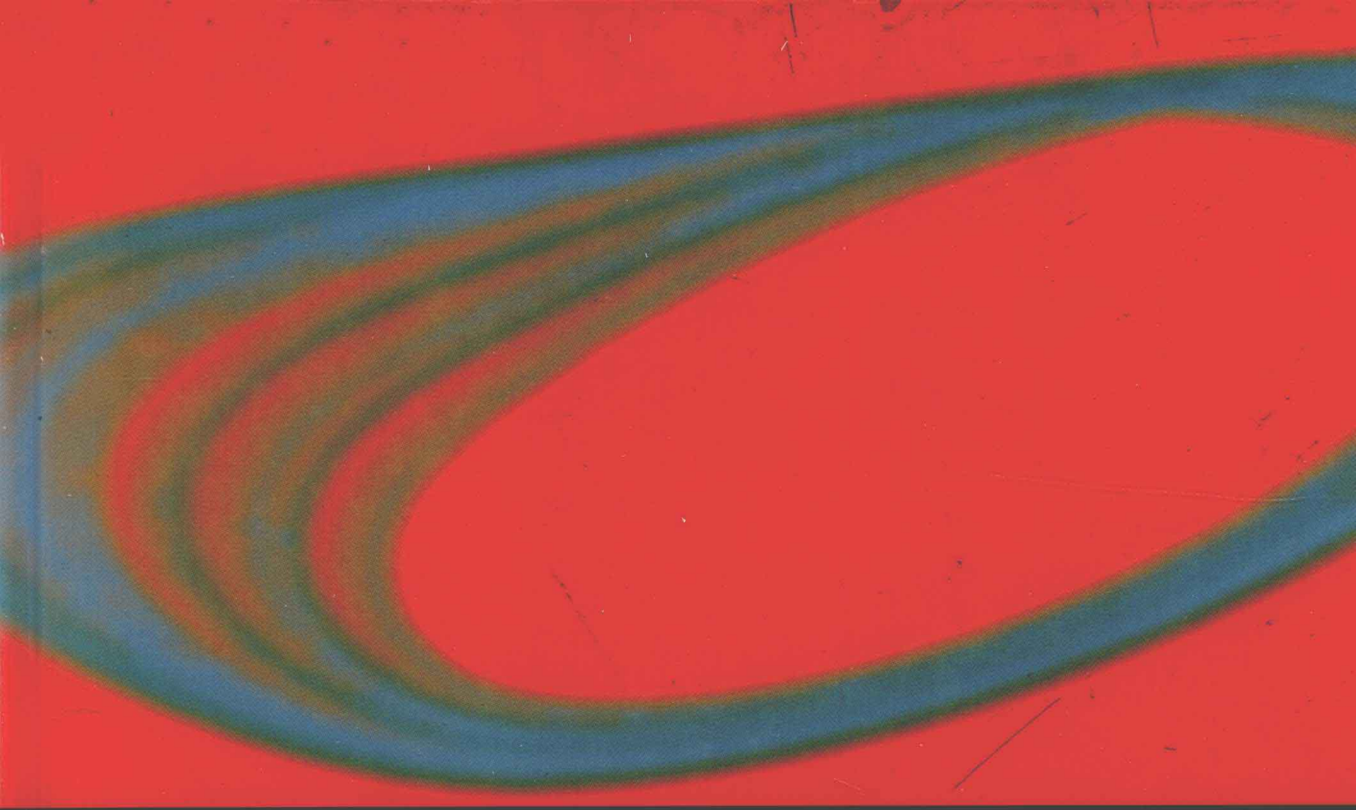

Controlling Chaos

Theoretical and Practical Methods in Non-linear Dynamics

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To my parents

Preface

More than two decades of intensive studies on nonlinear dynamics have posed the question on the practical applications of chaos. One of the possible answers is to control chaotic behavior in such a way as to make it predictable. Indeed, nowadays the idea of controlling chaos, which we try to explain in this book, is an appealing one.

