

Donald W. Katzner

# Walrasian Microeconomics

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An Introduction to  
the Economic Theory  
of Market Behavior

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## **An Introduction to the Economic Theory of Market Behavior**

**Donald W. Katzner**

UNIVERSITY OF MASSACHUSETTS, AMHERST



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# **Walrasian Microeconomics**

**in memory of  
TARA**

# Preface

It is difficult to approach the study of microeconomic theory today without some feelings of ambivalence. Certainly, one cannot help but be attracted by the magnetism of success. An uninitiated outsider is unlikely to expect anything but chaos in an economy motivated solely by the self-interest of individuals and firms. That economists are able to assert coherence rather than chaos, at least in theory, is an intellectual achievement of immense proportion. This, by itself, is sufficient justification for taking the subject seriously. But there are additional reasons as well. Current microeconomic theory serves as a basis for the major paradigm within which most present-day economists think and communicate. It provides insight into a variety of isolated collections of empirical data extracted from the real world. Its study yields an understanding of deep and useful analytical tools. Its fundamental notions of simultaneity (both static and dynamic) and general equilibrium apply far beyond the limited boundaries of purely microeconomic, and even economic, phenomena. And the vision from which it springs is intimately intertwined with the historic emphasis on individual imagination and behavior that has formed such an important part of modern Western thought and culture.

Still, doubts remain. Lurking somewhere in the shadows is the suspicion that maybe microeconomic theory is not all that relevant for today's world. The economy is not, after all, exactly what you would call perfectly competitive; it is not clear that actual consumer and firm activity is consistent with behavior based on maximization; and so on. Thus, as one might expect, serious challenges to the main arguments of microeconomic theory have been mounted. Both methodology and assumptions are presently under attack, and alternatives have been proposed. Where all of this will lead is anybody's guess, but none of it can detract from the accomplishments of microeconomic theory described above. The issue is no longer whether coherence in the face of self-interest is theoretically possible, for that has already been established in the affirmative. Rather, the question is, how can we best understand whatever coherence there might be in the actual economic world in

which we live? Thus, a final reason for studying microeconomic theory is to determine exactly what various forms of theoretical (microeconomic) coherence entail, so it becomes possible to check which, if any, of these forms has relevance for the real economy.

The present volume introduces many of the ideas and propositions of the main body of microeconomic theory in a unified way. It focuses on individuals, firms, markets, and their interactions. It looks upon consumer and firm behavior as the outcome of decisions taken for single periods, or instants, of time. And it tries to construct a complete, consistent and cohesive picture of the perfectly competitive economy. This emphasis on unity is the central theme of the book; a serious effort has been made to avoid anything that detracts from a tightly knit articulation of the single vision it describes. Hopefully, it imparts a whole image, one that stands on its own and that therefore is capable of evaluation.

For lack of a better term, the theory constructed here and the view of economic reality from which it springs are described by the adjective "Walrasian." Although there is still dispute as to whether subsequent development accurately reflects Walras' true vision,<sup>1</sup> such usage is more or less consistent with present practice by economists. The expression "general equilibrium" also is relevant, but because this expression is applied often in other contexts as well, it is not employed below.

The appearance of the word "Introduction" in the title of this book gives license for certain liberties of exposition. Thus attention frequently focuses on the two-commodity, two-person, or two-firm case, while the more general argument is left to the reader. Simplifying assumptions are invoked for expositional ease and convenience. For example, the supposition that all firms within any given industry are identical appears frequently; the requirement that factors of production supplied by individuals to firms are fixed is imposed in the discussions of imperfect competition (Chapter 10), welfare (Chapter 11), and capital (Chapters 12 and 13); and the complicated second-order maximization conditions necessary to the traditional analysis of welfare (Chapter 11) are avoided by assuming that they are always satisfied. Lastly, many topics are ignored altogether. The most notable of these include linear programming, joint outputs in production, nontâtonnement dynamic processes, dated commodities, informational problems, risk, uncertainty, continua of traders, game theory, and the core.

Another aspect of the present work requiring comment is the role of mathematics. The past 50 years have witnessed increasing application of mathematical tools to many areas of economic analysis. There are several explanations for this embrace of mathematics. Jevons<sup>2</sup> and Cournot<sup>3</sup> believed that

<sup>1</sup> See, for example, W. Jaff , "Walras's Economics as Others See It," *Journal of Economic Literature* 18 (1980), pp. 528–549.

<sup>2</sup> W. S. Jevons, *The Theory of Political Economy*, 5th ed. (New York: Kelley, 1965), pp. xxi–xxv (from the preface to the 2nd edition), and pp. 3–5.

