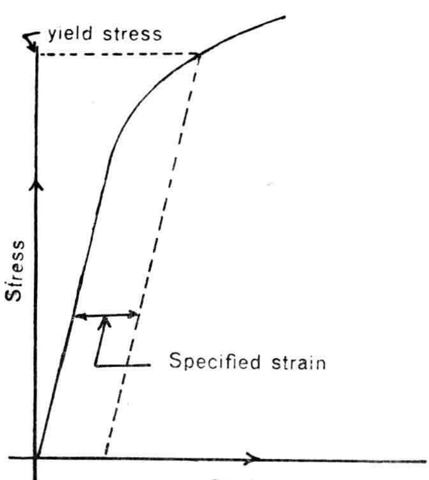

Fig. 1.5.

point of specified strain gives the yield point for it and the corresponding stress as the yield stress or the proof stress (Fig. 1.5). If the stress elongation is 0.1% then the corresponding stress is called 0.1% proof stress and so on.

Stress-strain curves for compression can similarly be plotted to determine the characteristic stresses such as proportional stress, yield stress and the ultimate stress. In case of steel these stresses are the same both in tension and in compression.

1.5. HOOKE'S LAW

It states that for materials subjected to simple tension or compression within elastic limit the stress is proportional to the strain. Mathematically it can be expressed as:

$$\frac{\text{Stress}}{\text{Strain}} = \text{Constant (called the Young's modulus or the modulus of elasticity and denoted by } E\text{)}.$$

or

$$\frac{p}{e} = E \qquad \ldots (1.3)$$

Young's modulus E may thus be defined as the simple tensile or compressive stress within elastic limits required to produce unit strain.

Since the unit of stress p is kg/cm² and the strain e being only a ratio the unit of E shall be kg/cm².

1.6. CHANGE IN LENGTH OF A BODY DUE TO APPLICATION OF LOAD ON IT

Consider the wire in Fig. 1.3 to be subjected to a pull of load P.

Let

l = original length of the wire

A = X-section area of wire

δl = change in length caused by the applied load

e = strain in wire due to applied load

p = stress intensity in the wire due to applied load

From Hooke's law we have:

$$\frac{p}{e} = E \qquad \ldots (\text{Eq. } 1.3)$$

or

$$e = \frac{p}{E} \qquad \ldots (1.4)$$

Substituting for e from Eq. 1.2 and for p from Eq. 1.1 we have:

$$\frac{\delta l}{l}=\frac{P}{AE}$$

or $$\delta l=\frac{Pl}{AE} \qquad \ldots (1.5)$$

1.7. FACTOR OF SAFETY

At stresses below the elastic limit the material does not retain any strain on unloading the member but beyond this limit a part of the strain usually remains after unloading the member. Also it is impossible to accurately foresee the external loads to which a structure is subjected. Moreover at times the available materials are victims of poor workmanships. These factors too are taken care of by adopting working stress that is lower than the maximum stress (usually the *ultimate stresses*) that the material is capable of taking. Thus if p is the ultimate stress for the material in a component then the *working stress* is $\frac{p}{n}$ where n is the factor of safety.

The magnitude of the factor of safety to be adopted depends upon the nature of loading, the homogeneity of materials used, the accuracy with which stresses in members and external forces can be evaluated, the degree of safety required and the degree of economy desired.

EXAMPLE 1.1. *A steel rod of 20 mm diameter and 500 cm long is subjected to an axial pull of 3000 kg. Determine (i) the intensity of stress, (ii) the strain, (iii) the elongation of rod. Take $E=2.1\times10^6$ kg/cm².*

SOLUTION. Diameter of rod = 20 mm = 2.0 cm.

Length of rod = 500 cm.

Load = 3000 kg. $E_s=2.1\times10^6$ kg/cm²

Cross-section area of rod is $A=\frac{\pi\times2.0^2}{4}=3.14$ cm²

(*i*) Intensity of stress is $p=\frac{P}{A}=\frac{3000}{3.14}=$ **955.41 kg/cm²**

(*ii*) Strain $e=\frac{p}{E}=\frac{955.41}{2.1\times10^6}$

$=$ **0.000455** (*Strain has no unit*)

(*iii*) Elongation $\delta l=\frac{Pl}{AE}=\frac{3000\times500}{3.14\times2.1\times10^6}$

$=$ **0.2275 cm.**

EXAMPLE 1.2. *A short hollow cast iron cylinder of wall thickness 1.0 cm is to carry a compressive load of 60 tonnes. Determine its outside diameter if the ultimate crushing stress for the material is 5400 kg/cm². Use a factor of safety of 6.* (*Banaras Hindu University, 1977*)

SOLUTION. Let outside diameter be D_o. Since wall thickness is 1.0 cm, the inside diameter of the cylinder is (D_o-2) cm (Fig. 1.6).

