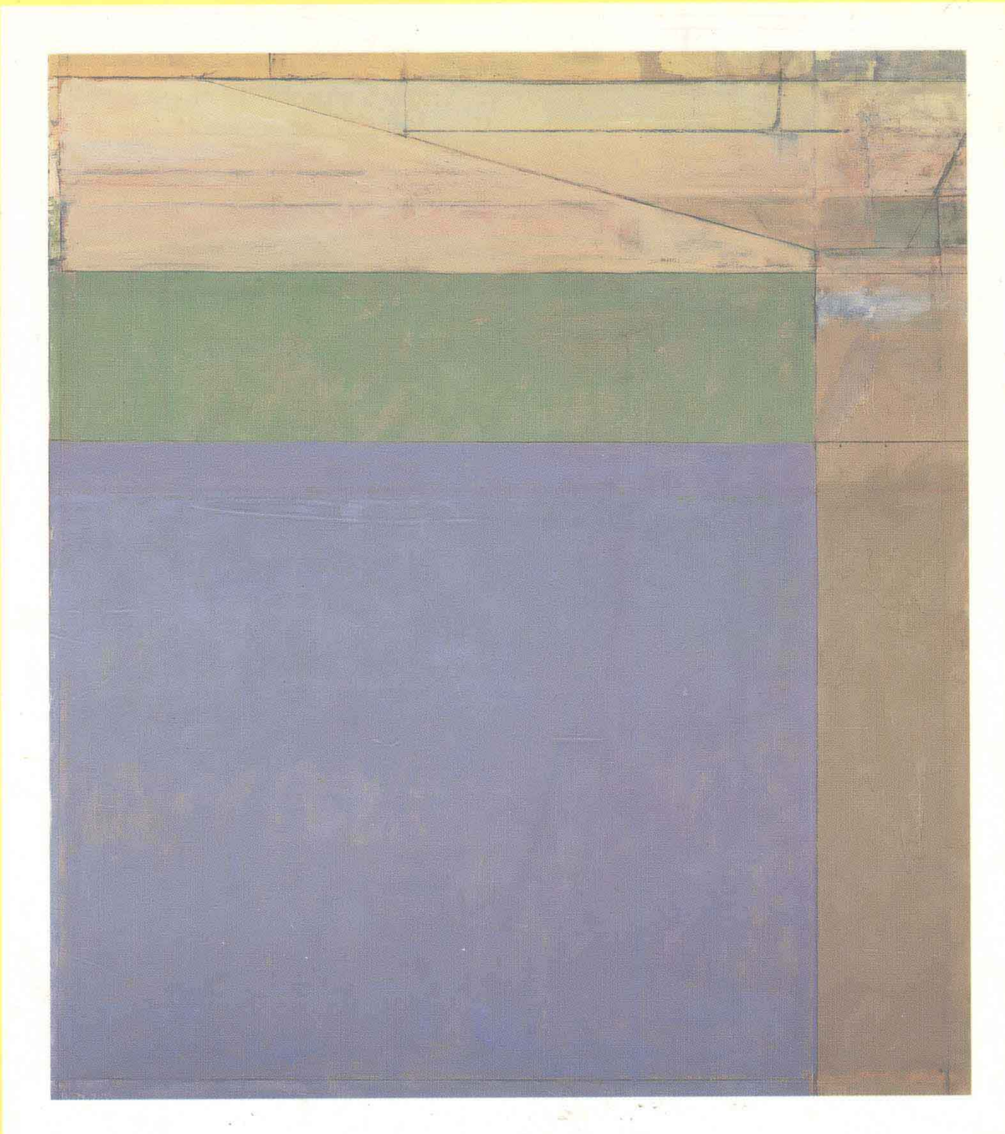
Elementary and Intermediate Algebra

A Combined Course

second edition



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Elementary and Intermediate Algebra

A Combined Course

Second Edition

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Preface

The primary goals of *Elementary and Intermediate Algebra: A Combined Course,* Second Edition, are to encourage students to develop their proficiency in algebra and to show how algebra is a modern modeling language for real-life problems.

Coverage This text was designed to be flexible with respect to the order of coverage of core algebra topics, adapting easily to a wide variety of course syllabi and teaching styles. It begins with Prerequisites, a review chapter. All or part of this material may be covered or it can be omitted. Graphing is first introduced in Chapter 4. The use of graphs encourages visualization to offer an opportunity for more conceptual understanding, strengthens graph-reading skills, and supports a smoother transition from concrete visual ideas to more abstract mathematics. Throughout the text, attention is given to geometry, collecting and interpreting data and statistics, and creating models, as well as to the NCTM Standards and Addenda and the AMATYC Guidelines.

Problem Solving A general problem-solving process for applied problems is stressed throughout the text: form a verbal model, label terms, create a mathematical model, solve, and check the answer in the original statement of the problem (see page 169). This problem-solving process helps students understand the problem, organize their work, and develop facility with verbal, analytical, graphical, and numerical approaches to problem solving. Students are also reminded of specific problem-solving strategies (see page 80) that are reinforced throughout the text in the exercises (see Exercises 81–82 on page 120 and Exercise 125 on page 843).

Exercises The comprehensive exercise sets offer students ample opportunity to practice algebraic techniques (see pages 160–161 and 579–581) and develop their conceptual and critical-thinking skills (see Exercise 45 on page 281, Exercises 75–82 on page 352, Exercise 93 on page 363, Exercises 89 and 90 on page 486, Exercise 41 on page 653, and Exercise 104 on page 765). The broad range of computational, conceptual, and applied problems in each exercise set is carefully graded to provide a smooth transition from routine to more challenging problems. Section and review exercises—as well as mid-chapter quizzes (see page 244) and chapter tests (see page 446)—consistently encourage student mastery of algebraic skills and concepts through practice and self-assessment.

Group Activities Each section ends with a Group Activity. This exercise reinforces students' understanding by exploring mathematical concepts in a variety of ways: You Be the Instructor, Extending the Concept, Problem Solving, Exploring with Technology, and Communicating Mathematically. Some Group Activities encourage interpretation or discovery of mathematical concepts and results (see pages 159, 213, and 760); some provide opportunities for problem posing and error analysis (see pages 22, 324, 594, and 772); and others reinforce methods of interpreting and constructing mathematical models, tables, and graphs (see pages 240, 634, 651, and 794). Designed to be completed in class or as homework assignments, the

Group Activities give students the opportunity to work cooperatively as they think, talk, and write about mathematics.

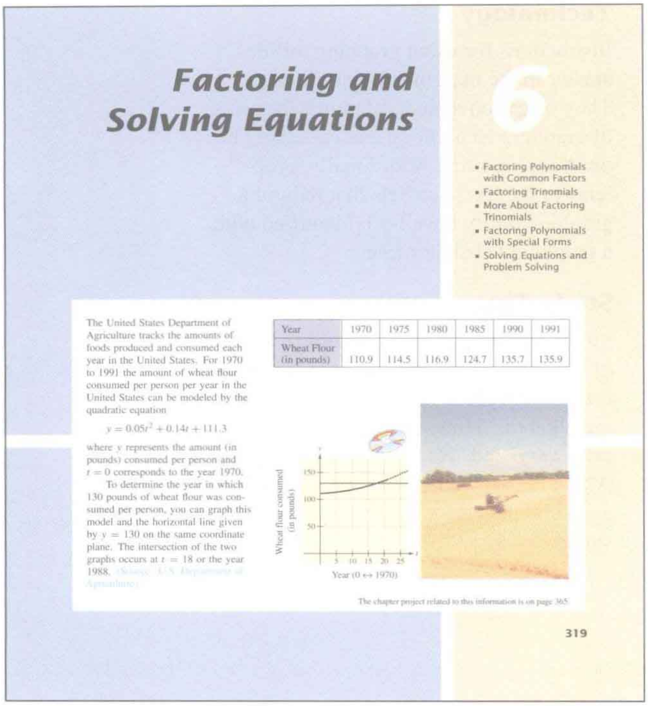
Technology Recognizing that graphing technology is becoming increasingly available, the text offers the opportunity to use graphing utilities throughout, but without requiring their use. This is achieved through a combination of features, including—at point of use—discovery opportunities that require scientific or graphing calculators (see pages 306 and 770), graphing utility instructions (see pages 412 and 575), and clearly labeled exercises that require the use of a graphing utility (see Exercise 48 on page 405 and Exercises 67–78 on pages 697 and 698).

Data Analysis/Modeling Throughout the text, students are offered opportunities to collect and interpret data, make conjectures, and construct mathematical models. Students are exposed to combining mathematical models to make related models (see Exercise 39 on page 455); encouraged to use mathematical models to make predictions and estimates from real data (see Exercise 78 on page 234, Exercise 43 on page 504, and Exercise 95 on page 638); invited to compare two or more models or compare actual data with a model (see Exercise 103 on page 765); and asked to use curve-fitting techniques to write their own models from data (see Exercise 98 on page 407, Exercises 37–39 on page 732, and Exercise 41 on page 745). Students are encouraged to use charts, tables, scatter plots, and graphs to summarize and interpret data.

