Image Modeling

Edited by AZRIEL ROSENFELD

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Preface

It has long been recognized in the field of image processing that the design of processing operations should be based on a model for the ensemble of images to be processed. This realization is becoming increasingly prevalent in the field of image analysis as well. Unfortunately, it is difficult to formulate realistic models for real-world classes of images; but progress is being made on a number of fronts, including models based on Markov processes, random fields, random mosaics, and stochastic grammars, among others. At the same time, analogous models are being developed in fields outside image processing, including stereology, mathematical morphology, integral geometry, statistical ecology, and theoretical geography. It is hoped that this volume, by focusing attention on the field of image modeling, will serve to stimulate further work on the subject, and will promote communication between researchers in image processing and analysis and those in other disciplines.

The papers in this volume were presented at a workshop on image modeling in Rosemont, Illinois on August 6–7, 1979. The workshop was sponsored by the National Science Foundation under Grant MCS-79-04414, and by the Office of Naval Research under Contract N00014-79-M-0070; their support is gratefully acknowledged. Three of the papers presented at the workshop are not included in this volume: B. Julesz, Differences between attentive (figure) and preattentive (ground) perception; W. K. Pratt and O. D. Faugeras, A Stochastic texture field model; W. R. Tobler, Generalization of image processing and modeling concepts to polygonal geographical data sets. All but the first of the papers in this book appeared in Volume 12 of the journal *Computer Graphics and Image Processing.*

Contents

Mosaic Models for Textures

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Traditionally the models of image texture have been classified as statistical or structural [15, 29, 30]. However, in [6, 9] we have suggested a classification of image models into *pixel-based* and *region-based* models, which we believe is more useful. The pixel-based models view individual pixels as the primitives of the texture. Specification of the characteristics of the spatial distribution of pixel properties constitutes the texture description [15, 28]. The region-based models conceive of a texture as an arrangement of a set of spatial subpatterns according to certain placement rules [30, 34]. Both the subpatterns and their placement may be characterized statistically.

Most of the models used in the past are pixel-based. These models have been proposed for images representing a variety of natural phenomena, including ocean waves and the earth's surface. However, for many images the region-based models appear to be more natural [1, 5, 37] than the pixel-based models, although relatively little research has been done on their development [9, 14]. In this paper we shall discuss a specific class of region-based models known as mosaic models, and shall review the work done on these models and their application to modeling textures.

1. MOSAIC MODELS

Mosaic models are defined in terms of planar random pattern generation processes. The characteristics of the patterns generated by a given process may be obtained from the definition of the process. These properties then determine the class of images for which the corresponding model is suitable. A variety of processes may be used to define mosaic models. We describe below briefly two classes of such processes that we have considered in our work. For details see [1–4, 8].

1.1. *Cell Structure Models*

Cell structure mosaics are constructed in two steps:

(a) Tessellate a planar region into cells. We shall consider only tessellations composed of bounded convex polygons.

(b) Independently assign one of m colors $c_1, c_2, \ldots, c_m$ to each cell according to a fixed set of probabilities

$$p_1, \ldots, p_m, \qquad \sum_{i=1}^{m} p_i = 1.$$

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Let $P_{ij}(d)$ denote the probability that one end of a randomly dropped needle of length d falls on color c_i given that the other end is in a region of color c_j. Let $W(d)$ be the probability that a randomly dropped needle of length d falls completely within a cell. Then it can be shown that

$$P_{ij}(d) = p_i(1 - W(d)) + \delta_{ij} W(d),$$

where δ is the Kronecker function.

Given the coloring process in step (b), the cell structure models form a family whose members differ in the manner in which the plane is tessellated. We shall now describe some members of this family that we have used, starting from the three regular tessellations and progressing toward some random ones.

1.1.1. Square model. This is an example of a cell structure model where the cells are of a uniform size. A square (checkerboard) model can be formed by the following procedure. First, choose the origin of an $x-y$ coordinate system on the plane with uniform probability density. Then tessellate the plane into square cells of side length b. Next, this "checkerboard" is rotated by an angle chosen with uniform probability from the interval $(0, 2\pi)$. The cells are now independently assigned one of the m tile types.

Modestino *et al.* [24, 26] have considered tessellations of the plane into rectangles and parallelograms. The lengths of the sides of the rectangles or the parallelograms are determined by two independent renewal processes defined along a pair of axes.

1.1.2. Hexagonal model. This model uses a network of identical hexagons to tessellate the plane. The hexagons can be oriented at any angle to the axes.

