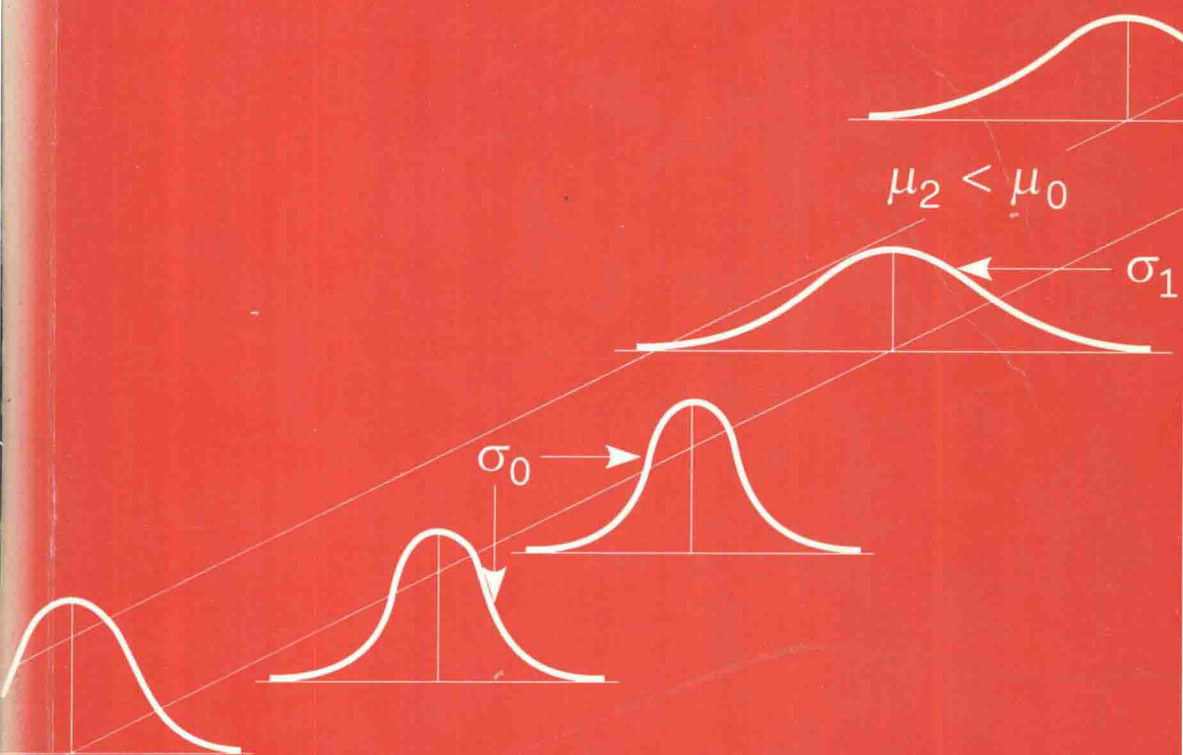


# INTRODUCTION TO STATISTICAL QUALITY CONTROL

Third Edition



Douglas C. Montgomery

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WILEY

**John Wiley & Sons (ASIA) Pte Ltd**

2 Clementi Loop #02-01

Singapore 129809



**ISBN 9971-51-361-7**

Montgomery

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Statistical  

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Quality  

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Control  

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Third Edition

Douglas C. Montgomery

*Arizona State University*



John Wiley & Sons, Inc.

New York • Chichester • Brisbane • Toronto • Singapore

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*Library of Congress Cataloging-in-Publication Data*  
Montgomery, Douglas C.

Introduction to statistical quality control / Douglas C. Montgomery.—3rd ed.  
p. cm.

Includes index.

ISBN 9971-51-361-7

1. Quality control—Statistical methods. 2. Process control—Statistical methods. I. Title.

TS156.M64 1996

658.5'62'015195—dc20

Printed and bound by Replika Press Pvt Ltd, 100% EOU, Delhi-110 040, India

10 9 8 7 6 5 4 3 2 1



## SPC Calculations for Control Limits

Notation:	UCL — Upper Control Limit	$\bar{\bar{x}}$ — Average of Measurements
	LCL — Lower Control Limit	$\bar{\bar{x}}$ — Average of Averages
	CL — Center Line	$R$ — Range
	$n$ — Sample Size	$\bar{R}$ — Average of Ranges
	PCR — Process Capability Ratio	USL — Upper Specification Limit
	$\sigma$ — Process Standard Deviation	LSL — Lower Specification Limit