This book is organized as follows. In Part I (Chapters 1–5) we describe basic methods of controlling chaos, while in Part II we reprint fundamental contributions to this field.

In Chapter 1 we give basic information about controlling procedures. Ideas of feedback and nonfeedback methods as well as of chaos synchronization are explained. Additionally, we describe Chua's circuit (a very simple electronic device) which will be used in several examples in the following chapters.

Chapter 2 describes feedback controlling methods in which unstable periodic orbits embedded in the chaotic attractor are stabilized. We discuss the Ott–Grebogi–Yorke and Pyragas methods and their connections with classical controlling methods. The method of controlling chaos by chaos, in which chaotic behavior can be modified by coupling via a feedback loop with another chaotic system, is also mentioned.

Nonfeedback methods are explained in Chapter 3. This approach is inevitably much less flexible than feedback methods, but in many practical systems it is easier to apply. We give methods in which chaos can be controlled through operating conditions or by system design. Additionally, taming chaos, entrainment and migration control procedures are discussed.

The synchronization chaos procedures of Chapter 4 allow two chaotic systems to have exactly the same response. The procedures of Pecora and Carroll and of continuous control are described. We also explain the idea of secure communication.

In Chapter 5 we discuss the problem of the selection and engineering implementation of the chaos controlling method for a particular practical problem.

Finally, in the references are listed the most important works on chaos controlling and synchronization.

Of the methods given in Chapters 2–5, it is mainly those which the author took a small part in developing that are described in detail. Other methods are only briefly described; full details of these can be found in the papers reprinted in Part II. The author has tried to select papers which, in his opinion, have had very significant impact on the development of the field.

This book is generally for those who have some introductory knowledge of nonlinear dynamics and who are interested in its potential applications. Knowledge of the classical control theory is not necessary to understand chaos controlling methods but could be of benefit to readers.

Finally, I would like to acknowledge the valuable comments of J. Brindley, C. Grebogi, L.O. Chua, M.S. El Naschie, L. Kocarev, V.S. Anishchenko, M. Ogorzalek and J. Wojewoda who have helped me in preparation of this work.

Tomasz Kapitaniak
Rosanów, 1996

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Contents

PART I:
General Outlook

1 Introduction

Chaos occurs widely in engineering and natural systems; historically it has usually been regarded as a nuisance and is designed out if possible. It has been noted only as irregular or unpredictable behavior, often attributed to random external influences. More recently, there have been examples of the potential usefulness of chaotic behavior, and we describe some of its potential usefulness in this book.

In Chapters 2 and 3 we review a number of methods by which undesirable chaotic behavior may be controlled or eliminated. More speculatively, we indicate ways in which the existence of chaotic behavior may be directly beneficial or exploitable.

We can divide chaos controlling approaches into two broad categories: firstly those in which the actual trajectory in the phase space of the system is monitored and some *feedback* process is employed to maintain the trajectory in the desired mode, and secondly *nonfeedback* methods in which some other property or knowledge of the system is used to modify or exploit chaotic behavior. Feedback methods do not change the controlled systems and stabilize unstable periodic orbits on strange chaotic attractors, while nonfeedback methods slightly change the controlled system, mainly by a small permanent shift of control parameter, changing the system behavior from chaotic attractor to periodic orbit which is close to the initial attractor. The main idea of both methods is illustrated in *Figure 1.1*.

We describe several methods by which chaotic behavior in a dynamical system may be modified, displaced in parameter space or removed. The Ott–Grebogi–Yorke (OGY) method (Ott *et al.*, 1990 – *Paper 1*) is extremely general, relying only on the universal property of chaotic attractors, namely that they have embedded within them infinitely many unstable periodic orbits (or even static equilibria). On the other hand, the method requires following the trajectory and employing a feedback control system which must be highly flexible and responsive; such a system in some experimental configurations may be large and expensive. It has the additional disadvantage that small amounts of noise may cause occasional large departures from the desired operating trajectory.