1.1.3. Triangular model. This is similar to (1) and (2) except that a triangular tessellation of the plane is used.

All three regular tessellations described above can be viewed as the result of a growth process from a set of nuclei placed at the points of an appropriate regular lattice. Assume all the nuclei start growing simultaneously along a circular frontier, at any given instant. At some later time the circles centered at neighboring lattice points come into contact. As the cells continue to grow, these points of contact become the midpoints of growing straight line segments along which the growth frontiers meet and the growth is stopped. Finally, the grown line segments form the sides of polygons that have the original nuclei as their centers. Expressions for $W(d)$ for these tessellations are known [1, 8, 17, 31, 32].

An interesting special case arises when we consider cells of unit area. Then the resulting mosaic is the realization of a random lattice point process defined by the coloring process. We shall now describe some random cell structure models.

1.1.4. Poisson line model. Consider a system of intersecting lines in the plane with random positions and orientations. Such a system when derived by the following Poisson process possesses fundamental properties of homogeneity and isotropy. A Poisson process of intensity τ/π determines points (θ, ρ) in the infinite rectangular strip $[0 \leq \theta < \pi, -\infty < \rho < \infty]$. Each of these points can be used to construct a line in the plane of the form $x \cos \theta + y \sin \theta - \rho = 0$, where ρ is the distance between the line and an arbitrarily chosen origin. This process is used to tessellate the plane into convex cells.

[1, 32] list some important characteristics of the Poisson line tessellation, such as the expected cell area, expected cell perimeter, expected number of cells meeting at a vertex, and expected total line length per unit area. A detailed discussion can be found in [19, 20, 36].

1.1.5. Voronoi model. This model is based upon a tessellation that is the result of a growth process similar to that used for the regular cell structure models described earlier except that the growth now starts at randomly located points. Each of these points spreads out to occupy a "Dirichlet cell" [13, 21, 22] consisting of all the points that are nearer to it than to any other nucleus. The random initial arrangement of the nuclei may result in cell edges with any of infinitely many slopes, and therefore, a random tessellation. The cells are then independently colored as usual to obtain a *Voronoi mosaic.* [1, 32] present some properties of the Voronoi tessellation. For details, see [13, 21, 22].

1.1.6. Delaunay model. The Delaunay tessellation is closely related to the Voronoi tessellation. *Delaunay triangles* [21–23] can be constructed in the Voronoi tessellation by joining all pairs of nuclei whose corresponding Voronoi polygons share an edge. Thus the vertices of Voronoi polygons are the circumcenters of the Delaunay triangles. The properties of Delaunay tessellations are discussed in [21–23].

1.2. Coverage Models

Coverage or "bombing" models constitute the second class of mosaic models that we have considered. A *coverage mosaic* is obtained by a random arrangement of a set of geometric figures ("bombs") in the plane.

We shall first define the class of binary coverage models. Consider a geometric figure in the plane and identify it by (i) the location of some distinguished point in the figure, e.g., its center of gravity, hereafter called the center of the figure, and (ii) the orientation of some distinguished line in the figure, e.g., its principle axis of inertia. Let a point process drop points on the plane, and let each point represent the center of a figure. If the points are replaced by their corresponding figures, the plane is partitioned into foreground (covered by the figures) and background.

A *multicolored coverage mosaic* is obtained by considering figures of more than one color. The color of a given figure is randomly chosen from a known vector of colors $\mathbf{c} = (c_1, c_2, \ldots, c_m)$ according to a predetermined probability vector $\mathbf{p} = (p_1, p_2, \ldots, p_m)$. Let c_0 denote the background color. Since, in general, the figures overlap, we must have a rule to determine the colors of the regions that are covered by figures of more than one color. We shall give one example of such a rule. Let us view the point process as dropping the centers sequentially in time. Each time a new point falls, the area covered by the associated figure is colored with the color of that figure irrespective of whether any part of the area has already been included in any of the previously fallen figures. The color of a region in the final pattern is thus determined by the color of the latest figure that covered it. (Note that we could just as well have allowed a figure to cover only an area not included in any of the previous figures.)

As in the case of the cell structure models, $P_{ij}(d)$ denotes the probability that one of the ends of a randomly dropped needle of length d falls in a region of color c_i given that the other end is in a region of color c_j, $0 \leq i, j \leq m$, where c_0 denotes the color of the background, the region not occupied by any of the figures. Some general properties of coverage models are discussed in [1, 12, 35].

2. PROPERTIES OF MOSAIC MODELS

A major part of our past effort has been devoted to relating properties of the patterns generated by mosaic models to the parameters occurring in their definitions. These results have then been used to fit the models described in Sections 1.1 and 1.2 to real textures. We now summarize the past work.

2.1. Geometric Properties of Components in Cell Structure Mosaics

Ahuja [1, 2] presents a detailed analysis of the geometric properties of components in the cell structure mosaics. To avoid the numerous details, we shall present here only a qualitative description of the basic approaches involved. A concise but more illustrative discussion appears in [5]. Some experimental results are presented in [7, 10].