<sup>3</sup> A. Cournot, *Researches into the Mathematical Principles of the Theory of Wealth*, N. T. Bacon trans. (New York: Kelley, 1960), pp. 2–5.

certain forms of reasoning in economics are mathematical in character. Walras<sup>4</sup> thought that to be scientific was, in part, to be mathematical, and he accurately predicted the explosion of mathematical usage by economists as their science progressed in the 20th century. More recently Debreu<sup>5</sup> cites linguistic convenience and the ability to obtain deeper understanding and analytical extensions that might not otherwise be possible. And Weintraub<sup>6</sup> argues that we comprehend our economic world by organizing what we see and by seeking connections between the things that are observed. In other words, we create mental structure. Because doing mathematics is creating mental structure in its purest form, mathematics has naturally become important in the pursuit of economic inquiry. But regardless of the efficacy of any of these explanations the fact remains that to understand and communicate ideas today, especially in microeconomic theory, requires the employment of mathematical methods. Succeeding chapters, therefore, must necessarily rely heavily on mathematical deduction. However, an attempt has been made to make “non-mathematical elements,” such as the role of assumptions and what happens when they are violated, stand out.

In addition, several pedagogic devices are included to aid the reader. First, the names of concepts that are formally defined in the text are often printed in italics where they are defined. Second, all important mathematical notions employed in the text but not defined therein are rigorously characterized in Appendix A. Third, this appendix provides a self-contained statement, occasionally with discussion, of the main mathematical ideas and propositions appearing in the book. No proofs are given, but references to standard treatments are cited throughout both text and appendix. Fourth, as the main argument of the book begins (Chapter 2), formality and rigor of thought are emphasized heavily. Attention is drawn to particularly important lines of reasoning by use of the theorem and proof form of exposition. As the reader comes to understand the disciplined thinking required in microeconomic theory, such properness and rigidity relax gradually into a more fluid and flexible style. By then the reader should be able to supply the formality and rigor on his or her own. Fifth, the mathematical prerequisites are kept to a minimum—namely, a thorough background in calculus and a little familiarity with elementary linear algebra. (Readers should be warned, however, that they will have to develop, if they have not already done so, some sophistication in their ability to reason mathematically.)

For the most part, ensuing discussion is self-contained and builds on itself. Concepts are defined and propositions are usually either proved (at least in the two-dimensional case) or a method of proof is indicated for the interested reader to follow up. Chapter 1 is an exception. This chapter, in providing background for

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<sup>4</sup> L. Walras, *Elements of Pure Economics*, W. Jaffé trans. (Homewood, Ill.: Irwin, 1954), pp. 47–48.

<sup>5</sup> G. Debreu, “Economic Theory in the Mathematical Mode,” *American Economic Review* 74 (1984), p. 275.

<sup>6</sup> E. R. Weintraub, *General Equilibrium Analysis: Studies in Appraisal* (Cambridge: Cambridge University Press, 1985), pp. 178, 179.



what is to come, necessarily introduces concepts whose formal definitions have to wait for later development. A second exception appears in Section 2.2 which relies on mathematics somewhat beyond the level of calculus. Chapter exercises act as foils against which the reader may test her or his understanding, and they provide supplements to and illustrations of textual material. A very broad range of difficulty is represented. The more challenging exercises are marked with an asterisk; the hardest of these are identified by a double asterisk. Many answers and hints may be found in Appendix B. Without any pretense to being complete, references are made in the text to the historical sources of some of the more important economic ideas. Occasional comments in footnotes document the vagaries of several of today's accepted connections between the origins of certain notions and our ancestral economists.

The present work was undertaken in response to the unique demands of the Ph.D. program in economics at the University of Massachusetts at Amherst. Not surprisingly, one purpose of the program's initial graduate-level, two-semester exposure to microeconomic theory is to bring its students to roughly the same level of competence in microeconomics as their counterparts tend to achieve after one year of study of microeconomic theory at other major institutions. The course covers most of the important topics and requires mastery of many standard analytic techniques. But in addition to this, and here is where uniqueness arises, students are expected to develop a concept of microeconomic, "neoclassical" theory as a whole so as to be able to compare the neoclassical construction to alternative paradigms of economic thought. It is my hope that the book will be useful to others with similar needs.

