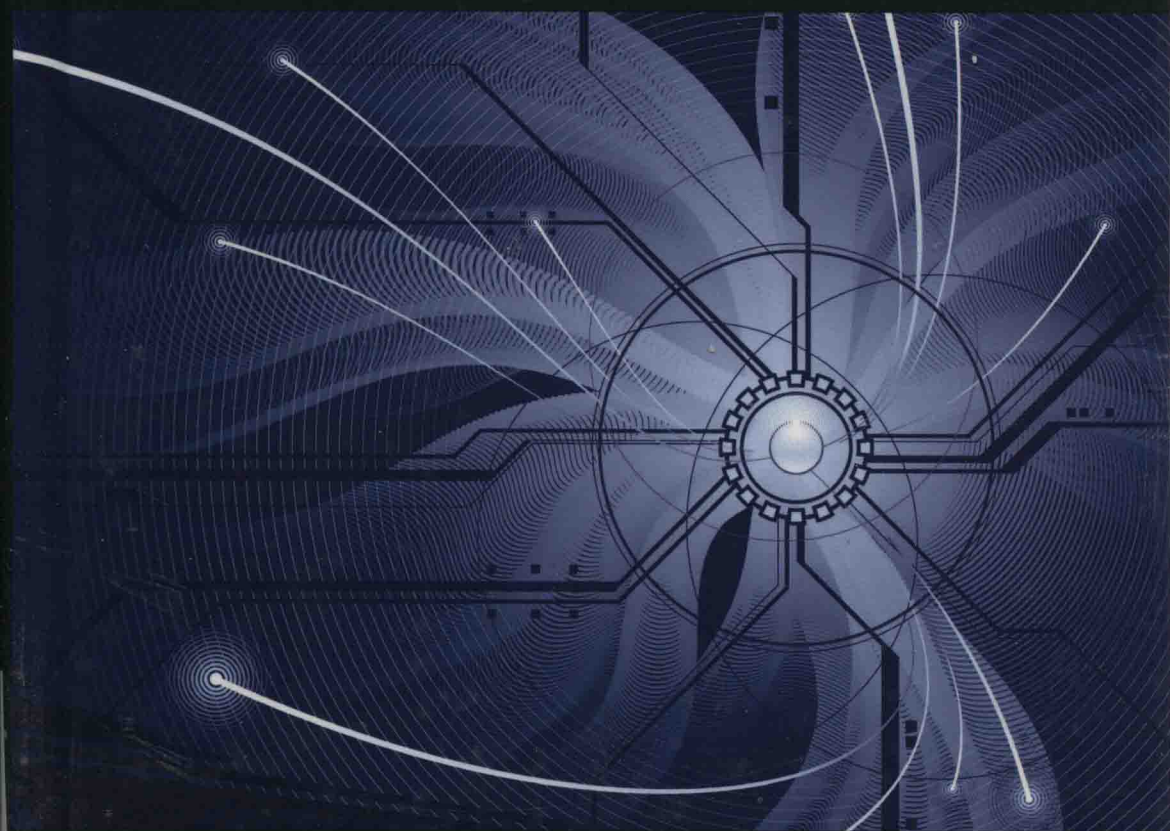


Third Edition

# Analysis of Turbulent Flows with Computer Programs



Tuncer Cebeci

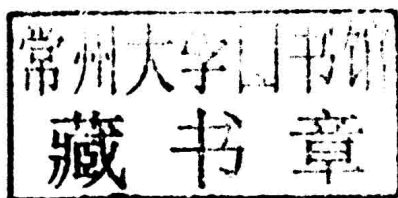


# Analysis of Turbulent Flows with Computer Programs

THIRD EDITION

Tuncer Cebeci

*Formerly Distinguished Technical Fellow,  
Boeing Company, Long Beach, California  
[horizonpublishing.net](http://horizonpublishing.net)*



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# Dedication

This book is dedicated to the memory of my beloved wife, Sylvia Holt Cebeci, my best friend of many years. She will always be with me, in my heart, in my memories and in all the loving ways she touched my life each day.

I will always remember her note to me on my birthday in 2011 prior to her death on May 4, 2012:

Dear TC,

The twilight times bring you automatic admission to a special club; to share the game of special 50's, 60's music; remembering your friends and death of a special friend; being a historian simply because you live long enough. You and I have been so fortunate to share a golden life everyday.

Love, Sylvia



# Preface to the Third Edition

The first edition of this book, *Analysis of Turbulent Boundary Layers*, was written in the period between 1970 and early 1974 when the subject of turbulence was in its early stages and that of turbulence modeling in its infancy. The subject had advanced considerably over the years with greater emphasis on the use of numerical methods and an increasing requirement and ability to calculate turbulent two- and three-dimensional flows with and without separation. The tools for experimentation were still the traditional Pitot tube and hot wire-anemometer so that the range of flows that could be examined was limited and computational methods still included integral methods and a small range of procedures based on the numerical solution of boundary layer equations and designed to match the limited range of measured conditions. There have been tremendous advances in experimental techniques with the development of non-intrusive optical methods such as laser-Doppler, phase-Doppler and particle-image velocimetry, all for the measurement of velocity and related quantities and of a wide range of methods for the measurement of scalars. These advances have allowed an equivalent expansion in the range of flows that have been investigated and also in the way in which they could be examined and interpreted. Similarly, the use of numerical methods to solve time-averaged forms of the Navier-Stokes equations, sometimes interactively with the inviscid-flow equations, has expanded, even more so with the rise and sometimes fall of Companies that wished to promote and sell particular computer codes. The result of these developments has been an enormous expansion of the literature and has provided a great deal of information beyond that which was available when the first edition was written. Thus, the topics of the first edition needed to be re-examined in the light of new experiments and calculations, and the ability of calculation methods to predict a wide range of practical flows, including those with separation, to be reassessed.