Applications To emphasize for students the connection between mathematical concepts and real-world situations, up-to-date, real-life applications are integrated throughout the text. Appearing as examples (see pages 203, 220, 693, and 794), exercises (see Exercise 56 on page 223, Exercise 98 on page 440, Exercise 41 on page 707, and Exercise 116 on page 806), group activities (see page 717), and projects (see page 748), these applications help students validate the material they are learning and offer them frequent opportunities to use and review their problem-solving skills. A wide range of disciplines is represented by the applications—including such areas as physics, chemistry, electronics, the social sciences, biology, and business—as well as the career interviews, covering areas such as insurance, real estate, architecture, engineering, graphic arts, business, education, scuba diving, biochemistry, and economics.

Connections In addition to highlighting the connections between algebra and areas outside mathematics through real-world applications, this text also emphasizes the connections between algebra and other branches of mathematics, such as probability (see page 465), geometry (see page 134), logic (see Appendix A), and statistics (see Appendix B). Too, many examples and exercises throughout the text reinforce the connections among graphical, numerical, and algebraic representations of important algebraic concepts (see Exercises 27 and 28 on page 606).

There are many other new features of *Elementary and Intermediate Algebra: A Combined Course,* Second Edition, as well, including Discovery, Chapter Opening Applications, Study Tips, Historical Notes, Mid-Chapter Quizzes, Chapter Summaries, Career Interviews, and Chapter Projects. These and other features of the Second Edition are described in greater detail on the following pages.



Factoring and Solving Equations

6

- Factoring Polynomials with Common Factors
- Factoring Trinomials
- More About Factoring Trinomials
- Factoring Polynomials with Special Forms
- Solving Equations and Problem Solving

The United States Department of Agriculture tracks the amounts of foods produced and consumed each year in the United States. For 1970 to 1991 the amount of wheat flour consumed per person per year in the United States can be modeled by the quadratic equation

$$y = 0.05t^2 + 0.14t + 111.3$$

where y represents the amount (in pounds) consumed per person and $t = 0$ corresponds to the year 1970.

To determine the year in which 130 pounds of wheat flour was consumed per person, you can graph this model and the horizontal line given by $y = 130$ on the same coordinate plane. The intersection of the two graphs occurs at $t = 18$ or the year 1988. (Source: U.S. Department of Agriculture)

Year	1970	1975	1980	1985	1990	1991
Wheat Flour (in pounds)	110.9	114.5	116.9	124.7	135.7	135.9

The chapter project related to this information is on page 365.

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Chapter Opener

Each chapter opens with a look at a real-life application that is explored in depth in the Chapter Project at the end of the chapter. Real data is shown using graphical, numerical, and algebraic techniques. In addition, a list of the section titles shows students how the topics fit into the overall development of algebra.

Section Outline

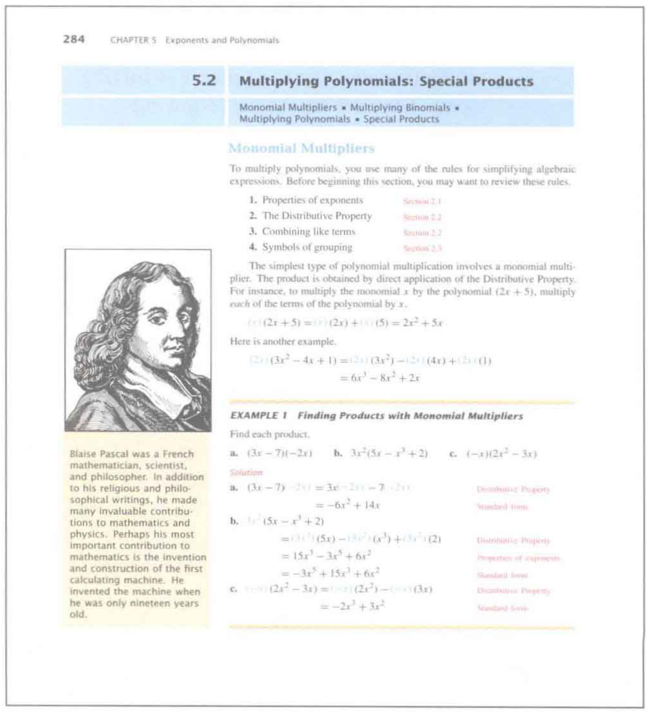
Each section begins with a list of the major topics covered in that section. These topics are also the subsection titles and can be used for easy reference and review by students.

Historical Notes

To help students understand that algebra has a past, historical notes featuring mathematical artifacts or mathematicians and their work are included in each chapter.

Notes

Notes anticipate students' needs by offering additional insight, pointing out common errors, and describing generalizations.



284 CHAPTER 5 Exponents and Polynomials

5.2 Multiplying Polynomials: Special Products

Monomial Multipliers • Multiplying Binomials • Multiplying Polynomials • Special Products

Monomial Multipliers

To multiply polynomials, you use many of the rules for simplifying algebraic expressions. Before beginning this section, you may want to review these rules.

1. Properties of exponents — Section 2.1
2. The Distributive Property — Section 2.2
3. Combining like terms — Section 2.2
4. Symbols of grouping — Section 2.3

The simplest type of polynomial multiplication involves a monomial multiplier. The product is obtained by direct application of the Distributive Property. For instance, to multiply the monomial x by the polynomial $(2x + 5)$, multiply *each* of the terms of the polynomial by x.

$$(x)(2x + 5) = (x)(2x) + (x)(5) = 2x^2 + 5x$$

Here is another example.

$$(2x)(3x^2 - 4x + 1) = (2x)(3x^2) - (2x)(4x) + (2x)(1) = 6x^3 - 8x^2 + 2x$$

Blaise Pascal was a French mathematician, scientist, and philosopher. In addition to his religious and philosophical writings, he made many invaluable contributions to mathematics and physics. Perhaps his most important contribution to mathematics is the invention and construction of the first calculating machine. He invented the machine when he was only nineteen years old.

EXAMPLE 1 Finding Products with Monomial Multipliers

Find each product.

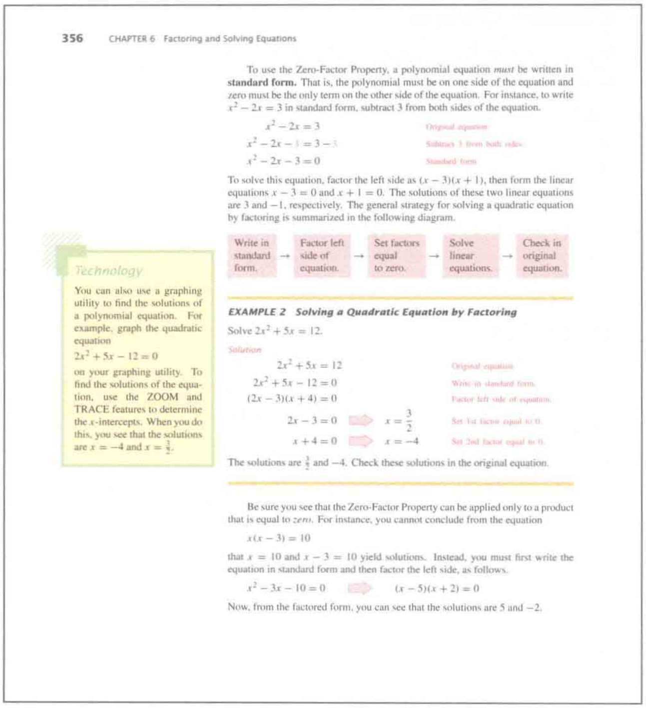
a. $(3x - 7)(-2x)$ **b.** $3x^2(5x - x^3 + 2)$ **c.** $(-x)(2x^2 - 3x)$

Solution

a. $(3x - 7)(-2x) = 3x(-2x) - 7(-2x)$ — Distributive Property
$= -6x^2 + 14x$ — Standard form

b. $3x^2(5x - x^3 + 2)$
$= (3x^2)(5x) - (3x^2)(x^3) + (3x^2)(2)$ — Distributive Property
$= 15x^3 - 3x^5 + 6x^2$ — Properties of exponents
$= -3x^5 + 15x^3 + 6x^2$ — Standard form

c. $(-x)(2x^2 - 3x) = (-x)(2x^2) - (-x)(3x)$ — Distributive Property
$= -2x^3 + 3x^2$ — Standard form



356 CHAPTER 6 Factoring and Solving Equations

To use the Zero-Factor Property, a polynomial equation *must* be written in **standard form.** That is, the polynomial must be on one side of the equation and zero must be the only term on the other side of the equation. For instance, to write $x^2 - 2x = 3$ in standard form, subtract 3 from both sides of the equation.

$$x^2 - 2x = 3 \quad \text{Original equation}$$
$$x^2 - 2x - 3 = 3 - 3 \quad \text{Subtract 3 from both sides}$$
$$x^2 - 2x - 3 = 0 \quad \text{Standard form}$$

To solve this equation, factor the left side as $(x - 3)(x + 1)$, then form the linear equations $x - 3 = 0$ and $x + 1 = 0$. The solutions of these two linear equations are 3 and -1, respectively. The general strategy for solving a quadratic equation by factoring is summarized in the following diagram.