To estimate the expected component area in a regular cell structure mosaic, let us first consider the colored regular lattice defined by cell centers, each having the same color as its cell. The expected number of points in a component of this lattice is obtained by viewing the component as a stack of overlapping identically colored runs in succeeding rows, formed as a result of a one-dimensional row-incremental Markov growth process. The statistics of the within-row components, or runs, are easy to obtain. The expected number of cells in a component of a regular mosaic is the same as the expected number of points in a component of the regular lattice. The expected area of the mosaic component is then obtained by using the known cell area. For the random models, the cell centers do not form a regular lattice. However, the expected number of neighbors of a cell and the expected number of cells meeting at a vertex are fixed for a given tessellation. A conjecture is presented that suggests that the expected area of a component in a random mosaic can be approximated by the expected area of a component in a regular mosaic that has the same cell area and number of cell neighbors as the corresponding expected values in the random mosaic.

The expected perimeter of a component is estimated in terms of the expected number of sides of a cell in the component that belong to the component border. Expected component perimeter follows from the known expected perimeter of a cell, the expected number of sides of a cell, and the expected number of cells in a component obtained as described above.

The problem of estimating the expected width of a component, i.e., the expected length of intercept on an arbitrary component due to a randomly located and oriented line transect, is also considered in [1, 2]. The probability that the number of cells along the intercept is n can be determined easily. Given the orientation of the transect, the total length of the intercept in a regular tessellation can then be expressed in terms of the cell size. For the random tessellations, the orientation of the transect need not be known, since the intercept length is independent of the direction in which it is measured. The expected intercept length can be found by considering different values of n.

2.2. Geometric Properties of Components in Coverage Mosaics

Estimation of the expected area, expected perimeter, and expected width of a component in a coverage mosaic is discussed in detail in [1, 3]. Here we shall briefly outline the approaches used without giving any mathematical details.

The computation of the expected component area is very similar to that for the cell structure models. A component is viewed as resulting from stacking of overlapping runs of figure centers. A run of centers is defined as the sequence of those successive centers within a row whose corresponding figures overlap. A run in a given row may overlap with a run in a distant row if the figures are sufficiently large. The expected total number of components in a given image is derived from a Markov formulation of the component growth process. The expected total area covered by the figures is easy to obtain in terms of the probability that an arbitrary point is isolated. These two results together provide the expected component area.

The estimation of the expected perimeter makes use of the estimate of the expected total length of that part of the border of a figure that is not covered by any other figure. This latter estimate can be made in terms of the expected number of uncovered segments along the border of a figure and the expected length of one such segment. Exact formulas are obtained for the Euclidean plane mosaics, but results for the grid case are approximate. In multicolored coverage patterns the perimeter is computed from borders between different colors and the background and between different colors. It is easy to see that the former is the same as in binary coverage patterns, where all bombs have the same color. Different colors share this border with the background according to their stationary probabilities. Similarly, the expected length of the border between a given color and other colors is the difference of its expected lengths of border with the background when the figures with the other colors are not dropped and when they are dropped. Different colors share this border according to their stationary probabilities.

Computation of expected width of a component is relatively more complex for coverage models. The intercept of a component along a transect consists of smaller intercepts due to many overlapping figures. The distribution of the length of each of these smaller intercepts can be obtained. The component intercept can then be interpreted as formed by a renewal process where the ends of the smaller intercepts define the renewal "times". The expected length of the intercept is given by the renewal equation. This approach, however, requires that the figures used be convex.

2.3. Spatial Correlation in Mosaics

We shall now review those properties of the patterns generated by mosaic models that involve relationships between the gray levels (or colors, etc.) at a pair of points at a given distance and orientation. Once again, we keep the description nonmathematical for brevity. For details, see [1, 4] and the other references cited below.

The joint probability density function for a pair of points in a cell structure mosaic can be expressed in terms of the probability that the two points belong to the same cell. For points chosen at a random orientation, this latter probability is only a function of the distance d between them, and was denoted $W(d)$ earlier. For the regular cell structure models and the Poisson line model, the analytic expressions for $W(d)$ are known. For the occupancy model, it has been shown [17] to involve the solution of a complicated double integral. Ahuja [1, 4] has empirically estimated $W(d)$ for the occupancy and the Delaunay models. Since then, Moore [27] also has conducted experiments with the occupancy model, and has estimated $W(d)$ for that model. For the coverage models, computation of the joint probability density involves point containment properties of certain regions determined by the figures involved and the separation and orientation of the points. For the multicolored coverage models, one has to consider further the cases in which these regions may have different colors.