Variables Data ( $\bar{X}$  and R Control Charts):

$\bar{x}$ Control Chart	$n$	$A_2$	$D_3$	$D_4$	$d_2$
UCL = $\bar{\bar{x}} + A_2\bar{R}$	2	1.880	0.000	3.267	1.128
LCL = $\bar{\bar{x}} - A_2\bar{R}$	3	1.023	0.000	2.574	1.693
CL = $\bar{\bar{x}}$	4	.729	0.000	2.282	2.059
	5	.577	0.000	2.115	2.326
$R$ Control Chart	6	.483	0.000	2.004	2.534
UCL = $\bar{R} D_4$	7	.419	.076	1.924	2.704
LCL = $\bar{R} D_3$	8	.373	.136	1.864	2.847
CL = $\bar{R}$	9	.337	.184	1.816	2.970
Capability Study	10	.308	.223	1.777	3.078
PCR = (USL - LSL)/(6 $\hat{\sigma}$ ); where $\hat{\sigma} = \bar{R}/d_2$					

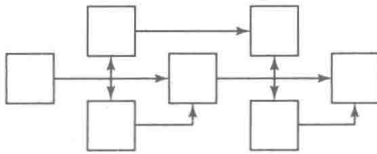
Attribute Data ( $p$ ,  $np$ ,  $c$ , and  $u$  Control Charts):

Control Chart Formulas

	$p$ (fraction)	$np$ (number of nonconforming)	$c$ (count of nonconformances)	$u$ (count of nonconformances/unit)
CL	$\bar{p}$	$n\bar{p}$	$\bar{c}$	$\bar{u}$
UCL	$\bar{p} + 3\sqrt{\frac{\bar{p}(1-\bar{p})}{n}}$	$n\bar{p} + 3\sqrt{n\bar{p}(1-\bar{p})}$	$\bar{c} + 3\sqrt{\bar{c}}$	$\bar{u} + 3\sqrt{\frac{\bar{u}}{n}}$
LCL	$\bar{p} - 3\sqrt{\frac{\bar{p}(1-\bar{p})}{n}}$	$n\bar{p} - 3\sqrt{n\bar{p}(1-\bar{p})}$	$\bar{c} - 3\sqrt{\bar{c}}$	$\bar{u} - 3\sqrt{\frac{\bar{u}}{n}}$
Notes	If $n$ varies, use $\bar{n}$ or individual $n_i$	$n$ must be a constant	$n$ must be a constant	If $n$ varies, use $\bar{n}$ or individual $n_i$

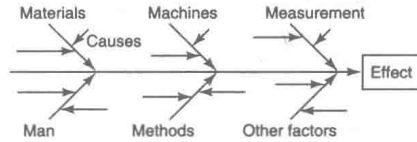
## Quality Improvement Tools

### Process Flow Diagram



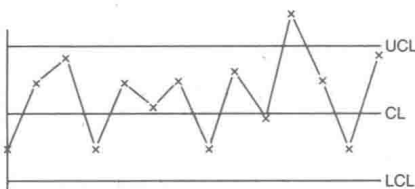
- Expresses detailed knowledge of the process
- Identifies process flow and interaction among the process steps
- Identifies potential control points

### Cause and Effect (Fishbone) Diagram



- All contributing factors and their relationship are displayed
- Identifies problem areas where data can be collected and analyzed

### Control Chart



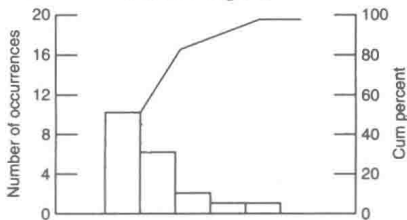
- Helps reduce variability
- Monitors performance over time
- Allows process corrections to prevent rejections
- Trends and out-of-control conditions are immediately detected

## Checksheet

A						
B						
C						
D						
E						
F						

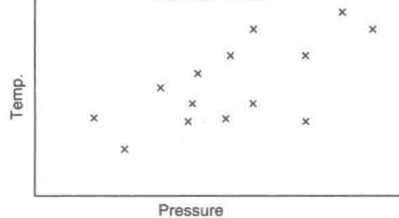
- Simplifies data collection and analysis
- Spots problem areas by frequency of location, type, or cause