Write in standard form. → Factor left side of equation. → Set factors equal to zero. → Solve linear equations. → Check in original equation.

Technology

You can also use a graphing utility to find the solutions of a polynomial equation. For example, graph the quadratic equation

$2x^2 + 5x - 12 = 0$

on your graphing utility. To find the solutions of the equation, use the ZOOM and TRACE features to determine the x-intercepts. When you do this, you see that the solutions are $x = -4$ and $x = \frac{3}{2}$.

EXAMPLE 2 Solving a Quadratic Equation by Factoring

Solve $2x^2 + 5x = 12$.

Solution

$$2x^2 + 5x = 12 \quad \text{Original equation}$$
$$2x^2 + 5x - 12 = 0 \quad \text{Write in standard form.}$$
$$(2x - 3)(x + 4) = 0 \quad \text{Factor left side of equation.}$$
$$2x - 3 = 0 \Rightarrow x = \frac{3}{2} \quad \text{Set 1st factor equal to 0.}$$
$$x + 4 = 0 \Rightarrow x = -4 \quad \text{Set 2nd factor equal to 0.}$$

The solutions are $\frac{3}{2}$ and -4. Check these solutions in the original equation.

Be sure you see that the Zero-Factor Property can be applied only to a product that is equal to *zero*. For instance, you cannot conclude from the equation

$$x(x - 3) = 10$$

that $x = 10$ and $x - 3 = 10$ yield solutions. Instead, you must first write the equation in standard form and then factor the left side, as follows.

$$x^2 - 3x - 10 = 0 \Rightarrow (x - 5)(x + 2) = 0$$

Now, from the factored form, you can see that the solutions are 5 and -2.

Technology

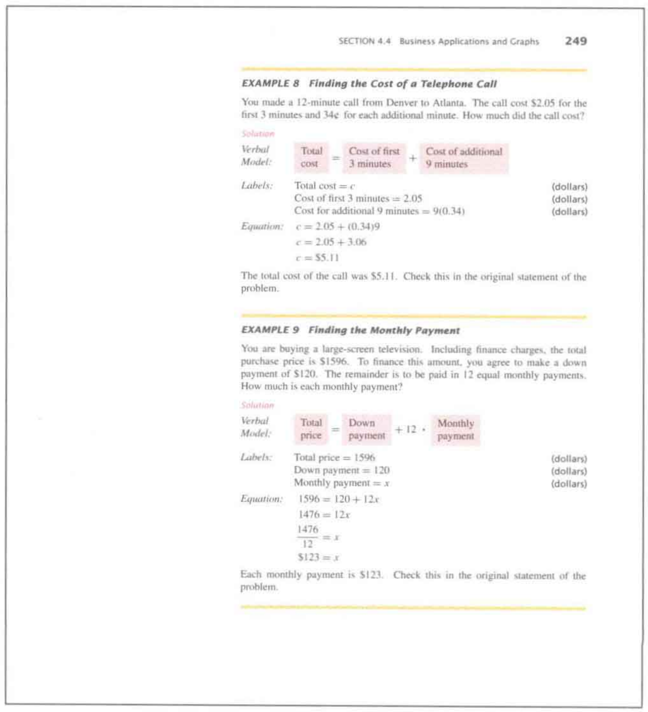
Instructions for using graphing utilities appear in the margin at point of use. They offer convenient reference for users of graphing technology and can easily be omitted if desired. Additionally, problems in the Exercise Sets that require a graphing utility have been identified with a graphing calculator icon.

Study Tips

Study Tips appear in the margin at point of use. They offer students specific, helpful, and insightful suggestions for studying algebra. "How to Study Algebra" on page xxvii and "Reading and Writing About Mathematics" on page xxx outline a general plan designed to improve student study skills.

Problem Solving

The text provides ample opportunity for students to develop their problem-solving skills. They are taught the following approach to solving applied problems: (1) Construct a verbal model; (2) Label variable and constant terms; (3) Construct an algebraic model; (4) Using the model, solve the problem; and (5) Check the answer in the original statement of the problem. This process has wide applicability, and it is used with verbal, analytical, graphical, and numerical approaches to problem solving. Identifying units of measure and checking solutions is emphasized, and many solutions have explanations and additional help in the form of comments adjacent to the computation. Color is also used to emphasize and clarify the solution steps.



SECTION 4.4 Business Applications and Graphs 249

EXAMPLE 8 Finding the Cost of a Telephone Call

You made a 12-minute call from Denver to Atlanta. The call cost $2.05 for the first 3 minutes and 34¢ for each additional minute. How much did the call cost?

Solution

Verbal Model: Total cost = Cost of first 3 minutes + Cost of additional 9 minutes

Labels: Total cost = c (dollars)
Cost of first 3 minutes = 2.05 (dollars)
Cost for additional 9 minutes = 9(0.34) (dollars)

Equation: $c = 2.05 + (0.34)9$
$c = 2.05 + 3.06$
$c = \$5.11$

The total cost of the call was $5.11. Check this in the original statement of the problem.

EXAMPLE 9 Finding the Monthly Payment

You are buying a large-screen television. Including finance charges, the total purchase price is $1596. To finance this amount, you agree to make a down payment of $120. The remainder is to be paid in 12 equal monthly payments. How much is each monthly payment?

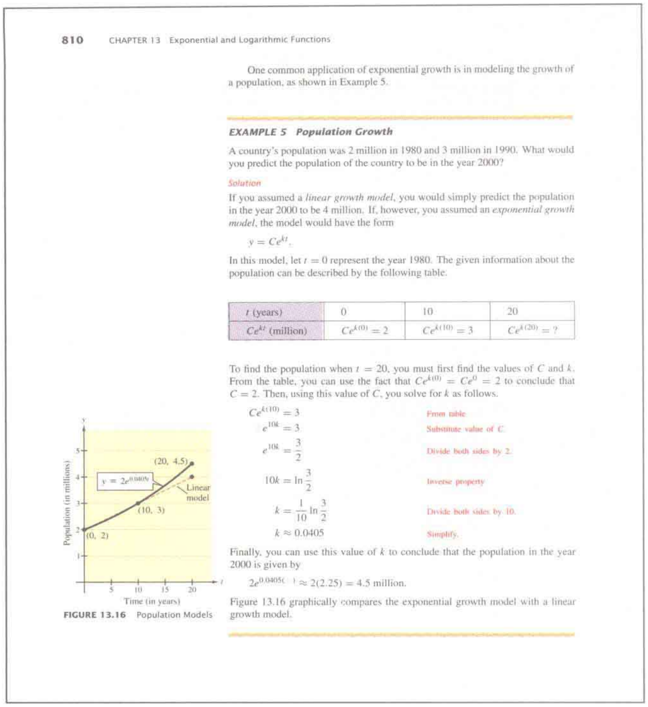
Solution

Verbal Model: Total price = Down payment + 12 · Monthly payment

Labels: Total price = 1596 (dollars)
Down payment = 120 (dollars)
Monthly payment = x (dollars)

Equation: $1596 = 120 + 12x$
$1476 = 12x$
$\frac{1476}{12} = x$
$\$123 = x$

Each monthly payment is $123. Check this in the original statement of the problem.



810 CHAPTER 13 Exponential and Logarithmic Functions

One common application of exponential growth is in modeling the growth of a population, as shown in Example 5.

EXAMPLE 5 Population Growth

A country's population was 2 million in 1980 and 3 million in 1990. What would you predict the population of the country to be in the year 2000?

Solution

If you assumed a *linear growth model*, you would simply predict the population in the year 2000 to be 4 million. If, however, you assumed an *exponential growth model*, the model would have the form

$$y = Ce^{kt}.$$

In this model, let $t = 0$ represent the year 1980. The given information about the population can be described by the following table.

t (years)	0	10	20
Ce^{kt} (million)	$Ce^{k(0)} = 2$	$Ce^{k(10)} = 3$	$Ce^{k(20)} = ?$

To find the population when $t = 20$, you must first find the values of C and k. From the table, you can use the fact that $Ce^{k(0)} = Ce^0 = 2$ to conclude that $C = 2$. Then, using this value of C, you solve for k as follows.

$Ce^{k(10)} = 3$ From table

$e^{10k} = 3$ Substitute value of C.

$e^{10k} = \frac{3}{2}$ Divide both sides by 2.

$10k = \ln\frac{3}{2}$ Inverse property

$k = \frac{1}{10}\ln\frac{3}{2}$ Divide both sides by 10.

$k \approx 0.0405$ Simplify.

Finally, you can use this value of k to conclude that the population in the year 2000 is given by

$$2e^{0.0405(\ )} \approx 2(2.25) = 4.5 \text{ million.}$$

Figure 13.16 graphically compares the exponential growth model with a linear growth model.

FIGURE 13.16 Population Models

Applications

Real-life applications are integrated throughout the text in examples and exercises. These applications offer students constant review of problem-solving skills and emphasize the relevance of the mathematics. Many of the applications use recent, real data, and all are titled for easy reference. Photographs with captions throughout the text also encourage students to see the link between mathematics and real life.

