
Predictive Control

with Constraints

J.M. Maciejowski



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Preface

Predictive Control, or Model-Based Predictive Control ('MPC' or 'MBPC') as it is sometimes known, is the only advanced control technique — that is, more advanced than standard PID control — to have had a significant and widespread impact on industrial process control. The main reason for this is that it is

- The only generic control technology which can deal routinely with equipment and safety constraints.

Operation at or near such constraints is necessary for the most profitable or most efficient operation in many cases. The penetration of predictive control into industrial practice has also been helped by the facts that

- Its underlying idea is easy to understand,
- Its basic formulation extends to multivariable plants with almost no modification,
- It is more powerful than PID control, even for single loops without constraints, without being much more difficult to tune, even on 'difficult' loops such as those containing long time delays.

Predictive control was developed and used in industry for nearly 20 years before attracting much serious attention from the academic control community. This community tended to ignore its potential for dealing with constraints, thus missing its main advantage. In addition, it tended to point to the fact that, when constraints are ignored, predictive control is equivalent to conventional, though generally 'advanced', linear control, and hence apparently nothing new. This is true, but again misses the important point that issues such as tunability and understandability are crucial for the acceptance of a control technology, at least in the process control environment. Fortunately, the academic community has for some years now appreciated that predictive control really does offer something new for control in the presence of constraints, and has provided much analysis, and new ideas, to such an extent that it has gone beyond current industrial

practice, and is preparing the ground for much wider application of predictive control — potentially to almost all control engineering problems. The constant increase in computing speed and power certainly makes that a real prospect.

In this book I have attempted to bring everything together. I have tried to convey the simplicity of the underlying concepts of predictive control, but also to show how it relates to existing control theory, and indeed how much more can be done with it when its use is informed by a knowledge of standard control techniques, such as state estimation, disturbance modelling, and frequency response analysis. Predictive control involves optimization, and I have included details of how to solve at least some of the optimization problems encountered. I have also tried to display the main directions of current research, and give some indication of likely future directions, both of research and of applications.

This book assumes that the reader has some previous exposure to control theory, such as a first course and some contact with state-space models. It will be suitable for graduate students taking systems and control courses, but I hope that it will also prove useful to practising industrial engineers. In order to increase its usefulness to non-students, and also to alleviate the problem of prerequisites for students, I have included a number of *Mini-Tutorials*. These are one-page summaries of topics, such as observers or Lyapunov equations, that are needed in order to understand the material at certain points. I believe that the Mini-Tutorials contain enough explanation to allow the reader to follow the developments in the book, but of course they are not meant to replace in-depth study of these important topics.

It is essential to have access to suitable software in order to solve any non-trivial predictive control problems, and to gain experience of how it actually works. This book assumes that the reader has access to *MATLAB*, together with its *Control System Toolbox* and *Model Predictive Control Toolbox*. Some simple *MATLAB* files have also been written, and some files which augment those available in the *Model Predictive Control Toolbox* — some of these also require the *Optimization Toolbox*. All such additional software is available on this book's Companion Web Site:

<http://www.booksites.net/maciejowski/>

The versions of software used for the preparation of this book were:

<i>MATLAB</i>	5.3.1
<i>Control System Toolbox</i>	4.2.1
<i>Model Predictive Control Toolbox</i>	1.0.4
<i>Optimization Toolbox</i>	2.0

My students Eric Kerrigan and Simon Redhead were responsible for most of the modifications to *Model Predictive Control Toolbox* functions which are available on the web site.

This book originated in a course of lectures given at the Faculty of Aerospace Engineering in Delft, during November and December 1997. I would like to thank Bob Mulder, Hans van der Vaart and Samir Bennani for inviting me to spend a sabbatical term at Delft, for making all the arrangements, for making my stay at Delft both pleasant and interesting, and above all for having enough vision to believe that a course on predictive

control was not out of place in an aerospace department. I would also like to thank Ton van den Boom for giving me feedback on the first few chapters, Rob de Vries (both of TU Delft) for some valuable help with Chapter 4, and Hans van der Vaart for providing me with the linearized model of the Citation aircraft which is used in a number of illustrative examples in the book. The use of an aircraft example may seem quirky, since predictive control has been used almost exclusively in the process industries. There are two reasons for this. Firstly, it reflects my conviction that predictive control has great potential in all application sectors, as a result of continuing increases in real-time computational possibilities. Secondly, most process control examples require considerable explanation of the context to those who are not chemical engineers, whereas most readers will be able to understand the (very simplified) aircraft example from their own experience. Of course there are also examples based on process control in the book, and the major case studies in Chapter 9 are both taken from process control.

I gave a graduate course based on the material in this book to the Centre for Process Control Design at Lund University, and received several very valuable suggestions from both faculty and students who attended that course. Andrew Ogden-Swift of Honeywell Hi-Spec Solutions, Sean Goodhart of Aspentech, David Sandoz of SimSci-Predictive Control, and Jacques Richalet of Adersa have all been very generous of their time, have provided me with details of their companies' products, and have discussed the practicalities of predictive control most valuably and stimulatingly. David Sandoz and David Clarke both provided extremely useful, but also encouraging, feedback at various stages of the development of the manuscript, and Fred Loquasto of UC Santa Barbara read the almost-final version in record time, and picked up several errors and infelicities.

Any errors remain my own responsibility, of course.

J.M. Maciejowski

14 October 2000

A Companion Web Site accompanies *Predictive Control* by Jan Maciejowski.

Visit the *Predictive Control* Companion Web Site at

www.booksites.net/maciejowski

Here you will find valuable teaching and learning material including:

For Students and Lecturers:

- MATLAB files allowing you to model and simulate many of the systems presented in the book
- Links to software needed to run the simulations (commercial Matlab-based software from The Mathworks, and free extensions and other software from the author)
- Links to websites related to predictive control

For Lecturers:

- Downloadable solution's manual



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