### Pareto Diagram



- Identifies most significant problems to be worked first
- Historically 80% of the problems are due to 20% of the factors
- Shows the vital few

### Scatter Plot



- Identifies the relationship between two variables
- A positive, negative, or no relationship can be easily detected

## Design of Experiments (DOE)

- Useful in process development and troubleshooting
- Identifies magnitude and direction of important process variable effects
- Greatly reduces the number of runs required to perform an experiment
- Identifies interaction among process variables
- Useful in engineering design and development
- Focuses on optimizing process performance

### Histogram



- The shape shows the nature of the distribution of the data
- The central tendency (average) and variability are easily seen
- Specification limits can be used to display the capability of the process





Introduction to  

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Statistical  

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Quality  

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Control  

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Third Edition



# About the Author

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Douglas C. Montgomery, Professor of Industrial and Management Systems Engineering at Arizona State University, received his B.S., M.S., and Ph.D. degrees from Virginia Polytechnic Institute, all in engineering. From 1969 to 1984 he was a faculty member of the School of Industrial & Systems Engineering at the Georgia Institute of Technology; from 1984 to 1988 he was at the University of Washington, where he held the John M. Fluke Distinguished Chair of Manufacturing Engineering, was Professor of Mechanical Engineering, and Director of the Program in Industrial Engineering.

Dr. Montgomery has research and teaching interests in engineering statistics including statistical quality control techniques, design of experiments, regression analysis and empirical model building, and the application of operations research methodology to problems in manufacturing systems. He has authored and coauthored many technical papers in these fields and is an author of ten other books. Dr. Montgomery is a Fellow of the American Society for Quality Control, a Fellow of the American Statistical Association, and a Fellow of the Institute of Industrial Engineers. He is a recipient of the Ellis R. Ott award, and both the Brumbaugh Award and the Shewell Award from the American Society for Quality Control. He is currently the editor of the *Journal of Quality Technology*.

In addition to his academic activities, Dr. Montgomery is a partner in Statistical Productivity Consultants, a Seattle-based consulting organization specializing in the application of statistical methods for quality and productivity improvement. Dr. Montgomery has served as a consultant in these areas to many major corporations.



# Preface

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This book is about the modern use of statistical methods for quality control and improvement. It provides comprehensive coverage of the subject from basic principles to state-of-the-art concepts and applications. The objective is to give the reader a sound understanding of the principles and the basis for applying those principles in a wide variety of both product and nonproduct situations. While statistical techniques are emphasized throughout, the book has a strong engineering and management orientation. Extensive knowledge of statistics is not a necessary prerequisite for using this book. Readers whose background includes a basic course in statistical methods will find this book easily accessible.

The book is an outgrowth of over 25 years of teaching, research, and consulting in the application of statistical methods in quality engineering and quality improvement. It is designed as a textbook for students enrolled in colleges and universities, who are studying engineering, management, statistics, and related fields and are taking a first course in statistical quality control. Such courses are often taught at the junior or senior level. I have also used the text materials extensively in training programs for professional practitioners, including quality and reliability engineers, manufacturing and development engineers, managers, procurement specialists, marketing personnel, technicians and laboratory analysts, inspectors, and operators. Many professionals have also used the material for self-study.

Chapter 1 is an introduction to the philosophy and basic concepts of quality improvement. It notes that quality has become a major business strategy and that organizations with successful quality-improvement programs can increase their productivity, enhance their market penetration, and achieve greater profitability and a strong competitive advantage. Some of the managerial and implementation aspects of total quality management (or TQM) are included.

Following the introductory chapter, the book is divided into four parts. Part I presents a description of statistical methods useful in quality improvement. Topics covered included sampling and descriptive statistics, the basic notions of probability and probability distributions, point and interval estimation of parameters, and statistical hypothesis testing. These topics are usually covered in a basic course in statistical methods; however, their presentation in this text is from the quality-engineering viewpoint. My experience has been that even readers with a strong statistical background will find the approach to

this material useful and somewhat different from that used in a standard statistics textbook.

