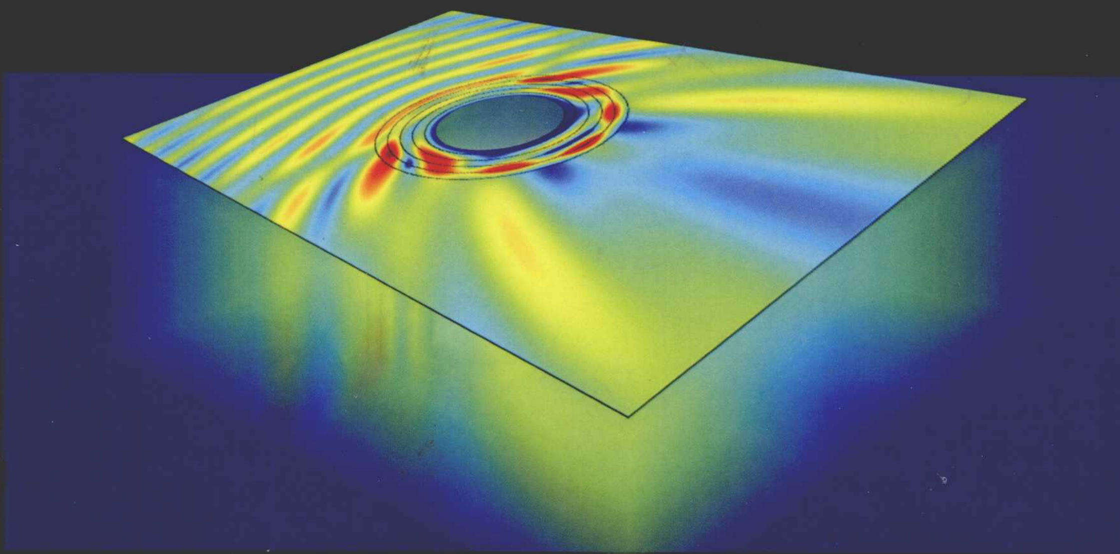
An Introduction to Metamaterials and Waves in Composites



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Preface

In the fall of 2006 I read a couple of articles on transformation-based cloaking in *Science* and heard a talk by Prof. Graeme Milton on cloaking due to anomalous resonances. I was intrigued by these new ideas; were they applicable to shocks, vibrations, seismic waves? An opportunity to learn more came early in 2007, when Prof. Milton offered a course called "Waves in Composites and Metamaterials." I soon realized that the contents of the course were worth preserving and disseminating to a broader audience. I started to work out the details of each lecture and posting them on my web-page at the University of Utah. The next three months were a juggling act between my research on computational plasticity and trying to understand metamaterial concepts. Later that year, I converted my notes into Wiki form and gave them a more permanent home at Wikiversity.* This book has grown out of those lecture notes.

The book has been written with beginning graduate students and advanced undergraduates in mind. The book will also be of use to engineers and researchers who wish to understand recent developments in the dynamics of composites. A background of elementary calculus and differential equations, linear algebra, complex analysis, and vector and tensor analysis is assumed. The reader unfamiliar with these topics can find excellent sources on the web that are sufficient to understand most of the book. The notation in the book has been kept as independent of coordinate system as possible. Components of vectors and tensors are introduced only when necessary and the Einstein summation convention is used extensively.

Special emphasis is placed on elastic media and acoustics, in part because of my background in solid mechanics but also because there is a large gap between the literature on the dynamics of elastic/acoustic composites and that on electromagnetic composite media. Previous knowledge of elasticity and electrodynamics will help in the navigation of this book, but is not essential. It is crucial that the reader develop a familiarity with electrodynamics during the reading of this book; many developments in the fields of phononic crystals and acoustic metamaterials have been based on previous developments in electromagnetic materials.

I have adopted a "direct," deductive, approach in this book in the sense that results do not appear magically. This, of course, has required that an amount of detail be included in the text. I suggest that the student work through some of the detail to gain an understanding but spend more time analyzing the results and developing concepts. The problems at the end of each chapter are designed to consolidate the

*The web page is http://en.wikiversity.org/wiki/Waves_in_composites_and_metamaterials.

understanding of techniques used in the book. Beyond general ideas, the book does not discuss the design of metamaterials and artificial crystals; that task is left to the readers of this book.