Examples

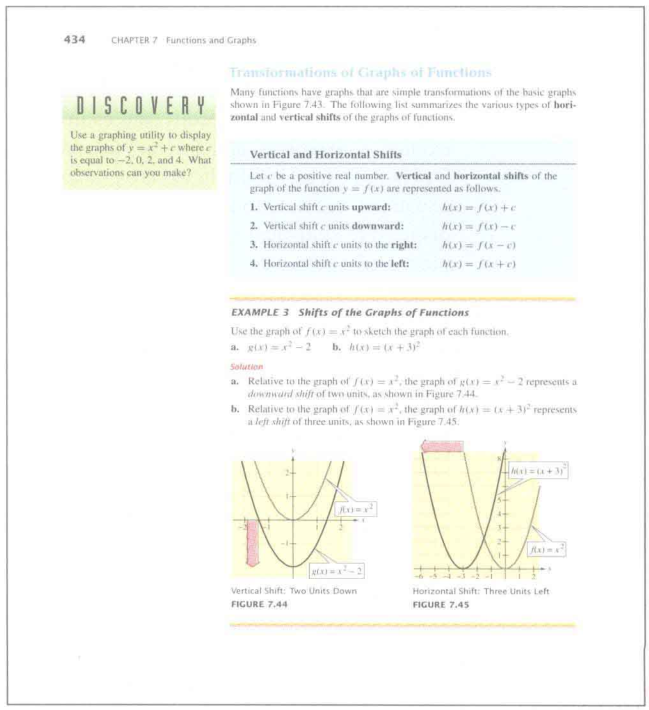
Each of the text examples was carefully chosen to illustrate a particular mathematical concept, problem-solving approach, or computational technique, and to enhance students' understanding. The examples in the text cover a wide variety of problem types, including computational, real-life applications (many with real data), and those requiring the use of graphing technology. Each example is titled for easy reference, and real-life applications are labeled. Many examples include side comments in color, which clarify the key steps of the solution process.

Discovery

Throughout the text, Discovery notes encourage active participation by students, often taking advantage of the power of technology (graphing calculators and scientific calculators) to explore mathematical concepts and discover mathematical patterns. Using a variety of approaches, including visualization, verification, pattern recognition, and modeling, students develop an intuitive understanding of algebraic topics.

Definitions and Rules

All of the important rules, formulas, guidelines, properties, definitions, and summaries are highlighted for emphasis. Each is also titled for easy reference.



434 CHAPTER 7 Functions and Graphs

DISCOVERY

Use a graphing utility to display the graphs of $y = x^2 + c$ where c is equal to -2, 0, 2, and 4. What observations can you make?

Transformations of Graphs of Functions

Many functions have graphs that are simple transformations of the basic graphs shown in Figure 7.43. The following list summarizes the various types of **horizontal** and **vertical shifts** of the graphs of functions.

Vertical and Horizontal Shifts

Let c be a positive real number. **Vertical** and **horizontal shifts** of the graph of the function $y = f(x)$ are represented as follows.

1. Vertical shift c units **upward:** $h(x) = f(x) + c$
2. Vertical shift c units **downward:** $h(x) = f(x) - c$
3. Horizontal shift c units to the **right:** $h(x) = f(x - c)$
4. Horizontal shift c units to the **left:** $h(x) = f(x + c)$

EXAMPLE 3 Shifts of the Graphs of Functions

Use the graph of $f(x) = x^2$ to sketch the graph of each function.

a. $g(x) = x^2 - 2$ **b.** $h(x) = (x + 3)^2$

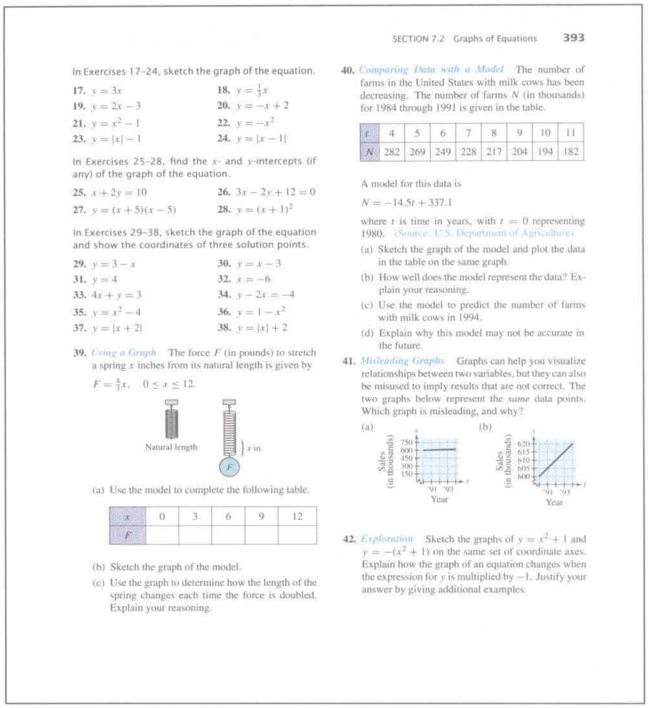
Solution

a. Relative to the graph of $f(x) = x^2$, the graph of $g(x) = x^2 - 2$ represents a *downward shift* of two units, as shown in Figure 7.44.

b. Relative to the graph of $f(x) = x^2$, the graph of $h(x) = (x + 3)^2$ represents a *left shift* of three units, as shown in Figure 7.45.

Vertical Shift: Two Units Down
FIGURE 7.44

Horizontal Shift: Three Units Left
FIGURE 7.45



SECTION 7.2 Graphs of Equations 393

In Exercises 17–24, sketch the graph of the equation.

17. $y = 3x$
18. $y = \frac{1}{3}x$
19. $y = 2x - 3$
20. $y = -x + 2$
21. $y = x^2 - 1$
22. $y = -x^2$
23. $y = |x| - 1$
24. $y = |x - 1|$

In Exercises 25–28, find the x- and y-intercepts (if any) of the graph of the equation.

25. $x + 2y = 10$
26. $3x - 2y + 12 = 0$
27. $y = (x + 5)(x - 5)$
28. $y = (x + 1)^2$

In Exercises 29–38, sketch the graph of the equation and show the coordinates of three solution points.

29. $y = 3 - x$
30. $y = x - 3$
31. $y = 4$
32. $x = -6$
33. $4x + y = 3$
34. $y - 2x = -4$
35. $y = x^2 - 4$
36. $y = 1 - x^2$
37. $y = |x + 2|$
38. $y = |x| + 2$

39. *Using a Graph* The force F (in pounds) to stretch a spring x inches from its natural length is given by

$F = \frac{4}{3}x, \quad 0 \le x \le 12.$

(a) Use the model to complete the following table.

x	0	3	6	9	12
F					

(b) Sketch the graph of the model.

(c) Use the graph to determine how the length of the spring changes each time the force is doubled. Explain your reasoning.

40. *Comparing Data with a Model* The number of farms in the United States with milk cows has been decreasing. The number of farms N (in thousands) for 1984 through 1991 is given in the table.

t	4	5	6	7	8	9	10	11
N	282	269	249	228	217	204	194	182

A model for this data is

$N = -14.5t + 337.1$

where t is time in years, with $t = 0$ representing 1980. (Source: U.S. Department of Agriculture)

(a) Sketch the graph of the model and plot the data in the table on the same graph.

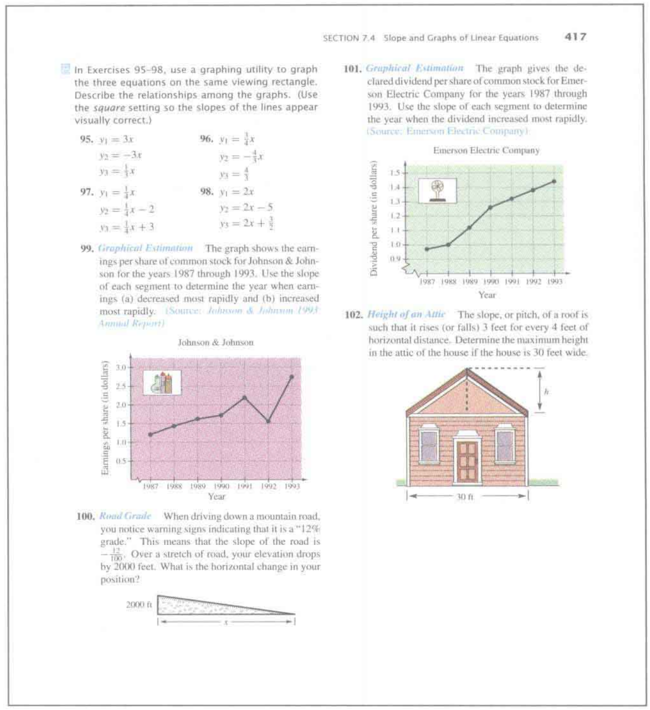
(b) How well does the model represent the data? Explain your reasoning.

(c) Use the model to predict the number of farms with milk cows in 1994.

(d) Explain why this model may not be accurate in the future.