Part II contains six chapters on statistical process control (or SPC). While the entire range of SPC tools are extensively discussed, the primary focus is on the control chart. The control chart is certainly not new, but its use in modern-day business and industry is of tremendous value. Furthermore, the combination of sensing and measurement technology, the widespread availability of powerful microcomputers, and good modern SPC software has made the implementation of SPC at the workplace a standard practice in many businesses. SPC will play an even greater role in the U.S. industry over the next 20 years than it has in the last 50.

Part III contains three chapters that show how statistically designed experiments can be used for process design, development and improvement. Chapter 10 presents the fundamental concepts of designed experiments and introduces the reader to some of the data analysis methods employed. While the treatment of the subject is not extensive and is no substitute for a formal course in experimental design, it will enable the reader to appreciate more sophisticated examples of experimental design. Chapter 11 illustrates factorial and fractional factorial designs, with particular emphasis on the two-level system of designs. These designs are used extensively in industry for factor screening and process characterization. Chapter 12 introduces response surface methods and designs, illustrates evolutionary operation (EVOP) for process monitoring, and gives an overview of Taguchi's contributions to quality engineering. I've tried to present my view that Taguchi has made many valuable contributions to quality improvement philosophy, but that his technical methods are often ineffective and inefficient, and can be improved. Chapters 10, 11, and 12 emphasize the important interrelationship between statistical process control and experimental design for process improvement.

Part IV contains two chapters dealing with acceptance sampling. The focus is on lot-by-lot acceptance sampling, although there is some discussion of continuous sampling and MIL STD 1235C in Chapter 14. Other sampling topics presented include various aspects of the design of acceptance-sampling plans, a discussion of MIL STD 105E, MIL STD 414 (and their civilian counterparts, ANSI/ASQC Z1.4 and ANSI/ASQC Z1.9), and other techniques such as chain sampling and skip-lot sampling.

Throughout Parts II and IV, guidelines are given for selecting the proper type of statistical process-control or sampling technique to use in a wide variety of product and nonproduct situations. There are also extensive references to journal articles and other technical literature that should assist the reader in applying the methods described.

## CHANGES IN THE THIRD EDITION

Based on my own teaching experiences and extensive feedback from other users of the text, I have made numerous changes in this edition of the book. A major change is in the order of presentation of variables and attributes control charts. Chapter 7 on cumulative sum (CUSUM) and exponentially weighted moving average (EWMA) control charts has been completely rewritten. The tabular or algorithmic CUSUM is emphasized, and the

V-mask approach is only briefly mentioned. Hopefully, the SPC software providers will finally eliminate the V-mask as well. Chapter 8 is extensively updated and rewritten and includes coverage of short-run SPC methods, group control charts for multiple-stream processes, acceptance control charts and control charts with modified limits, much new material on multivariate quality control, an expanded treatment of SPC with autocorrelated data, the economic design of control charts (which formerly was covered in a separate chapter), a discussion of the interface between SPC and engineering control, and a brief survey of several other topics. The coverage of process-capability analysis in Chapter 9 has been expanded to include more useful information on process capability indices (confidence intervals), and techniques for assessing gage capability. Throughout Part II I have provided illustrations of typical computer software output that supports quality engineering activities.

Part III on process improvement with designed experiments has been completely reorganized and expanded. I originally included this material in the previous edition at the request of many instructors who find it necessary to introduce students in their quality control courses to the fundamentals of experimental design because this topic is not covered in their basic statistics course. I have expanded this material to give more details of its use in quality and process improvement. I have also illustrated how the computer is used in the analysis of data from designed experiments. I realize that it is possible to present this material without introducing the student to the analysis of variance (indeed, some books have done that); however, I chose to use the analysis of variance because when the students plan and conduct experiments in the real world they will use a computer package that uses this method. It's a disservice to the students to teach them otherwise.

Part IV on acceptance sampling has been shortened, because most modern courses deemphasize this topic. I reduced the discussion of the details of the military standard sampling plans, eliminated some of the tables from these plans, and condensed or eliminated the discussion of several other topics.

All of the examples in this book utilize data from real applications. In some cases, I have disguised the data or the application situation so that proprietary information will be protected.

## ACKNOWLEDGMENTS

Many people have generously contributed their time and knowledge of quality improvement to this book. I would like to thank Dr. Bill Woodall, Dr. Joe Sullivan, Dr. George Runger, Mr. Eric Ziegel, Dr. Joe Pignatiello, Dr. John Ramberg, Dr. Ernie Saniga, and Dr. Jim Alloway for their thorough and insightful reviews on a draft of the third edition. These individuals shared many of their ideas and teaching experiences with me, leading to substantial improvements in the book.