The nonfeedback approach is inevitably much less flexible, and requires more prior knowledge of equations of motion. On the other hand, to apply such a method, we do not have to follow the trajectory. The control procedures can be applied at any time and we can switch from one periodic orbit to another without returning to the chaotic behavior, although after each switch, transient chaos may be observed. The lifetime of this transient chaos strongly

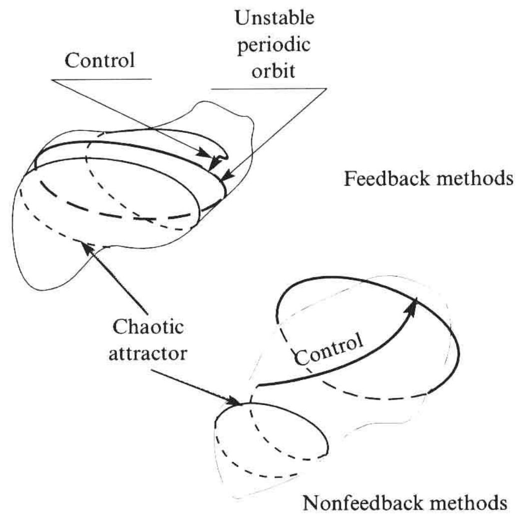

Figure 1.1 General idea of feedback and nonfeedback controlling methods.

depends on initial conditions. Moreover, in a nonfeedback method we do not have to wait until the trajectory is close to an appropriate unstable orbit; in some cases this time can be quite long. The dynamic approach can be very useful in mechanical systems, where feedback controllers are often very large (sometimes larger than the control system). In contrast, a dynamical absorber having a mass of order 1% of that of the control system is able, as we will show in the example of Chapter 3, to convert chaotic behavior to periodic over a substantial region of parameter space. Indeed, the simplicity by which chaotic behavior may be changed in this way, and the possibility of an easy access to different periodic orbits, may actually motivate the search for, and exploitation of, chaotic behavior in practical systems. This prompts us to pose a final question – how can we exploit chaos in real systems? The OGY method, at least in theory, gives access to the wide range of possible behavior encompassed by the unstable periodic (and other) orbits embedded in a chaotic attractor. Moreover, the sensitivity of the chaotic regime to both initial conditions and parameter values means that the desired effects may be produced by fine tuning. Thus, we may actually wish to design chaos into a system, in order to exploit this adaptability. Nonfeedback methods can, in principle, give us advice on the design, whether we wish to design chaos out or in. Additionally, they enable us to choose regions of design parameter space or operating parameter space within which chaos will occur and will be acceptable. An example of practical use might be the minimalization of metal fatigue by switching from a necessary strictly periodic operation of the fully loaded conditions, where repeated stresses are applied at certain places, to a noisy periodicity (rather like a healthy heartbeat) under idling conditions.

The essential property of a chaotic trajectory is that it is not asymptotically stable. Closely correlated initial conditions have trajectories which quickly become uncorrelated. Despite this obvious disadvantage, it has been established that control leading to the synchronization of two chaotic systems is possible. In Chapter 4 we describe basic synchronization procedures and discuss its potential application to secure communications.

Methods described in Chapters 2–4 are illustrated by the example of the controlling chaos in Chua's circuit (Chua *et al.*, 1986; Chua, 1993) shown in *Figure 1.2*. Chua's circuit contains three linear energy-storage elements (an inductor and two capacitors), a linear resistor, and a single nonlinear resistor N_R, namely Chua's diode with a three-segment piecewise-linear v–i characteristic defined by

$$f(v_{C_1}) = m_0 v_{C_1} + \tfrac{1}{2}(m_1 - m_0)(|v_{C_1} + 1| - |v_{C_1} - 1|) \qquad (1.1)$$

where the slopes in the inner and outer regions are m_o and m_1 respectively (*Figure 1.3*).