41. *Misleading Graphs* Graphs can help you visualize relationships between two variables, but they can also be misused to imply results that are not correct. The two graphs below represent the *same* data points. Which graph is misleading, and why?

42. *Exploration* Sketch the graphs of $y = x^2 + 1$ and $y = -(x^2 + 1)$ on the same set of coordinate axes. Explain how the graph of an equation changes when the expression for y is multiplied by -1. Justify your answer by giving additional examples.



SECTION 7.4 Slope and Graphs of Linear Equations 417

In Exercises 95–98, use a graphing utility to graph the three equations on the same viewing rectangle. Describe the relationships among the graphs. (Use the *square* setting so the slopes of the lines appear visually correct.)

95. $y_1 = 3x$, $y_2 = -3x$, $y_3 = \frac{1}{3}x$
96. $y_1 = \frac{3}{4}x$, $y_2 = -\frac{4}{3}x$, $y_3 = \frac{4}{3}$
97. $y_1 = \frac{1}{4}x$, $y_2 = \frac{1}{4}x - 2$, $y_3 = \frac{1}{4}x + 3$
98. $y_1 = 2x$, $y_2 = 2x - 5$, $y_3 = 2x + \frac{3}{2}$

99. *Graphical Estimation* The graph shows the earnings per share of common stock for Johnson & Johnson for the years 1987 through 1993. Use the slope of each segment to determine the year when earnings (a) decreased most rapidly and (b) increased most rapidly. (Source: *Johnson & Johnson 1993 Annual Report*)

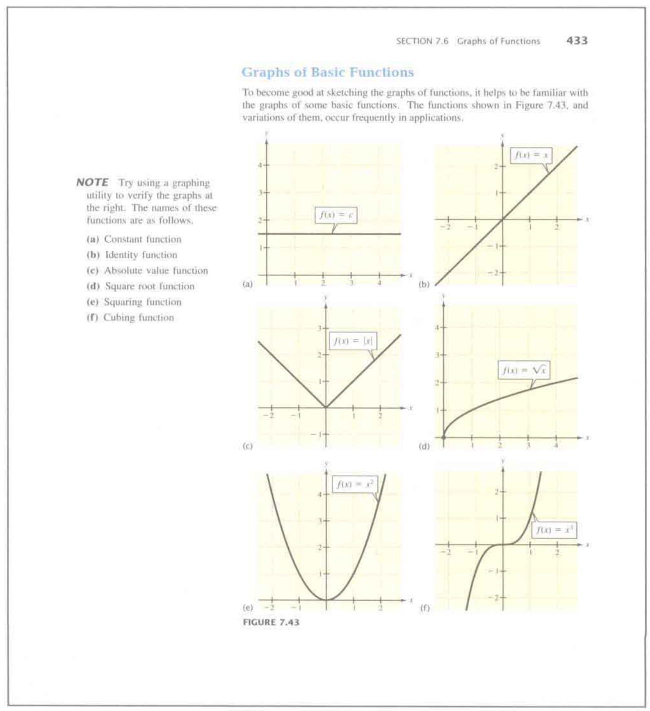
100. *Road Grade* When driving down a mountain road, you notice warning signs indicating that it is a "12% grade." This means that the slope of the road is $-\frac{12}{100}$. Over a stretch of road, your elevation drops by 2000 feet. What is the horizontal change in your position?

101. *Graphical Estimation* The graph gives the declared dividend per share of common stock for Emerson Electric Company for the years 1987 through 1993. Use the slope of each segment to determine the year when the dividend increased most rapidly. (Source: Emerson Electric Company)

102. *Height of an Attic* The slope, or pitch, of a roof is such that it rises (or falls) 3 feet for every 4 feet of horizontal distance. Determine the maximum height in the attic of the house if the house is 30 feet wide.

Graphics

Visualization is a critical problem-solving skill. To encourage the development of this ability, the text has numerous figures in examples, exercises, and answers to odd-numbered exercises. Included are graphs of equations and functions, geometric figures, displays of statistical information, scatter plots, and numerous screen outputs from graphing technology. All graphs of equations and functions, computer- or calculator-generated for accuracy, are designed to resemble students' actual screen outputs as closely as possible. Graphics are also used to emphasize graphical interpretation, comparison, and estimation.



SECTION 7.6 Graphs of Functions 433

Graphs of Basic Functions

To become good at sketching the graphs of functions, it helps to be familiar with the graphs of some basic functions. The functions shown in Figure 7.43, and variations of them, occur frequently in applications.

NOTE Try using a graphing utility to verify the graphs at the right. The names of these functions are as follows.

(a) Constant function
(b) Identity function
(c) Absolute value function
(d) Square root function
(e) Squaring function
(f) Cubing function

FIGURE 7.43

Group Activities **Extending the Concept**

A Mathematical Riddle What is the largest number that can be written using the three digits 2, 3, and 4? The number 432 seems to be the obvious answer. However, if you allow the digits to be exponents, then you can obtain numbers that are much larger than 432. For instance, consider the numbers

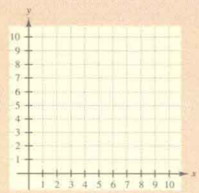
$$(32)^4 = 1{,}048{,}576$$

and

$$3^{24} \approx 282{,}430{,}000{,}000.$$

In your group, see who can create the largest number using the three digits 2, 3, and 4.

Group Activities **Extending the Concept**



Using Inequalities Try the following activity. One person picks a point with whole number coordinates on a grid like the one at left without revealing the coordinates. A second person writes the equation of a line passing through the grid region. The first person graphs the line on the grid and indicates whether the secret point lies above, below, or on the line. Continue writing and graphing lines until the second person is able to guess the coordinates of the secret point. Switch roles and try again. What is the fewest number of turns your team required to guess the point?

Group Activities

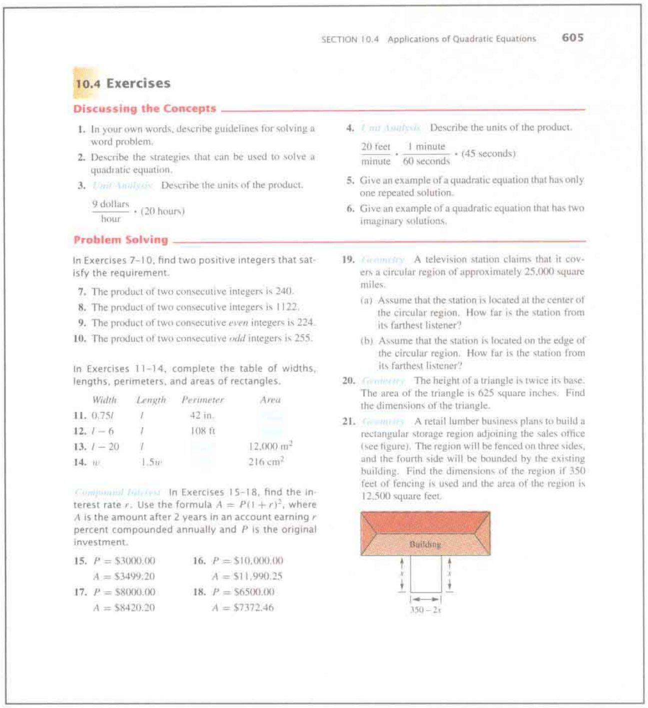
The Group Activities that appear at the end of sections reinforce students' understanding by approaching mathematical concepts in a variety of ways: Communicating Mathematically, You Be the Instructor, Extending the Concept, Problem Solving, and Exploring with Technology. Designed to be completed as group projects in class or as homework assignments, the Group Activities give students opportunities for interactive learning and to think, talk, and write about mathematics.

Group Activities **Problem Solving**

Fitting a Quadratic Model The data in the table represents the United States government's annual net receipts y (in billions of dollars) from individual income taxes for the year x from 1990 through 1992, where $x = 0$ corresponds to 1990. (Source: U.S. Department of the Treasury)

x	0	1	2
y	467	468	476

Use a system of three linear equations to find a quadratic model that fits the data. According to your model, what were the annual net receipts from individual income taxes in 1993? The actual annual net receipts for 1993 were $510 billion. How does the value obtained from your quadratic model compare? Suppose you had been involved in planning the 1993 federal budget and had used this model to estimate how much federal income could be expected from 1993 individual income taxes. When you review the actual 1993 tax receipts and see that the model wasn't completely accurate, how do you evaluate the model's prediction performance? Are you satisfied with it? Why or why not?


SECTION 10.4 Applications of Quadratic Equations 605

10.4 Exercises

Discussing the Concepts

1. In your own words, describe guidelines for solving a word problem.
2. Describe the strategies that can be used to solve a quadratic equation.
3. *Unit Analysis* Describe the units of the product. $\frac{9 \text{ dollars}}{\text{hour}} \cdot (20 \text{ hours})$
4. *Unit Analysis* Describe the units of the product. $\frac{20 \text{ feet}}{\text{minute}} \cdot \frac{1 \text{ minute}}{60 \text{ seconds}} \cdot (45 \text{ seconds})$
5. Give an example of a quadratic equation that has only one repeated solution.
6. Give an example of a quadratic equation that has two imaginary solutions.