The style and content of this book have been influenced by several people, particularly Prof. Graeme Milton, Prof. William Pariseau, Prof. Robert Smith, and Prof. Rebecca Brannon of the University of Utah, and Prof. Andrew Norris of Rutgers University. Special thanks are due to my wife Champa who suffered through a year of lost weekends and evenings while this book was being written. I would like to thank Dr. Emilio Calius for writing part of the introduction, Dr. Eric Wester for showing me his approach to impedance tube calculations, Dr. Bryan Smith for helping with cloaking simulations, and our librarian Alison Speakman for giving me quick access to the older literature on the dynamics of composites. Thanks also to my editor, Dr. John Navas, who persisted and made this book possible.

All errors in this book are, of course, mine and I will be grateful if readers inform me of any errors that they find.

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Introduction

> Traditionally, the answer to a given problem is obtained by copying suitable equations, submodels, and boundary conditions with their appropriate solution techniques from available sources. This is called "matching" and may result in a good first-step learning experience; however, it should be augmented later on by more independent work ...
>
> C. KLEINSTREUER, *Two-phase flow: Theory and applications*, 2003.

Let us consider the propagation of light through glass, clear water, or any other reasonably transparent media; the propagation of sound through plasterboard walls or steel bulkheads; the propagation of vibration through aluminum airframes; or the propagation of radio waves through the earth's atmosphere. What all of these situations have in common is that they involve the interaction of waves with a medium, be it solid, fluid, or gas. If the structure of the medium is just right, these interactions can have interesting effects. The colors in a peacock's feathers are caused by light interacting with a periodic solid material; a rainbow is produced by the interaction of light with spherical water droplets; a mirage is caused by the interaction of light with our layered atmosphere. If we have special instruments we can observe similar effects for sound and elastic vibrations. But to what extent can we control these phenomena?

That light can be bent must have been known since soon after humans started fishing, and even the controlled bending of light through lenses has been known for a long time. Translucent crystals of various kinds have been found among both Neanderthal and Cro-Magnon remains that are tens of thousands of years old. The oldest polished lens artifacts seem to be plano-convex (flat on one side and convex on the other) rock crystal lenses that may have been used as magnifying glasses, or as burning-glasses to start fires by concentrating sunlight. The earliest written records of lenses date to Ancient Greece, with Aristophanes' play *The Clouds* (424 BC) mentioning a burning-glass (a biconvex lens used to focus the sun's rays to produce fire). The oldest lens artifact is the Nimrud lens, which is over three thousand years old, dating back to ancient Assyria. Clearly, attempts to control the propagation of waves has a long history.

Ordinary matter is fundamentally discrete. All naturally occurring media known to man are composed of discrete molecules, which in turn are made up of atoms. An atom consists of electrons, positively charged protons, and neutrons and, even further down in the length scale, quarks. A central assumption in classical mechanics and electrodynamics is that the discrete nature of matter can be overlooked, provided the length scales of interest are large compared to the length scales of discrete molecular structure. Thus, matter at sufficiently large length scales can be treated as a continuum in which all physical quantities of interest, including density, stiffness, magnetic permeability, and electric permittivity are continuously differentiable.

Wave propagation in periodic structures such as natural crystals and in layered media has been studied since the pioneering work of Kelvin, Rayleigh, and Maxwell in the late 1800s. But, since the geometries of these structures could be varied only slightly, there was limited control over the effects that could be achieved. In 1987, the work of Yablonovitch and John triggered an interest in the creation of artificial crystals that could be designed to exhibit particular wave phenomena. However, most of these artificial photonic or phononic crystals were designed to operate at wavelengths of the order of the lattice parameter until the development of metamaterials.

Metamaterials

The name metamaterials emerged in the late 1990s and was first used officially by the Defense Advanced Research Projects Agency (DARPA) Symposium on Meta-Materials in 1999. It was coined by the pioneers in the field by using the prefix *meta*, which can be translated from the Greek as beyond, to imply beyond conventional materials. Metamaterials are a new class of complex composite materials that have created considerable excitement because they can be engineered to exhibit any desired electromagnetic, acoustic, or mechanical effective properties up to and including such exotic behaviors as negative refraction, negative bulk modulus, or negative mass under certain excitations. The physics of metamaterials and their interaction with waves of all kinds can be extremely counter-intuitive, causing strong criticism and debate as well as an explosion of research.