In this case the state equations for dynamics of our scheme of *Figure 1.2* are as follows:

$$\begin{aligned} C_1 \frac{dv_{C_1}}{dt} &= G(v_{C_2} - v_{C_1}) - f(v_{C_1}) \\ C_2 \frac{dv_{C_2}}{dt} &= G(v_{C_1} - v_{C_2}) + i_L \\ L \frac{di_L}{dt} &= v_{C_2} \end{aligned} \qquad (1.2)$$

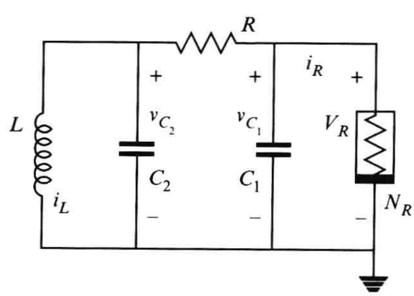

Figure 1.2 Chua's circuit.

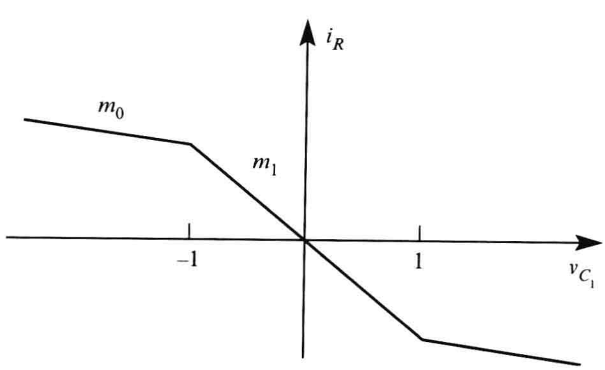

Figure 1.3 i_R–v_{C_1} characteristic of non-linear resistor.

where $G = 1/R$.

Introducing new variables, $x = v_{C_1}$, $y = v_{C_2}$, $z = i_L/G$, $\alpha = C_2/C_1$ and $\beta = C_2/LG^2$, we can rewrite *Equation (1.2)* in dimensionless form:

$$\begin{aligned} \dot{x} &= \alpha[y - x - f(x)] \\ \dot{y} &= x - y + z \\ \dot{z} &= -\beta y \end{aligned} \tag{1.3}$$

It is well-known that for $R = 1.64$ kΩ, $C_1 = 10$ nF, $C_2 = 99.34$ nF, $m_1 = -0.76$ mS, $m_0 = 0.41$ mS and $L = 18.46$ mH, Chua's circuits operate on the chaotic double-scroll Chua's attractor shown in *Figure 1.4*.

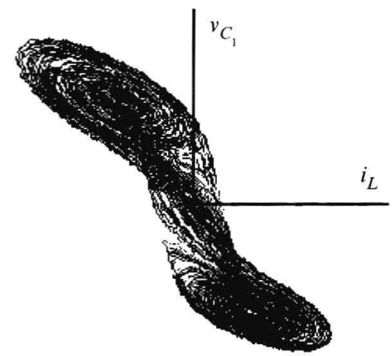

Figure 1.4 Double scroll attractor.

The chaotic dynamics of Chua's circuit have been widely investigated (e.g. Madan, 1993). One of the main advantages of this system is the very good accuracy between numerical simulations of *Equation (1.3)* and experiments on real electronic devices. Experiments with this circuit are very easy to perform, even for nonspecialists.

The problem of controlling chaos in engineering systems is discussed in Chapter 5. Some basic modifications of controlling procedures which allow their direct practical implementation are described.

In Part II we have reprinted a selection of important contributions to the problem of controlling and synchronization of chaotic systems.

2 Controlling chaos through feedback

2.1 Ott–Grebogi–Yorke method

Ott, Grebogi and Yorke (Ott *et al.*, 1990 – *Paper 1*; Romeiras *et al.*, 1992 – *Paper 2*) have, in an important series of papers, proposed and developed a method by which chaos can always be suppressed by shadowing one of the infinitely many unstable periodic orbits (or perhaps steady states) embedded in the chaotic attractor.

Basic assumptions of this method are as follows.