Problem Solving

In Exercises 7–10, find two positive integers that satisfy the requirement.

7. The product of two consecutive integers is 240.
8. The product of two consecutive integers is 1122.
9. The product of two consecutive *even* integers is 224.
10. The product of two consecutive *odd* integers is 255.

In Exercises 11–14, complete the table of widths, lengths, perimeters, and areas of rectangles.

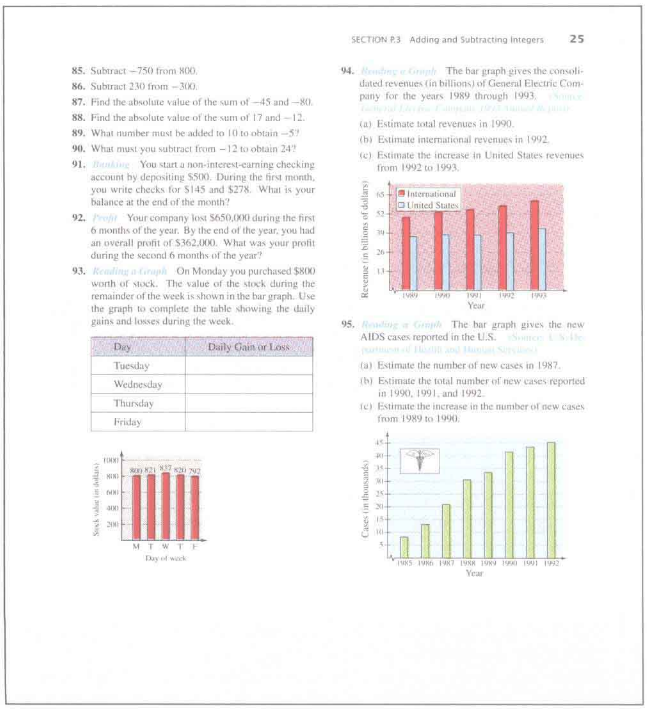
	Width	Length	Perimeter	Area
11.	$0.75l$	l	42 in.	
12.	$l - 6$	l	108 ft	
13.	$l - 20$	l		12,000 m^2
14.	w	$1.5w$		216 cm^2

Compound Interest In Exercises 15–18, find the interest rate r. Use the formula $A = P(1 + r)^2$, where A is the amount after 2 years in an account earning r percent compounded annually and P is the original investment.

15. $P = \$3000.00$, $A = \$3499.20$
16. $P = \$10{,}000.00$, $A = \$11{,}990.25$
17. $P = \$8000.00$, $A = \$8420.20$
18. $P = \$6500.00$, $A = \$7372.46$

19. *Geometry* A television station claims that it covers a circular region of approximately 25,000 square miles.
(a) Assume that the station is located at the center of the circular region. How far is the station from its farthest listener?
(b) Assume that the station is located on the edge of the circular region. How far is the station from its farthest listener?
20. *Geometry* The height of a triangle is twice its base. The area of the triangle is 625 square inches. Find the dimensions of the triangle.
21. *Geometry* A retail lumber business plans to build a rectangular storage region adjoining the sales office (see figure). The region will be fenced on three sides, and the fourth side will be bounded by the existing building. Find the dimensions of the region if 350 feet of fencing is used and the area of the region is 12,500 square feet.

Building; x; x; $350 - 2x$


SECTION P.3 Adding and Subtracting Integers 25

85. Subtract −750 from 800.
86. Subtract 230 from −300.
87. Find the absolute value of the sum of −45 and −80.
88. Find the absolute value of the sum of 17 and −12.
89. What number must be added to 10 to obtain −5?
90. What must you subtract from −12 to obtain 24?
91. *Banking* You start a non-interest-earning checking account by depositing \$500. During the first month, you write checks for \$145 and \$278. What is your balance at the end of the month?
92. *Profit* Your company lost \$650,000 during the first 6 months of the year. By the end of the year, you had an overall profit of \$362,000. What was your profit during the second 6 months of the year?
93. *Reading a Graph* On Monday you purchased \$800 worth of stock. The value of the stock during the remainder of the week is shown in the bar graph. Use the graph to complete the table showing the daily gains and losses during the week.

Day	Daily Gain or Loss
Tuesday	
Wednesday	
Thursday	
Friday	

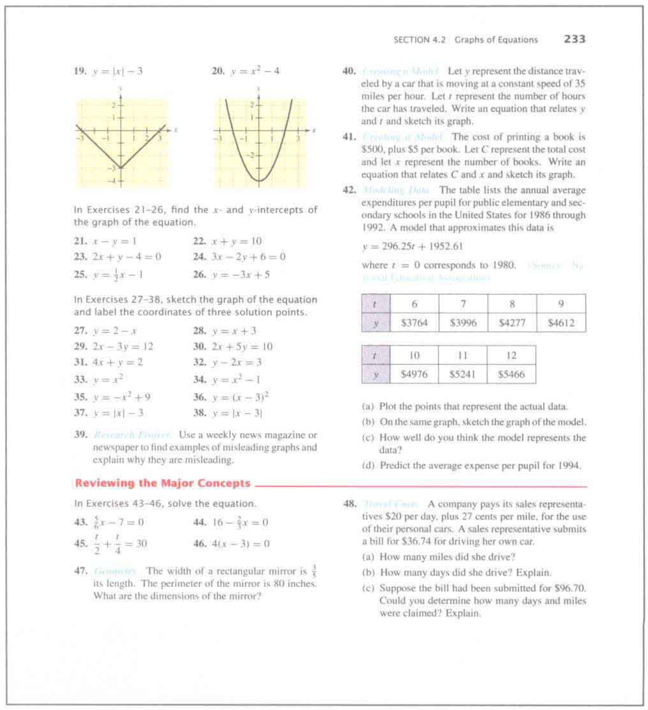
Stock value (in dollars); 800 821 837 820 792; M T W T F; Day of week

94. *Reading a Graph* The bar graph gives the consolidated revenues (in billions) of General Electric Company for the years 1989 through 1993.
(a) Estimate total revenues in 1990.
(b) Estimate international revenues in 1992.
(c) Estimate the increase in United States revenues from 1992 to 1993.

Revenue (in billions of dollars); International; United States; 1989 1990 1991 1992 1993; Year

95. *Reading a Graph* The bar graph gives the new AIDS cases reported in the U.S. (Source: U.S. Department of Health and Human Services)
(a) Estimate the number of new cases in 1987.
(b) Estimate the total number of new cases reported in 1990, 1991, and 1992.
(c) Estimate the increase in the number of new cases from 1989 to 1990.

Cases (in thousands); 1985 1986 1987 1988 1989 1990 1991 1992; Year


SECTION 4.2 Graphs of Equations 233

19. $y = |x| - 3$
20. $y = x^2 - 4$

In Exercises 21–26, find the x- and y-intercepts of the graph of the equation.

21. $x - y = 1$
22. $x + y = 10$
23. $2x + y - 4 = 0$
24. $3x - 2y + 6 = 0$
25. $y = \frac{1}{2}x - 1$
26. $y = -3x + 5$

In Exercises 27–38, sketch the graph of the equation and label the coordinates of three solution points.

27. $y = 2 - x$
28. $y = x + 3$
29. $2x - 3y = 12$
30. $2x + 5y = 10$
31. $4x + y = 2$
32. $y - 2x = 3$
33. $y = x^2$
34. $y = x^2 - 1$
35. $y = -x^2 + 9$
36. $y = (x - 3)^2$
37. $y = |x| - 3$
38. $y = |x - 3|$
39. *Research Project* Use a weekly news magazine or newspaper to find examples of misleading graphs and explain why they are misleading.
40. *Creating a Model* Let y represent the distance traveled by a car that is moving at a constant speed of 35 miles per hour. Let t represent the number of hours the car has traveled. Write an equation that relates y and t and sketch its graph.
41. *Creating a Model* The cost of printing a book is \$500, plus \$5 per book. Let C represent the total cost and let x represent the number of books. Write an equation that relates C and x and sketch its graph.
42. *Modeling Data* The table lists the annual average expenditures per pupil for public elementary and secondary schools in the United States for 1986 through 1992. A model that approximates this data is

$y = 296.25t + 1952.61$

where $t = 0$ corresponds to 1980.

t	6	7	8	9
y	\$3764	\$3996	\$4277	\$4612

t	10	11	12
y	\$4976	\$5241	\$5466

(a) Plot the points that represent the actual data.
(b) On the same graph, sketch the graph of the model.
(c) How well do you think the model represents the data?
(d) Predict the average expense per pupil for 1994.

Reviewing the Major Concepts

In Exercises 43–46, solve the equation.

43. $\frac{5}{6}x - 7 = 0$
44. $16 - \frac{2}{3}x = 0$
45. $\frac{t}{2} + \frac{t}{4} = 30$
46. $4(x - 3) = 0$
47. *Geometry* The width of a rectangular mirror is $\frac{3}{5}$ its length. The perimeter of the mirror is 80 inches. What are the dimensions of the mirror?
48. *Travel Costs* A company pays its sales representatives \$20 per day, plus 27 cents per mile, for the use of their personal cars. A sales representative submits a bill for \$36.74 for driving her own car.
(a) How many miles did she drive?
(b) How many days did she drive? Explain.
(c) Suppose the bill had been submitted for \$96.70. Could you determine how many days and miles were claimed? Explain.