Metamaterials have a long prehistory, dating back at least as far as the Lycurgus cup from the 4th century AD that uses metallic nanoparticle colloids embedded in glass to dramatically change its color as a function of the illumination angle. But scientific research in this area only started in the late 19th century, as Floquet, Rayleigh, Bose, and others investigated waves in periodic systems of various kinds. Although materials that exhibited reversed physical characteristics were first described theoretically by Veselago in 1967, it was not until the late 1990s that practical designs of such materials were discovered.

In our view metamaterials involve inclusions and inter-inclusion distances that are much smaller than a wavelength. As long as this condition is met, the dimensions of the metamaterials' internal structure are independent of the wavelength they interact with, and determined mainly by practical considerations. Consequently such media can be described by homogenization and effective media concepts. They typically involve coupling of the waves with resonances of some kind. On the other hand, the phononic and photonic crystals (also called band-gap materials) involve length scales that are on the order of half a wavelength or more and are described by Bragg reflection and other periodic media concepts. The key dimensions of a band gap material are directly linked to the wavelengths that it will strongly interact with. However, there is often no clear distinction between metamaterials and photonic/phononic crystals and the same structure may behave as one or the other, depending on the wavelength and frequency of the incident waves.

Modern developments

Metamaterials have a prehistory dating back many centuries; but modern development really started in the mid-1990s when it was realized that split-ring resonators and thin wire structures provided the means of constructing electromagnetic metamaterials with a negative refractive index, which was first demonstrated experimentally by Smith et al. at the University of California in 2000 (Smith et al., 2000). Research in this area, although active, remained something of a niche until Pendry's seminal paper (Pendry, 2000), published that same year, in which he described how metamaterials could enable the perfect lens, one whose imaging resolution is not limited by diffraction, which would have profound consequences for microscopy, spectroscopy, and micro-fabrication. Also in 2000, a group of researchers in Hong Kong reported the first elastic metamaterial and its ability to greatly affect acoustic transmission in a narrow frequency band (Liu et al., 2000). The composite material was later shown to exhibit negative density at those frequencies (Liu et al., 2005).

By 2006 both Leonhardt (Leonhardt, 2006b) and Pendry (Pendry et al., 2006) had developed theories of electromagnetic cloaking using metamaterials, and later that year Schurig and Smith, now at Duke University, experimentally demonstrated cloaking of a small region at one microwave frequency (Schurig et al., 2006a). Theoretical concepts for acoustic cloaking were first published in 2007 (Cummer and Schurig, 2007, Chen and Chan, 2007a), but experimental results were not available until 2010, elastic structures being more complex as they can support shear as well as longitudinal wave modes. Cloaks to protect marine structures from waves (Farhat et al., 2008) and buildings from earthquakes (Brun et al., 2009) have also been proposed recently. Progress in cloaking is continuing, with the first optical frequency metamaterial being produced in 2008 and the first cloak that operates over a range of frequencies in the microwave spectrum being demonstrated in 2010. There are still major challenges, such as the issue of losses, which is driving interest in superconducting metamaterials (Kurter et al., 2010) and how to achieve the desired effects over an usefully wide range of frequencies.

Recent developments and future directions

Although cloaking and super lensing continue to be major foci of metamaterials research, the field has begun to broaden significantly over the past couple of years. A team at Caltech is designing metamaterial coatings to improve the effectiveness of solar cells; several groups are looking at applications involving THz focusing and imaging, others at wireless communications in the 100 Mhz to 10 GHz range, some at controlling heat transmission through phonons; yet others are using metamaterials to simulate black holes and other aspects of the structure of the universe by exploiting the analogy between metamaterials' ability to distort electromagnetic space and gravity's distortions of spacetime. A team at Nanjing's Southeast University reported the first microwave black hole in 2009 (Cheng et al., 2009, Cheng et al., 2010).