Over the years since the first edition was published, I have received assistance and ideas from a great many people. A complete list of colleagues with whom I have interacted in various academic or consulting projects over the years would be impossible to enumerate. However, some of the major contributors and their professional affiliations



are as follows: Dr. J. Bert Keats, Dr. Mary R. Anderson, Dr. Dwayne A. Rollier, and Dr. Norma F. Hubele, Arizona State University; Mr. Seymour M. Selig, formerly of the Office of Naval Research; Dr. Lynwood A. Johnson, Dr. Russell G. Heikes, Dr. David E. Fyffe, and Dr. H. M. Wadsworth, Jr., Georgia Institute of Technology; Dr. Richard L. Storch, University of Washington; Dr. William H. Woodall, University of Alabama; Dr. Cynthia A. Lowry, Texas Christian University; Dr. Christina M. Mastrangelo, the University of Virginia; Dr. Smiley Cheng, Dr. John Brewster, Dr. Brian Macpherson, and Dr. Fred Spiring, the University of Manitoba; Dr. Joseph D. Moder, University of Miami; Dr. Erwin M. Saniga, University of Delaware; Dr. John S. Ramburg, University of Arizona; Dr. Frank B. Alt, and Dr. George C. Runger, University of Maryland; Dr. Kenneth E. Case, Oklahoma State University; Mr. Daniel R. McCarville, Ms. Lisa Custer, and Mr. Robert Stuart, Motorola; Dr. Dale Sevier, Hybritech; Mr. John A. Butora, Mr. Leon V. Mason, Mr. Lloyd K. Collins, Mr. Dana D. Leshner, Mr. Roy E. Dent, Mr. Mark Fazey, Ms. Kathy Schuster, Mr. Dan Fritze, Dr. J. S. Gardiner, Mr. Ariel Rosentrater, Mr. Lolly Marwah, Mr. Ed Schleicher, Mr. Armin Weiner, and Ms. Elaine Baechtle, IBM; Mr. Thomas C. Bingham, Mr. K. Dick Vaughn, Mr. Robert LeDoux, Mr. John Black, Mr. Jack Wires, Dr. Julian Anderson, Mr. Richard Alkire, and Mr. Chase Nielsen, The Boeing Company; Ms. Karen Madison, Mr. Don Walton, and Mr. Mike Goza, Alcoa; Mr. Harry Peterson-Nedry, Ridgecrest Vineyards and The Chehalem Group; Dr. Russell A. Boyles, Precision Castparts Corporation; Dr. Sadre Khalessi and Mr. Franz Wagner, Signetics Corporation; Mr. Larry Newton and Mr. C. T. Howlett, Georgia Pacific Corporation; Mr. Robert V. Baxley, Monsanto Chemicals; Dr. Craig Fox, Dr. Thomas L. Sadosky, Mr. James F. Walker, and Mr. John Belvins, The Coca-Cola Company; Mr. Bill Wagner and Mr. Al Pariseau, Litton Industries; Mr. John M. Fluke, Jr., John Fluke Manufacturing Company; Dr. William DuMouchel, Columbia University; Dr. Pat Spagon and Dr. Paul Tobias, Semitech; and Ms. Janet Olson, BBN Software Products Corporation. I would also like to acknowledge the contribution of my partner in Statistical Productivity Consultants, Mr. Sumner S. Averett. All of these individuals and many others have contributed to my knowledge of the quality improvement field.

Ms. Charity Robey, the editor at John Wiley who I worked with in preparing the third edition of this book, deserves much credit. She has tolerated the peculiarities of authorship far better than did other editors with whom I have worked. Ms. Cheryl Jennings made many valuable contributions by her careful checking of the manuscript and proof materials.

I thank the various professional societies and publishers who have given permission to reproduce their materials in my text. Permission credit is acknowledged at appropriate places in this book.

I am also indebted to several organizations, including the Office of Naval Research, the National Science Foundation, the Aluminum Company of America, and the IBM Corporation. These organizations sponsored my academic research activities in quality improvement for a number of years. Finally, I would like to thank the many users of the first and second editions of this book including students, practicing professionals, and my academic colleagues. Many of the changes and (hopefully) improvements in this edition of the book are the direct result of your feedback.

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