(a) The dynamics of the system can be described by an n-dimensional map of the form.

$$\zeta_{n+1} = f(\zeta_n, p) \tag{2.1}$$

This map, in the case of continuous-time systems, can be constructed, e.g. by introducing a transversal surface of section for system trajectories (Poincaré map).

(b) p is some accessible system parameter which can be changed in some small neighborhood of its nominal value p^*.

(c) For this value p^* there is a periodic orbit within the attractor around which we would like to stabilize the system.

(d) The position of this orbit changes smoothly with changes in p, and there are small changes in the local system behavior for small variations of p.

Let ζ_F be a chosen fixed point of the map f of the system existing for the parameter value p^*. In the close vicinity of this fixed point with good accuracy we can assume that the dynamics are linear and can be expressed approximately by

$$\zeta_{n+1} - \zeta_F = \boldsymbol{M}(\zeta_n - \zeta_F) \tag{2.2}$$

The elements of the matrix $\boldsymbol{M}$ can be calculated using the measured chaotic time series and analyzing its behavior in the neighborhood of the fixed point. Further, the eigenvalues λ_s, λ_u and eigenvectors e_s, e_u of this matrix can be found. These eigenvectors determine the stable and unstable directions in the small neighborhood of the fixed point.

Denoting by f_s, f_u the contravariant eigenvectors ($f_s e_s = f_u e_u = 1$, $f_s e_u = f_u e_s = 0$) we can find the linear approximation valid for small $|p_n - p^*|$:

$$\zeta_{n+1} = p_n g + (\lambda_n e_n f_n + \lambda_s e_s f_s)(\zeta_n - p_n g) \tag{2.3}$$

where

$$g = \frac{\partial \zeta_F(p)}{\partial p}\bigg|_{p+p^*}$$

Because ζ_{n+1} should fall on the stable manifold of ζ_F, choose p_n such that $f_u \zeta_{n+1} = 0$:

$$p_n = \frac{\lambda_n \zeta_n f_u}{(\lambda_u - 1) g f_u} \tag{2.4}$$

The OGY algorithm is schematically explained in *Figure 2.1*, and its main properties are as follows.

(a) No model of dynamics is required. One can use either full information from the process or a delay coordinate embedding technique using single variable experimental time series. An extremely interesting development in this direction has been described by Dressler and Nitsche (1992 – *Paper 3*).
(b) Any accessible variable (controllable) system parameter can be used as the control parameter.
(c) In the absence of noise and error, the amplitude of applied control signal must be large enough (exceed a threshold) to achieve control.
(d) Inevitable noise can destabilize the controlled orbit, resulting in occasional chaotic bursts.
(e) Before settling into the desired periodic mode, the trajectory exhibits chaotic transients, the length of which depends on the actual starting point.

In Ogorzalek (1993b) the OGY method has been applied to control chaos in Chua's circuit (*Figure 1.2, Equation (1.2)*). Using a specific software package (Dabrowski *et al.*, 1992), unstable periodic orbits embedded in the

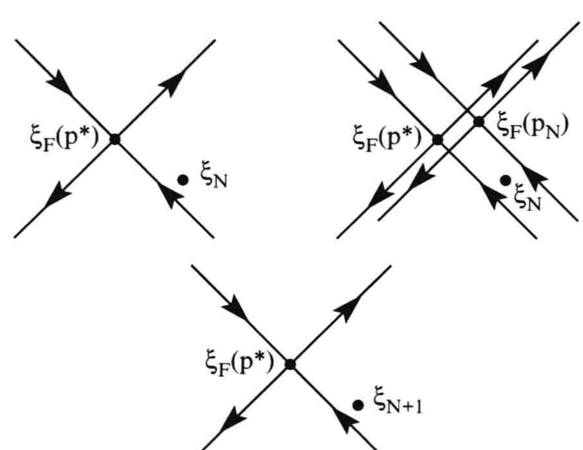

Figure 2.1 Idea of Ott–Grebogi–Yorke method.