Research worldwide has focused mainly on electromagnetic metamaterials because there are many kinds of modern devices that require interactions with electromagnetic waves. Applications range from health care to communications, from power generation and conversion to semiconductor manufacturing, from stealth applications for the military to security. Electromagnetic metamaterial antennas that offer improved performance in smaller sizes appeared in 2009 and many other commercial applications will undoubtedly follow.

But there is a gap in published research on elastic and acoustic metamaterials when compared with that on electrodynamic metamaterials. Though deep sub-wavelength acoustic imaging has been demonstrated (Zhu et al., 2010), there is a paucity of experimental realizations of elastic metamaterial designs and applications. For instance, there is a need in the construction industry for new approaches to sound insulation and for earthquake protection, vibration cloaking of sensitive components (perhaps for protecting electrodynamic metamaterial components), and mechanical sensors and actuators by controlling the vibrations of beams and plates. As our understanding of the capabilities of metamaterials improves, we can expect the opportunities for exciting new research and applications to multiply.

The book

This book deals mainly with theoretical aspects of metamaterials, periodic composites, and layered composites. The first chapter introduces the reader to elasticity, acoustics, and electrodynamics in media. Concepts such as an anisotropic, tensorial mass density, frequency-dependent material properties, and dissipation and constraints imposed by causality are introduced from the beginning. The second chapter deals with plane wave solutions to the wave equations that describe elastic, acoustic, and electromagnetic waves. Reflection and refraction at plane interfaces are explored for various situations and transmission through slabs is discussed. The third chapter deals with the plane wave expansion of sources and with scattering from curved interfaces, specifically spheres and cylinders. Multiple scattering is also explained in brief.

Electrodynamic metamaterials are covered in the fourth chapter. Particular emphasis is placed on homogenization of metamaterials as proposed by Pendry and co-workers and an effort is made to give some physical insight into metamaterials using the Drude model. Perfect lensing and negative refractions are also discussed.

The fifth chapter is on elastodynamic and acoustic metamaterials. Simple spring-mass models are used to motivate the possibility of negative and anisotropic mass density. The Milton-Willis theory of frequency-dependent mass is explained and a gyrocontinuum model with a frequency-dependent moment of inertia is discussed. Helmholtz resonator models are used to show that effective dynamic elastic moduli can be negative. The Willis equations, which appear to be a general descriptor of composites with microstructure, are discussed in detail and a spring-mass model that exhibits Willis behavior is examined. The last section of the chapter deals with extremal materials such as negative Poisson's ratio and pentamode materials.

Chapter 6 deals with transformation-based cloaking. The invariance of the conductivity equations and of Maxwell's equation under coordinate transformations is derived. Examples of transformations are discussed and aspects of acoustic cloaking are introduced. We then show that the equations of elastodynamics transform into the Willis equations under coordinate transformations. The special behavior of pentamode materials and their application to acoustic cloaking is also examined.

The next chapter deals with periodic composites. The Bloch-Floquet theorem is introduced and applied to the problems of finding the effective behavior of composites in the quasistatic limit. The quasistatic equations are found to be identical to the equations of elastostatics and electrostatics, but with complex parameters. The Hashin effective medium solution is used to show how results from elastostatics can be extended to dynamic problems using analytic continuation. Finally, Brillouin zones and band gaps in periodic structures are discussed in the context of lattice models of elastic structures.

The final chapter involves layered media. Wave propagation in smoothly varying layered media and approximate solutions to problems involving such media are examined. The propagator matrix is introduced and used to show that a periodic layered medium can exhibit anisotropic density. Quasistatic homogenization of laminates is explored and hierarchical laminates are shown to possess remarkable properties.

Many of the ideas in this book are yet to be realized experimentally and we hope that the readers of this book will explore some of these ideas and bring them to technological maturity.

Contents

1

Elastodynamics, Acoustics, and Electrodynamics

> We, however, working as we do to advance a single department of science, can devote but little of our time to the simultaneous study of other branches. As soon as we enter upon any investigation, all our powers have to be concentrated on a field of narrowed limit.
>
> HERMANN VON HELMHOLTZ, On the aim and progress of physical science, 1869.