Exercises

Exercise are grouped into four categories: Discussing the Concepts, Problem Solving, Reviewing the Major Concepts, and Additional Problem Solving. The numerous computational, conceptual, and applied problems include multi-part, exploration and discovery, writing, estimation, numeracy, geometry, and challenging exercises, as well as real-life applications, mathematical modeling, graphical comparisons, data interpretation and analysis, fitting a line to data, and exercises that require graphing technology. Designed to build competence, skill, and understanding, each part of the exercise set is graded in difficulty to allow students to gain confidence as they progress. Detailed solutions to all odd-numbered exercises are given in the Student Solutions Guide, and answers to all odd-numbered exercises appear in the back of the text.

Geometry

Geometric formulas and concepts are reviewed throughout the text. For reference, common formulas are listed inside the back cover of this text.

CAREER INTERVIEW

Lisa M. Deitemeyer

Civil Engineer

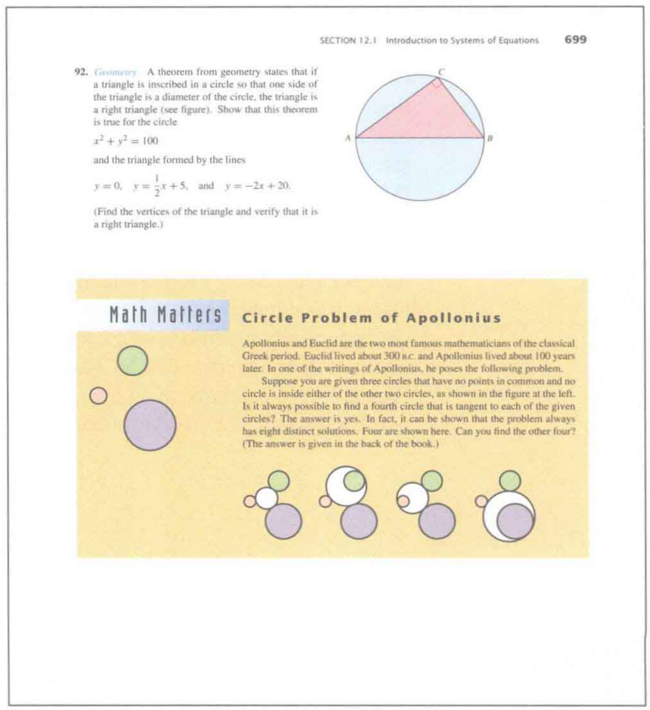
Johnson-Brittain & Associates, Inc.

Tucson, AZ 85701

Johnson-Brittain does highway design work, primarily for the Arizona Department of Transportation. I am responsible for drainage design of roadways and intersections. It is important that water properly drain off the road surface to avoid flooding problems. One strategy for removing excess water is to use a pipe drainage system that empties into a retention pond. When designing a pipe system and choosing pipe size, I use the equation $V = Q/A$ to find the velocity V of water moving at flow rate Q (volume per unit time) through a given pipe of cross-sectional area A. Finding the water velocity is very important. If it is too fast, erosion can occur in the retention pond. If it is too slow, sedimentation can clog the pipe. As you can see, algebra is very important to my work. I am always solving for different variables that are needed for drainage design.

Career Interviews

Appearing in each chapter, Career Interviews with people who use algebra in their jobs help students understand that algebra is a modern, problem-solving language.



SECTION 12.1 Introduction to Systems of Equations 699

92. *Geometry* A theorem from geometry states that if a triangle is inscribed in a circle so that one side of the triangle is a diameter of the circle, the triangle is a right triangle (see figure). Show that this theorem is true for the circle

$$x^2 + y^2 = 100$$

and the triangle formed by the lines

$$y = 0, \quad y = \frac{1}{2}x + 5, \quad \text{and} \quad y = -2x + 20.$$

(Find the vertices of the triangle and verify that it is a right triangle.)

Math Matters **Circle Problem of Apollonius**

Apollonius and Euclid are the two most famous mathematicians of the classical Greek period. Euclid lived about 300 B.C. and Apollonius lived about 100 years later. In one of the writings of Apollonius, he poses the following problem.

Suppose you are given three circles that have no points in common and no circle is inside either of the other two circles, as shown in the figure at the left. Is it always possible to find a fourth circle that is tangent to each of the given circles? The answer is yes. In fact, it can be shown that the problem always has eight distinct solutions. Four are shown here. Can you find the other four? (The answer is given in the back of the book.)

Math Matters

Each chapter contains a Math Matters feature that engages student interest by discussing a historical note or mathematical problem. For those features that pose a question, the answers appear in the back of the text.

Mid-Chapter Quizzes

Each chapter contains a Mid-Chapter Quiz with answers in the back of the text. This feature allows the student to perform a self-assessment midway through the chapter.

344 CHAPTER 6 Factoring and Solving Equations

MID-CHAPTER QUIZ

Take this quiz as you would take a quiz in class. After you are done, check your work against the answers given in the back of the book.

In Exercises 1–4, find the missing factor.

1. $\frac{2}{3}x - 1 = \frac{1}{3}(\quad)$
2. $x^2y - xy^2 = xy(\quad)$
3. $y^2 + y - 42 = (y+7)(\quad)$
4. $2x^2 - x - 1 = (x-1)(\quad)$

In Exercises 5–16, factor the polynomial.

5. $10x^2 + 70$
6. $2a^3b - 4a^2b^2$
7. $x(x+2) - 3(x+2)$
8. $t^3 - 3t^2 + t - 3$
9. $y^2 + 11y + 30$
10. $u^2 + u - 30$
11. $x^3 - x^2 - 30x$
12. $2x^2y + 8xy - 64y$
13. $3v^2 - 4v - 2$
14. $6 - 13z - 5z^2$
15. $6x^2 - x - 2$
16. $10s^4 - 14s^3 + 2s^2$
17. Find all integer values of b such that the polynomial

   $x^2 + bx + 12$

   can be factored. Describe the method you used.
18. Find two values of c such that

   $x^2 - 10x + c$

   can be factored. Describe the method you used.
19. Find all possible products of the form

   $(3x + m)(x + n)$

   such that $mn = 6$. Describe the method you used.
20. The area of the rectangle shown in the figure is $x^2 + 4x - 5$. Find its length.

Chapter Project

Chapter Projects, referenced in the chapter opener, are engaging applications that use real data, graphs, and modeling to enhance students' understanding of mathematical concepts. Designed as individual or group projects, they offer additional opportunities to think, discuss, and write about mathematics. Many projects include research assignments that give students the opportunity to collect, analyze, and interpret their own data. Each Chapter Project is also available in an interactive, multimedia, CD-ROM format.

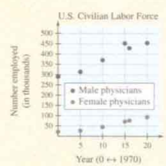
Chapter Project 441

CHAPTER PROJECT: Working Men and Women

Data is frequently represented in three ways: *numerically* by a table, *graphically*, or *algebraically* by an equation, formula, or algebraic model. Because each type of representation has its own advantages, the best way to present data is to use a mix of all three representations.

In gathering data, it may be easiest to first organize the data in a table. Finding patterns or trends in the data may be easier to recognize with a graph. And, finally, making future predictions may be easier with an algebraic model.

For instance, the table below shows the numbers of men and women (in thousands) who were practicing physicians from 1970 through 1990. The data is represented graphically by the scatter plot at the left. While investigating the questions below, you will be asked to find algebraic models to represent the data, and then to use the models to interpret the data. (Source: American Medical Association)



Year	1970	1975	1980	1985	1986	1990
Women Physicians	21.4	27.2	44.7	71.9	76.8	93.3
Men Physicians	289.5	313.1	370.2	452.3	429.0	454.0

1. ***Graphical Reasoning*** Use the scatter plot to write a verbal description of the data. Discuss any trend or pattern that is evident from the scatter plot.
2. ***Linear Modeling*** Use graph paper to redraw the scatter plot. Approximate each of the data sets with a line. Then find an equation for each line.
3. ***Linear Modeling*** Use the linear regression program on a graphing utility to find a linear model for each data set. Compare the results with those obtained in Question 2.
4. ***How Well Does It Fit?*** When the regression program in Question 3 is run, it will display a correlation coefficient r that measures how well the linear model fits the data. The closer r is to 1, the better the model fits the data. Which of the two models fits its data better? Does your answer seem reasonable from the graphical point of view?
5. ***Prediction*** Predict the numbers of women and men physicians in 1995. Discuss different ways that you could obtain the prediction. Which method do you prefer? Why?
6. ***Prediction*** Extend the lines you drew in Question 2 until they intersect. What interpretation can you make? Is the interpretation realistic in the context of the data?
7. ***Research Project*** Use your school's library or another reference source to find data for the numbers of men and women in an occupation. Organize the data numerically, graphically, and algebraically. What can you conclude?