Apart from classroom pedagogy, however, thinking of microeconomic neoclassical theory as a totality naturally becomes important in its own right as the theory is questioned and as it develops in response to criticism. Only from the holistic perspective is one able to speculate and ponder the meaning of it all. Reflections on meaning might include contemplations on the significance of the theory's assumptions, implications, and conclusions; on the merits of the uses to which it might be put; and on its power and limitations as a metaphor for understanding microeconomic activity. Such contemplations involve nothing less than the evaluation of the theory itself. And whether we like it or not, evaluation is inevitable. Indeed, it goes on all the time. Evaluation is both necessary for and an integral part of the evolution of any science.

I owe a considerable debt to those who have contributed to subsequent chapters in their numerous stages of development. Ivor F. Pearce helped to clarify my thinking on many aspects of microeconomic theory during frequent, sometimes long, and always very stimulating conversations over a period of an academic year. His patience and insight were truly inspiring. In addition to teaching me much of what I know about capital theory, Douglas Vickers made more cogent and useful comments on various drafts and supplied more ideas for revision than any author has any right to expect—even from a friend. Randall Bausor, Frederick A. Curtis, Philip E. Mirowski, E. Roy Weintraub, Robert Paul Wolff, and especially John P.

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*Amherst, Massachusetts*

D. W. K.

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# Chapter

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# 1

## Introduction

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Many years ago, Alfred Marshall eloquently characterized economics, in part, as the “study of mankind in the ordinary business of life . . .” [7, p. 1]. Today this sweeping idea is strikingly out of style. The present age, with its emphasis on precision of thought and numerical measurement of fact, seems to have settled, at least insofar as Walrasian microeconomics is concerned, for the more confining view of Robbins; namely, that economics “is the science which studies human behavior as a relationship between ends and scarce means which have alternative uses” [11, p. 16]. Although they overlap considerably, the chasm separating various facets of these conceptions runs wide and deep. Whereas Marshall really did attempt to deal “not with an abstract or ‘economic’ man; but with a man of flesh and blood” [7, p. 27], all that appears to have survived in Walrasian analysis is *homo economicus*. The exorcising of the “flesh and blood,” no doubt, is one reason for the frequent complaints that the Walrasian theory of market behavior

suffers, perhaps fatally, from the unreality of its hypotheses and the uselessness or inapplicability of its conclusions.

Against this background, it is clearly important to know exactly what the Walrasian theory says and what it does not say; what it can do and what it cannot do; what its assumptions are and the implications that may be drawn from them. A thorough understanding of these matters may then serve as a platform from which the merits and losses of straying from the Marshallian perspective may be surveyed. But before rushing too far ahead, it is well to begin with a description of the thing under investigation.

## 1.1 A DESCRIPTION OF THE ECONOMY

Every person entering into a scientific inquiry arrives with certain concepts, understandings, beliefs, values, and biases inherited from past experience. This legacy derives from all earlier interactions of social, economic, ideological, psychological, and other forces on the individual, and is taken as given. It is the basis upon which one selects what is important for study and how what is important is to be analyzed.

So, too, with economists. Looking out on any society, the economist sees an innumerable array of economic facts and events. It is impossible to take every one into account. Some must be discarded. Others are organized and grouped into categories according to simplified characteristics commonly shared. As indicated above, criteria for organizing facts in any particular way derive from the economist's acquired knowledge, values, and so on. The process is called abstraction. Its purpose is to enable him to focus on those elements chosen as important.

The following description of the economy is an abstraction that often serves as a foundation for the Walrasian theory of market behavior. It should be regarded as preliminary and subject to subsequent amplification and modification.

The first elements to be discussed are goods. A *good* is a material thing or service that has the capacity of directly satisfying human wants, or can be used to produce something having that capacity, or both. The term *commodity* is synonymous with that of good. *Final goods* are passed immediately into the hands of individuals to satisfy wants. *Resources* are nonproduced goods needed to produce other goods. Occasionally goods produced in the past and productively used in the present, such as machinery, are referred to as resources. *Intermediate goods* are currently produced goods also employed in the productive process. Quantities of goods are measured in appropriate units, and all like goods measured in the same units are treated as homogeneous. Two goods of one general class, but of different qualities or in different geographic locations, appear as distinct goods measured in their own units. No distinction is made between a good at one point in time and the same good at any other point in time.

Goods are bought and sold at prices expressed in terms of, say, dollars per unit of the good in question. Not every good, however, need be scarce. Some, such

as air, may (ignoring the presence of pollution) exist in sufficient supply so as to satisfy all human wants relating to them. The prices of these goods are zero. A good has a positive price only when, relative to wants, there is not enough of it to go around.