The second edition, entitled *Analysis of Turbulent Flows*, undertook the necessary reappraisal, reformulation and expansion, and evaluated the calculation methods more extensively but also within the limitations of two-dimensional equations largely because this made explanations easier and the book of acceptable size. In addition, it was written to meet the needs of graduate students as well as engineers and so included homework problems that were more sensibly formulated within the constraints of two independent variables. References to more complex flows, and particularly those with separation, were provided and the relative merits of various turbulence models considered.

The third edition, entitled *Analysis of Turbulent Flows with Computer Programs*, keeps the structure of the first and second editions the same. It expands the solution of the boundary-layer equations with transport-equation turbulence models, considers the solution of the boundary-layer equations with flow separation and provides computer programs for calculating attached and separating flows with several turbulence models.

The second edition and the contents of this new edition should be viewed in the context of new developments such as those associated with large-eddy simulations (LES) and direct numerical solutions (DNS) of the Navier-Stokes equations. LES existed in 1976 as part of the effort to represent meteorological flows and has been rediscovered recently as part of the recognition of the approximate nature of solutions of time-averaged equations as considered here. There is no doubt that LES has a place in the spectrum of methods applied to the prediction of turbulent flows but we should not expect a panacea since it too involves approximations within the numerical method, the filter between time-dependent and time-average solutions and small-scale modeling. DNS approach also has imperfections and mainly associated with the computational expense which implies compromises between accuracy and complexity or, more usually, restriction to simple boundary conditions and low Reynolds numbers. It is likely that practical aerodynamic calculations with and without separation will continue to make use of solutions of the inviscid-flow equations and some reduced forms of the Navier-Stokes equations for many years, and this book is aimed mainly at this approach.

The first and second editions were written with help from many colleagues. AMO Smith was an enthusiastic catalyst and ideas were discussed with him over the years. Many colleagues and friends from Boeing, the former Douglas Aircraft Company and the McDonnell-Douglas Company, have contributed by discussion and advice and included K. C. Chang and J. P. Shao. Similarly, Peter Bradshaw, the late Herb Keller of Cal Tech and the late Jim Whitelaw of Imperial College have helped in countless ways.

Indian Wells

*Tuncer Cebeci*

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- 1. Integral Methods.**
- 2. Differential Method with CS Model for two-dimensional flows with and without heat transfer and infinite swept-wing flows.**
- 3. Hess-Smith Panel Method with and without viscous effects.**
- 4. Zonal Method for  $k$ - $\epsilon$  Model and solution of  $k$ - $\epsilon$  Model equations with and without wall functions.**
- 5. Differential Method for SA Model and for a Plane Jet.**
- 6. Differential Method for inverse and interactive boundary-layer flows with CS Model.**





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# Chapter 1

## Introduction

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### 1.1 Introductory Remarks

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Turbulence in viscous flows is described by the Navier–Stokes equations, perfected by Stokes in 1845, and now soluble by Direct Numerical Simulation (DNS). However, computing capacity restricts solutions to simple boundary conditions and



## 2 Analysis of Turbulent Flows with Computer Programs

moderate Reynolds numbers and calculations for complex geometries are very costly. Thus, there is need for simplified, and therefore approximate, calculations for most engineering problems. It is instructive to go back some eighty years to remarks made by Prandtl [1] who began an important lecture as follows:

*What I am about to say on the phenomena of turbulent flows is still far from conclusive. It concerns, rather, the first steps in a new path which I hope will be followed by many others.*

*The researches on the problem of turbulence which have been carried on at Göttingen for about five years have unfortunately left the hope of a thorough understanding of turbulent flow very small. The photographs and kine-to-graphic pictures have shown us only how hopelessly complicated this flow is ...*

Prandtl spoke at a time when numerical calculations made use of primitive devices – slide rules and mechanical desk calculators. We are no longer “hopeless” because DNS provides us with complete details of simple turbulent flows, while experiments have advanced with the help of new techniques including non-obtrusive laser-Doppler and particle-image velocimetry. Also, developments in large-eddy simulation (LES) are also likely to be helpful although this method also involves approximations, both in the filter separating the large (low-wave-number) eddies and the small ‘sub-grid-scale’ eddies, and in the semi-empirical models for the latter.

Even LES is currently too expensive for routine use in engineering, and a common procedure is to adopt the decomposition first introduced by Reynolds for incompressible flows in which the turbulent motion is assumed to comprise the sum of mean (usually time-averaged) and fluctuating parts, the latter covering the whole range of eddy sizes. When introduced into the Navier–Stokes equations in terms of dependent variables the time-averaged equations provide a basis for assumptions for turbulent diffusion terms and, therefore, for attacking mean-flow problems. The resulting equations and their reduced forms contain additional terms, known as the Reynolds stresses and representing turbulent diffusion, so that there are more unknowns than equations. A similar situation arises in transfer of heat and other scalar quantities. In order to proceed further, additional equations for these unknown quantities, or assumptions about the relationship between the unknown quantities and the mean-flow variables, are required. This is referred to as the “closure” problem of turbulence modeling.

The subject of turbulence modeling has advanced considerably in the last seventy years, corresponding roughly to the increasing availability of powerful digital computers. The process started with ‘algebraic’ formulations (for example, algebraic formulas for eddy viscosity) and progressed towards methods in which partial differential equations for the transport of turbulence quantities (eddy viscosity, or the Reynolds stresses themselves) are solved simultaneously with reduced forms of the