Chapter Summary

The Chapter Summary reviews the skills covered in the chapter. Section references for the major topics make this an effective study tool, and correlation to the review exercises offers guided practice.

Review Exercises

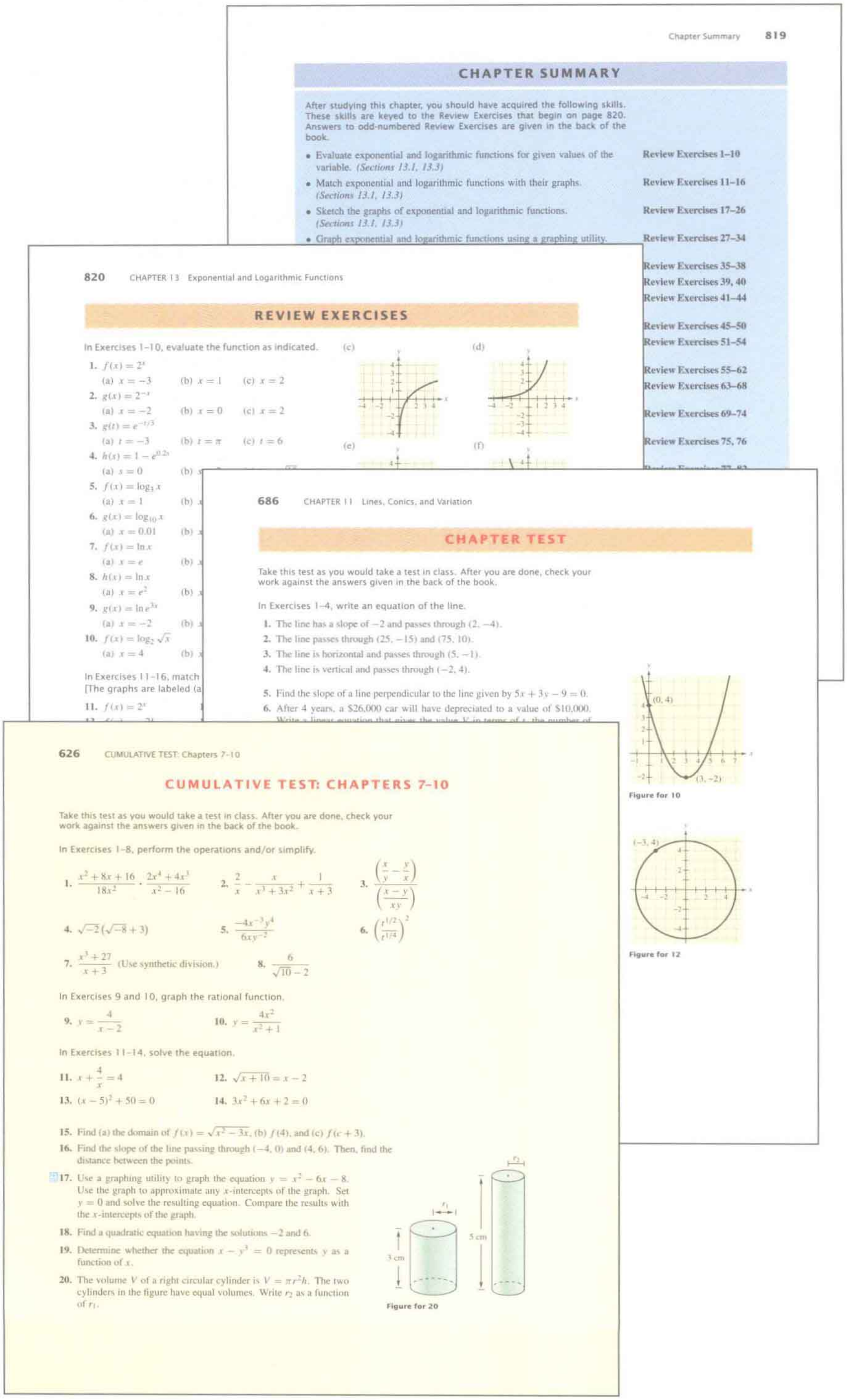
The Review Exercises at the end of each chapter offer the student an opportunity for additional practice. Each set of review exercises includes both computational and applied problems covering a wide range of topics.

Chapter Test

Chapter Tests allow students to assess their own level of success.

Cumulative Tests

The Cumulative Tests that appear after Chapters 3, 6, 10, and 13 help students judge their mastery of previously covered material, as well as reinforce the knowledge students have been accumulating throughout the text—preparing them for other exams and for future courses.



Supplements

Elementary and Intermediate Algebra: A Combined Course, Second Edition, by Larson and Hostetler, is accompanied by a comprehensive supplements package. All items are keyed to the text.

Printed Resources

Student Solutions Guide

- Detailed, step-by-step solutions to all odd-numbered section exercises (except Discussing the Concept) and review exercises
- Detailed, step-by-step solutions to all Mid-Chapter Quiz, Chapter Test, and Cumulative Test questions

Graphing Technology Keystroke Guide: Algebra by Benjamin N. Levy

- Keystroke instructions for Texas Instruments, Sharp, Casio, and Hewlett-Packard graphing calculators
- Examples with step-by-step solutions
- Extensive graphics screen output
- Technology tips

Instructor's Guide

- Solutions to even-numbered exercises
- Answers to all Group Activities, Technology Boxes, Discovery Boxes, and Chapter Projects

Test Item File and Resource Guide

- Printed test bank with approximately 4000 test items (multiple-choice, open-ended, and writing) coded by level of difficulty
- Technology-required test items coded for easy reference
- Bank of chapter test forms with answer keys
- Two final exams
- Transparency masters
- Notes to the Instructor, which includes information on standardized tests such as the Texas Academic Skills Program (TASP), Florida College Level Academic Skills Test (CLAST), and the California State University Entry Level Mathematics (ELM) Examination and provides a list of skills covered by the test and the corresponding section(s) in the text where the topic can be found, as well as notes on contemporary instructional strategies such as alternative assessment and cooperative learning

Media Resources

Tutor (IBM, Macintosh)
- Extensive additional practice

Videotapes by Dana Mosely
- Comprehensive coverage keyed to the text by section
- Detailed explanation of important concepts
- Numerous examples and applications, often illustrated via computer-generated animations
- Discussion of study skills
- For media resource centers; also available for student purchase

D. C. Heath Interactive Math Series CD-ROM Projects
- Real-life applications in an interactive, multimedia CD-ROM format
- IBM PC for Windows; Macintosh
- See page xvi for a description.

Computerized Testing
- Test-generating software for both IBM and Macintosh computers
- Approximately 4500 test items
- Also available as a printed test bank

CD-ROM Projects

for Elemtentary and Intermediate Algebra: A Combined Course, Second Edition

To accommodate a variety of teaching and learning styles, a series of real-life applications is available in a multimedia, interactive CD-ROM format. Suitable for individual or group assignments, these projects reinforce a variety of mathematical concepts. For each text chapter project is a CD-ROM project, allowing students to explore interactively questions that expand upon the topic and goals of the text project. Students have the opportunity to discover the nature of data sets through exploration, using a combination of graphical, numerical, and algebraic approaches in a guided learning environment. Throughout the text, you will notice a CD-ROM icon that reminds you of the availability of this multimedia software in conjunction with the chapter projects.

These multimedia projects broaden the scope of the text by offering additional opportunities for finding patterns and drawing conclusions, covering related topics and concepts, and providing practice with interpreting graphs, charts, and tables. The multimedia format provides access to extensive real data sets and facilitates hands-on data manipulation for practicing data analysis and modeling techniques. In addition, the projects include animations, color photographs, and audio enhancements.

Each multimedia project is presented in four parts: Introduction, Data, Exploration, and Exercises. The Introduction explains the goals of the project and the background of the project topic. The Data section presents all of the data that may be manipulated in the context of the project in a format that is appropriate to the placement in the text; additional history or pertinent facts may often be found in this section. The Exploration section enables students to manipulate data and discover certain facts about or patterns within the data. For example, the Transportation project allows students to use graphs to find patterns and interactively experiment with placing a line on a scatter plot of actual data to approximate a best fitting line. The Exercises section is a set of questions designed to guide the student to the types of discoveries that may be made from exploration of the data. For example, with the Transportation project students are asked to interpret slopes and y-intercepts, consider predictions, and compare various models.

The CD-ROM Projects for *Elementary and Intermediate Algebra: A Combined Course*, Second Edition, are available for use with multimedia Macintosh or IBM with Windows computers. They cover the following topics:

Chapter P	Population Growth Patterns	**Chapter 8**	Air Resistance and Parachutes
Chapter 1	Musical Sound	**Chapter 9**	Fractals
Chapter 2	Temperature	**Chapter 10**	Gravitation
Chapter 3	Solar Eclipses	**Chapter 11**	Transportation
Chapter 4	Sporting Goods Sales	**Chapter 12**	Retail Sales of Companies
Chapter 5	Chemistry and Color	**Chapter 13**	Half-Life and Radioactivity
Chapter 6	Food Consumption	**Chapter 14**	Banking and Personal Finance
Chapter 7	Working Men and Women		