The joint probability density function can be used to derive many joint pixel properties. The autocorrelation function is a commonly used second-order statistic. For cell structure models, it is the same as the function $W(d)$, and therefore is known for all of the models we have considered. Modestino *et al.* [24] present an integral for the autocorrelation function for their generalized checkerboard model where the cell sides have exponentially distributed lengths. They also present the corresponding expression for the power spectral density. The second-order properties of the parallelogram tessellation model are given in [25]. We may note here that Modestino *et al.* assign normally distributed gray levels to the cells such that the gray levels of adjacent cells are correlated. This is in contrast to the process described in Section 1.1, in which the gray levels of the cells are independent. For the coverage models, the autocorrelation function is obtained by a straightforward application of its definition in conjunction with the known joint probability density function.

The variogram [16, 18], the expected squared difference between the colors of a randomly chosen pair of points, is another useful second-order property, similar to the autocorrelation function. The joint probability density functions for point pairs are used to obtain the variograms for the individual models.

The gradient density is a useful measure of the spatial variation of color in Euclidean plane patterns. For grid patterns generated by mosaic models, Ahuja [1, 4] relates the digital edge density (analogous to the gradient density) to the perimeter results for the Euclidean plane patterns. The orientation distribution of the edges is known from the underlying tessellation (cell structure models) or the shapes of the figures (coverage models). Approximate responses of several digital edge operators, such as horizontal, vertical, and Roberts, when applied to mosaics, are given.

2.4. Fitting Mosaic Models to Textures

In [33] some preliminary experiments on fitting mosaic models to real textures are described. Predicted variograms were computed for two models, checkerboard and Poisson line, and were fitted to the actual variograms of ten texture samples from Brodatz's album [11]. These textures were also thresholded, and the average component width was computed. This width agreed very closely with the width predicted by the better fitting model in each case.

Some further experiments on mosaic model fitting are reported in [7, 10]. Samples of four Brodatz textures (wool, raffia, sand, and grass) [11] and three terrain textures were segmented, and average component area and perimeter were computed. Values predicted by six cell structure models (checkerboard, hexagonal, triangular, Poisson line, occupancy, and Delaunay) were also computed. (Predictions were also made for the square bombing model, but they were very poor in all cases.) For each texture, the model parameters were adjusted to make the area predictions match the observed values, and the resulting errors in predicted perimeter were tabulated; and vice versa. The minimum area error and minimum perimeter error models for each texture were the same in nearly all cases, and were consistent from sample to sample for nearly all the textures.

REFERENCES

1. N. Ahuja, Mosaic models for image analysis and synthesis, Ph.D. dissertation, Department of Computer Science, University of Maryland, College Park, Maryland, 1979.
2. N. Ahuja, Mosaic models for images, 1: geometric properties of components in cell structure mosaics, *Inform. Sci.* **23,** 1981, 69–104.
3. N. Ahuja, Mosaic models for images, 2: geometric properties of components in coverage mosaics, *Inform. Sci.* **23,** 1981, 159–200.
4. N. Ahuja, Mosaic models for images, 3: spatial correlation in mosaics, *Inform. Sci.* **24,** to appear.
5. N. Ahuja and A. Rosenfeld, Mosaic models for textures, *IEEE Trans. Pattern Analysis Machine Intelligence* **3,** 1981, 1–11.
6. N. Ahuja and A. Rosenfeld, Image models, in *Handbook of Statistics*, Vol. 2 (P. R. Krishnaiah, Ed.), North-Holland, New York, to be published.
7. N. Ahuja and A. Rosenfeld, Fitting mosaic models to textures, in *Image Texture Analysis* (R. M. Haralick, Ed.), Plenum, New York, to be published.
8. N. Ahuja and B. Schachter, *Pattern Models*, Wiley, New York, to be published.
9. N. Ahuja and B. Schachter, Image models, *Comput. Surveys*, to appear.
10. N. Ahuja, T. Dubitzki, and A. Rosenfeld, Some experiments with mosaic models for images, *IEEE Trans. Systems, Man, Cybernet.* **SMC-10,** 1980, 744–749.
11. P. Brodatz, *Textures: A Photographic Album for Artists and Designers*, Dover, New York, 1966.
12. D. Dufour, Intersections of random convex regions, Stanford University, Dept. of Statistics, T.R. 202, 1973.
13. E. N. Gilbert, Random subdivisions of space into crystals, *Ann. Math. Stat.* **33,** 1962, pp. 958–972.
14. R. M. Haralick, Statistical and structural approaches to texture, in *Proc. 4th Int. Joint Conf. Pattern Recognition, November 1978*, pp. 45–69.
15. J. K. Hawkins, Textural properties for pattern recognition, in *Picture Processing and Psychopictorics* (B. S. Lipkin and A. Rosenfeld, Eds.), pp. 347–370, Academic Press, New York, 1970.
16. C. Huijbregts, Regionalized variables and quantitative analysis of spatial data, in *Display and Analysis of Spatial Data* (J. Davis and M. McCullagh, Eds.), pp. 38–51, Wiley, New York, 1975.