Vibrations, sound, and light involve the propagation of waves. In the case of vibrations, these waves carry information about small changes in the shape of an elastic body. Vibrations are elastic waves. Our sense of touch can be used to track these elastic changes in shape as a function of time. Sound is the propagation of small changes in pressure through a fluid and our ears may be used to track these waves. Light is the propagation of small disturbances in electric and magnetic fields and we can use our eyes to track these changes. But for certain frequencies our senses are no longer adequate and specialized instruments are needed to sense vibrations, sound, and light.

Vibrations and seismic waves are special types of elastic waves. These waves are governed by the equations of elastodynamics. Acoustics deals with various types of sound waves. As you can guess, elastodynamics and acoustics are closely related. Light is a special type of electromagnetic wave and its propagation is described by Maxwell's equations of electrodynamics. Electromagnetic waves are quite different from elastic and acoustic waves in that they can travel through vacuum. But, remarkably, all three types of waves can be described by a similar set of equations and hence these disparate phenomena can be studied simultaneously.

The governing equations of linear elastodynamics, acoustics, and electrodynamics are the starting point of our study of waves in heterogeneous media and composites.* We will explore these governing equations and look at ways in which these equations

*Detailed derivations of the equations of linear elasticity can be found in Atkin and Fox (1980) and Slaughter (2002). Expositions on elastodynamics can be found in Achenbach (1973) and Harris (2001). An accessible introduction to the applications of elastodynamics in the study of vibrations can be found in Volterra and Zachmanoglou (1965). For seismic waves and problems related to seismology a good starting point is Aki and Richards (1980). Excellent introductions to acoustics can be found in Reynolds (1981) and Skudrzyk (1971), and numerous solved problems are given in Morse and Ingard (1986). The governing equations of electrodynamics are discussed in detail in Feynman et al. (1964) and Jackson (1999).

can be simplified. We will start with direct notation and then explore specialized coordinate systems. Fourier transforms will be used to convert the time-dependent equations into frequency-dependent form. We will discuss dissipation and the constraints placed by causality on the solutions of the governing equations. We will also identify the similarities between the equations of elastodynamics, acoustics, and electrodynamics. These similarities will allow us to use some of the same techniques to solve problems in these seemingly disparate fields.

1.1 A note about notation

The notation used in this book is based on Ogden (1997). Scalar quantities are denoted by italic letters (a) and Greek letters (ϕ). Vector quantities are represented with bold font lower-case letters ($\mathbf{v}$) and in some cases as upper-case bold letters ($\mathbf{E}$). Second-order tensor quantities are written with bold italic ($\boldsymbol{E}$) or with bold Greek ($\boldsymbol{\sigma}$). Third-order tensors are written with calligraphic fonts ($\mathcal{S}$) and fourth-order tensors are written in bold sans-serif fonts ($\mathsf{\mathbf{C}}$).

The scalar product of two vectors is indicated by $\mathbf{u}\cdot\mathbf{v}$, the vector product by $\mathbf{u}\times\mathbf{v}$, and the tensor product by $\mathbf{u}\otimes\mathbf{v}$. The tensor product is a second-order tensor. The action of a second-order tensor on a vector is represented by $\boldsymbol{S}\cdot\mathbf{v}$. The definition of the tensor product is

$$(\mathbf{u}\otimes\mathbf{v})\cdot\mathbf{w} = (\mathbf{v}\cdot\mathbf{w})\mathbf{u}\ .$$

The transpose of a second-order tensor is denoted by $\boldsymbol{S}^T$ and is defined via the relation

$$\mathbf{v}\cdot(\boldsymbol{S}^T\cdot\mathbf{u}) = \mathbf{u}\cdot(\boldsymbol{S}\cdot\mathbf{v})\ .$$

The trace of a second-order tensor is an invariant and can be defined with respect to an orthonormal basis $(\mathbf{e}_1,\mathbf{e}_2,\mathbf{e}_3)$ as

$$\text{tr}\boldsymbol{S} = S_{ii} = \mathbf{e}_i\cdot(\boldsymbol{S}\cdot\mathbf{e}_i)$$

where summation over repeated indices is implied. The determinant of a second-order tensor is likewise an invariant and can be expressed as

$$\det(\boldsymbol{S}) = e_{ijk}\, S_{i1}\, S_{j2}\, S_{k3}$$

where e_{ijk} is the permutation tensor which is defined as

$$e_{ijk} = \begin{cases} 1 & \text{for even permutations, i.e., } 123,\ 231,\ 312 \\ -1 & \text{for odd permutations, i.e., } 132,\ 321,\ 213 \\ 0 & \text{otherwise.} \end{cases}$$