A somewhat old-fashioned but still useful classification of resources is based on the categories of land, labor, capital, and enterprise. Land often includes natural resources (like oil) as well as land itself, while labor covers a variety of skilled and unskilled work the population is able to perform. The term capital may refer to money or to physical objects such as buildings and machinery. Although it turns out to be irrelevant in the present context (see Section 12.3), enterprise (or entrepreneurship) has historically been defined as the willingness and ability to assume the risk of organizing and operating a business endeavor.<sup>1</sup> The usual names for the rewards accruing to the owners of these resources are, respectively, rents, wages, interest, and profits.

The terms “rent” and “interest,” however, must be employed with care. Clearly the natural resource portion of land is not rented but sold in the usual way. Land itself, as well as physical capital objects, can either be rented or sold. (The relationship between sale and rental prices is described in Section 12.1.) By contrast, money capital is borrowed and then restored, with a fee paid for its use. This fee is the source of the word “interest.” To avoid confusion, the following conventions are adopted here: Land is rented and not sold. Physical capital is sold and not rented; that is, as physical capital is exchanged, it warrants a market price, but no rate of return is attributed to it. The fee for borrowing money capital is referred to as the return on money capital rather than as interest.

The economic activity of human beings in society takes two forms. On one hand, individuals provide land, labor, money capital (that is, savings) and enterprise in exchange for income. On the other hand, they use income to obtain goods satisfying their wants. (People neither purchase physical capital goods for their own consumption nor sell them to firms for use in production.) In either case, although she may be subject to certain constraints, the individual is free to choose: to work or not, which job, how to spend her income, and so on. The presence of alternatives implies that decisions must be made, and the actual alternative selected usually appears to be in the best interest of the person making the decision. Thus the individual, or *consumer*, as she is often called, is an important decision-making unit in the economy. Households, that is groups of individuals acting as one unit, are counted as single individuals.

A second major type of decision-making unit is the firm. A *firm* is an institution that, using money capital, employs certain goods to produce certain other goods and also operates in its own self-interest. The goods produced are

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<sup>1</sup> Walras perceived the reward for entrepreneurial activity as unreliable. Indeed, there is none at equilibrium. Hence entrepreneurs must make their living elsewhere as landowners, laborers, or lenders [15, pp. 225, 227].



referred to as *outputs*; those employed in production are given the name *inputs*, or *factors*. There are four classes of factors drawn from the four categories of resources described above: land, labor, physical capital, and enterprise. Money capital, although a resource required by the firm for its day-to-day operation, is not used in production and is therefore not a factor. Rather, money capital provides the purchasing power that gives the firm command over factors of production. As part of the process of abstraction, firms are taken to produce only a single output. Each firm must choose the output it produces, the inputs it employs, and the method by which it transforms inputs into output. In so doing, the firm is both aided and confined by existing technology, that is, the pool of all knowledge concerning the methods of obtaining outputs from inputs. Collections of firms with similar outputs are *industries*.

Another important distinction is between flows and stocks. *Flows* are quantities that pass through markets (the concept of a market is characterized momentarily) at definable rates per unit of time. *Stocks* are accumulations of prior flows. Thus the current output of firms, and the current purchases of final goods and the sale of factors (except physical capital) by consumers are all flows. Moreover, the machines currently owned by firms constitute stocks, whereas the purchases of new machines represent flows. Likewise, the money capital currently tied up in a firm is a stock and any current addition to that money capital is a flow.

All goods flowing among individuals and (or) firms, as well as payments for them (expressed in terms of a unit of account) pass through markets. A *market* is nothing more than an institutional arrangement facilitating such exchanges. As there are many different kinds of institutional arrangements, markets may assume a variety of forms. In any event, consumers buy final goods from firms and sell resources to them through markets. Firms borrow money capital and buy inputs from consumers (resources), buy inputs including physical capital from firms (intermediate goods), and sell their outputs (final and intermediate goods) to consumers and firms. Observing the pattern of flows through markets it is possible to see what goods are being produced and in what quantities, how and with what quantities of which inputs they are being produced, how much and to which firms consumers are supplying their labor and other resources, the distribution of income across individuals, and the way final commodities are apportioned among consumers. A schematic diagram of these flows is provided by Fig. 1.1. It should be noted that the inner flow of payments in the diagram exactly offsets the value of the outer flow of goods (including money capital), that the purchase and sale of intermediate goods (including physical capital) takes place within the box labeled "firms," and that in moving around the circles from consumers to firms and back again, no payments or goods are lost or leak out of the system. Thus there is no production for inventories, no government, and no trade with foreign countries. Of course, this picture may be expanded to include these elements, but for the time being there is much to be said for keeping matters as simple as possible.

Still further aspects of the economic environment need mention. For example, institutional rules governing property rights and the exchange of commodities