Area of cross-section

$$=\frac{\pi\,[D_o^2-(D_o-2)^2]}{4}$$

$$=\frac{\pi(4D_o-4)}{4}=\pi(D_o-1)\text{ cm}^2$$

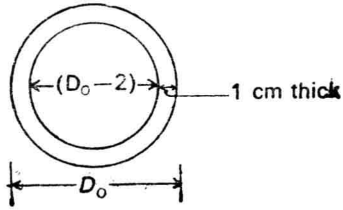

Fig. 1.6

Crushing load for the column

$$=5400\times\pi(D_o-1)\text{ kg}$$

$$=5.4\times\pi(D_o-1)\text{ tonne}$$

Factor of safety $=6$

$\therefore$ Safe load $=\dfrac{5.4\times\pi(D_o-1)}{6}=60$

or $D_o=\dfrac{60\times6}{5.4\times\pi}+1=$ **22.22 cm.**

EXAMPLE 1.3. *A 20 cm long steel tube of 15 cm internal diameter and 1 cm thick is surrounded closely by a brass tube of same length and thickness. The tubes carry an axial load of 15T. Estimate the load carried by each.* $E_s=2.1\times10^6$ *kg/cm*^{2}; $E_b=1\times10^6$ *kg/cm*^{2}.

SOLUTION. Outer and inner diameters of steel tube are $(15+2)=17$ cm and 15 cm respectively and for brass tube these are $(17+2)=19$ cm and 17 cm respectively.

$\therefore$ Cross-sectional area of steel tube is

$$A_s=\frac{\pi(17^2-15^2)}{4}=50.265\text{ cm}^2$$

Cross-sectional area of brass tube is

$$A_b=\frac{\pi(19^2-17^2)}{4}=56.549\text{ cm}^2$$

Young's modulus for steel is $E_s=2.1\times10^3$ kg/cm^2

Young's modulus for brass is $E_b=1\times10^6$ kg/cm^2

Length of each tube is $l=20$ cm

Axial load is $P=15000$ kg

Let P_s and P_b be the loads shared by steel and brass tubes respectively.

Thus $P_s+P_b=15000$ . . . (i)

Changes in length for both the tubes are the same.

$$\therefore\quad \frac{P_s l}{A_s E_s}=\frac{P_b l}{A_b E_b}$$

or

$$\frac{P_s}{P_b}=\frac{A_s E_s}{A_b E_b}=\frac{50.265\times2.1\times10^6}{56.549\times1\times10^6}=1.867$$

$\therefore \qquad P_s = 1.867\, P_b$

From equation (*i*) we have:

$$P_b = \mathbf{5231.95\ kg}$$

and $\qquad P_s = \mathbf{9768.05\ kg.}$

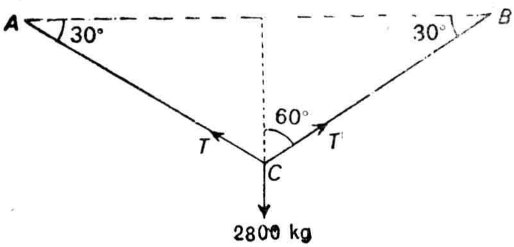

Fig. 1.7

EXAMPLE 1.4. *A vertical load of 2800 kg is supported by two inclined steel wires AC and BC, each 6.0 metre long as shown in Fig. 1.7. If allowable working stress in wire in tension is 700 kg/cm² determine the required cross sectional area of each wire and the vertical displacement of the point C. Take* $E = 2.0 \times 10^6$ *kg/cm².*

SOLUTION. Load *P* will cause tension in both the wires *AC* and *BC* and by symmetry the tensions in both the wires shall be equal, say *T*. Balancing the forces in vertical direction at joint *C* we have, by resolving:

$$2T \cos 60 = 2800 = 2T \times \tfrac{1}{2}$$

$$\therefore \qquad T = 2800 \text{ kg}$$

Allowed stress in wire is $p = 700$ kg/cm².

Required *X*-section area *A* of each wire

$$= \frac{2800}{700} \qquad \left(\because P = p \times A \text{ or } A = \frac{P}{p}\right)$$

$$= \mathbf{4.0\ sq\ cm.}$$

Displacement of point C. Let the point *C* displace to *C'* (Fig. 1.8). Since the displacement is small $\angle C'AB$ can still be taken to be 30°.

Draw $CD \perp AC'$.

Now $AC = AD$ and DC' is the elongation of *AC*.

Pull in the member *AC* is 2800 kg and its *X*-section area is 4.0 cm².

Elongation DC' is given by equation 1.5

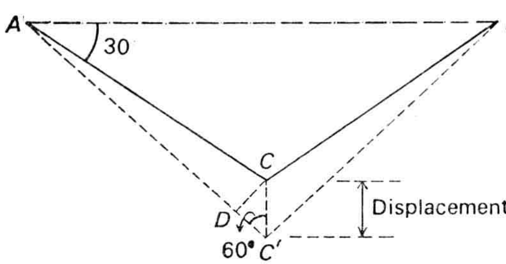

Fig. 1.8

$$DC' = \delta l = \frac{Pl}{AE} = \frac{2800 \times 600}{4 \times 2.0 \times 10^6} = 0.21 \text{ cm}$$