The inner product of two second-order tensors is a second-order tensor given by $\boldsymbol{S}\cdot\boldsymbol{T}$, in index notation $S_{ij}T_{jk}$. If $\det\boldsymbol{S}\neq 0$ then there exists a unique tensor called the inverse of $\boldsymbol{S}$ and denoted by $\boldsymbol{S}^{-1}$ such that

$$\boldsymbol{S} \cdot \boldsymbol{S}^{-1} = \boldsymbol{S}^{-1} \cdot \boldsymbol{S} = \boldsymbol{1}$$

where $\boldsymbol{1}$ is the identity tensor that takes a second-order tensor to itself. The contraction of two indices between two tensors is represented as $\boldsymbol{S} : \boldsymbol{T}$ and $\mathbf{C} : \boldsymbol{S}$, in index notation, $S_{ij}T_{ij}$ and $C_{ijk\ell}S_{k\ell}$. The magnitude of a vector is $\|\mathbf{v}\| = \sqrt{\mathbf{v} \cdot \mathbf{v}}$ and the magnitude of a second-order tensor is $\|\boldsymbol{S}\| = \sqrt{\boldsymbol{S}^T : \boldsymbol{S}}$. We use the notations $\mathbf{v} \cdot \boldsymbol{S}$ and $\boldsymbol{S}^T \cdot \mathbf{v}$ interchangeably. For higher-order tensors, the quantity $\mathbf{v} \cdot \mathbf{C} \cdot \mathbf{u}$ is equivalent in index notation to $C_{ijk\ell}v_i u_\ell$.

The notation used for differential operators is similar to that used in vector calculus. However, the definitions are slightly different. The gradient of a tensor field of any order is defined by its action on a vector $\mathbf{a}$ as

$$[\boldsymbol{\nabla}\boldsymbol{S}(\mathbf{x})] \cdot \mathbf{a} = \frac{d}{d\alpha}\boldsymbol{S}(\mathbf{x} + \alpha\mathbf{a})\Big|_{\alpha=0} = (\mathbf{a} \cdot \boldsymbol{\nabla})\boldsymbol{S}(\mathbf{x}) \, .$$

The divergence of a vector field is defined as $\boldsymbol{\nabla} \cdot \mathbf{v} = \mathrm{tr}(\boldsymbol{\nabla}\mathbf{v})$ and the divergence of a second-order tensor field is defined as

$$[\boldsymbol{\nabla} \cdot \boldsymbol{S}(\mathbf{x})] \cdot \mathbf{a} = \boldsymbol{\nabla} \cdot [\boldsymbol{S}(\mathbf{x}) \cdot \mathbf{a}] \, .$$

The curl of a vector field is defined by

$$[\boldsymbol{\nabla} \times \mathbf{v}(\mathbf{x})] \cdot \mathbf{a} = \boldsymbol{\nabla} \cdot [\mathbf{v}(\mathbf{x}) \times \mathbf{a}] \, .$$

The Laplacian of a scalar field is denoted by ∇^2 and is defined as

$$\nabla^2\phi = \boldsymbol{\nabla} \cdot (\boldsymbol{\nabla}\phi) = (\boldsymbol{\nabla} \cdot \boldsymbol{\nabla})\phi \, .$$

The Laplacian of a vector field is defined as

$$\nabla^2\mathbf{v} = \boldsymbol{\nabla} \cdot (\boldsymbol{\nabla}\mathbf{v})^T = (\boldsymbol{\nabla} \cdot \boldsymbol{\nabla})\mathbf{v} \, .$$

We use the symbols $a := b$ and $b =: a$ to indicate that the quantity a is being defined to be equal to b.

1.2 Elastodynamics

Consider the body Ω_0 with boundary Γ_0 shown in Figure 1.1. Points in the body can be located using the position vector $\mathbf{x}$. Let Ω be a subpart of the body with boundary Γ. Let the unit vector $\mathbf{n}$ be the outward normal to the surface Γ. The region Ω can be in the interior of the body or share a part of the surface of the body. Let there be