A TEXT/WORKBOOK

BEGINNING ALGEBRA

FOURTH EDITION

CHARLES P. McKEAGUE

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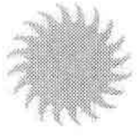

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Preface to the Instructor

This fourth edition of *Beginning Algebra: A Text/Workbook* retains the same basic format and style as the third edition. The book can be used in either a traditional lecture class or in a self-paced situation. In a lecture-format class, each section of the book can be discussed in a forty-five to fifty minute class session. In a self-paced situation, the practice problems in the margins allow the student to become actively involved with the material before working the problems in the problem set.

Features of the Book

Chapter Openers New to this edition, each chapter begins with a two-page opener that includes the following four elements:

1. *Introduction* Each chapter opens with an introduction in which a real-world application, historical example, or a link between topics is used to stimulate interest in the chapter. Whenever possible, these introductions are expanded on later in the chapter and then carried through to topics found further on in the book.
2. *Overview* A general overview of the chapter follows the chapter introduction. The overview lists the important topics that will be covered in the chapter, along with their connection to one another and to topics covered previously in the text. Most of the overviews end with a list of topics from previous chapters that students need to know in order to be successful in the current chapter.
3. *Study Skills* Found in the first six chapter openers is a list of study skills intended to help students become organized and efficient with their time. The study skills point students in the direction of success. They are intended to benefit students in this course and throughout their college careers. These skills are more detailed than the general study skills listed in the Preface to the Student.
4. *Content and Objectives* A section by section list of learning objectives concludes the chapter openers. These objectives are listed again at the beginning of the problem sets.

Practice Problems The practice problems, with their answers and solutions, are the key to moving students through the material in this book. In the margin, next to each example, is a practice problem. The practice problems are to be worked by students after they have read through the corresponding example. The

answer to each practice problem is at the bottom of the page, while solutions to most practice problems are in the back of the book.

Organization of the Problem Sets Five main ideas are incorporated into each problem set.

1. *Drill* There are enough problems in each set to ensure student proficiency in the material.
2. *Progressive Difficulty* The problems increase in difficulty as the problem set progresses.
3. *Odd-Even Similarities* Each pair of consecutive problems is similar. Since the answers to the odd-numbered problems are listed in the back of the book, the similarity of the odd-even pairs of problems allows your students to check their work on an odd-numbered problem and then to try the similar even-numbered problem.
4. *Application Problems* Students are always curious about how the algebra they are learning can be applied, but at the same time many students are apprehensive about attempting application problems. I have found that they are more likely to put some time and effort into trying application problems if they do not have to work an overwhelming number of them at one time and if they work on them every day. For these reasons, I have placed a few application problems toward the end of almost every problem set in the book.
5. *Review Problems* As was the case in the third edition, each problem set, beginning with Chapter 2, contains a few review problems. Where appropriate, the review problems cover material that will be needed in the next section. Otherwise they cover material from the previous chapter. That is, the review problems in Chapter 5 cover the important points covered in Chapter 4. Likewise, the review problems in Chapter 6 review the important material from Chapter 5. If you give tests on two chapters at a time, you will find this to be a time-saving feature. Your students will review one chapter as they study the next chapter.

Objectives in the Problem Sets New to this edition, each problem set begins with a list of learning objectives cross-referenced to the examples and problems to which they pertain. These are the same objectives that are listed in the chapter openers.

Blueprint for Problem Solving New to this edition, the Blueprint for Problem Solving is a detailed outline of the steps needed to successfully attempt application problems. Intended as a guide to problem solving in general, the blueprint overlays the solution process to all the application problems in the first few chapters of the book. As students become more familiar with problem solving, the steps in the blueprint are streamlined.

Research Projects Scattered throughout this edition are problems for students to research and then report on. Although they appear at the end of a number of the problem sets, they are not intended to be part of the student's daily assignment. In my classes, these are the problems I use for extra credit. In most cases, I require students to type their reports, just as they would type an essay in their English classes. Do not be concerned if you are not familiar with the topics shown in the research projects; the idea behind these projects is to have your students do the research, and then tell you what they have learned.

Chapter Summaries Each chapter summary lists the new properties and definitions found in the chapter. The margins in the chapter summaries contain examples that illustrate the topics being reviewed.

Chapter Reviews Each chapter ends with a set of review problems that cover all the different types of problems found in the chapter. The chapter reviews are longer and more extensive than the chapter tests.

Chapter Tests Each chapter test contains a representative sample of the problems covered in the chapter.

Changes in the Fourth Edition

In addition to the chapter openers, research projects, Blueprint for Problem Solving and learning objectives mentioned above, the followings items are also new to this edition.

Increased Visualization of Topics This edition contains many more diagrams, charts, and graphs than the previous edition. The purpose of this is to give students additional information, in visual form, to help them understand the topics we cover.

Facts from Geometry Many of the important facts from geometry are now listed under this heading. In most cases, an example or two accompanies each of the facts to give students a chance to see how topics from geometry are related to the algebra they are learning.

Number Sequences An introductory coverage of number sequences is integrated throughout Chapter 1 and then expanded and continued in Chapters 2 and 3. I find that there are many interesting topics I can cover if students have some experience with number sequences. It is also the easiest way to demonstrate inductive reasoning.

Tables, Histograms, Scatter Diagrams, and Line Graphs Beginning in Chapter 1 and then continued in Chapters 3 and 4, students are required to analyze information from tables. In addition to simply reading a table, they are given practice converting data in tabular form to data in graphical form that includes histograms and scatter diagrams. (Why not? Histograms, scatter diagrams, and line graphs are what they see if they read a newspaper or magazine.) From there they move on to graph ordered pairs and linear equations on a rectangular coordinate system. Section 3.1 has been rewritten completely to accommodate this new feature.

Unit Analysis Chapter 6 now contains problems requiring students to convert from one unit of measure to another. The method used to accomplish the conversions is the method they will use if they take a chemistry class. Since this method is similar to the method we use to multiply rational expressions, unit analysis is covered in Section 6.2 with multiplication and division of rational expressions.

Challenging Problems and Application Problems More challenging problems have been added to many of the problem sets. Some of the new problems are in the drill problem category, while many others are new, more realistic, application problems.

Supplements to the Textbook

This fourth edition of *Beginning Algebra: A Text/Workbook* is accompanied by a number of useful supplements.

- **Videotape Package** Free to adopters, the videotape package consists of 9 VHS videotapes, one for each chapter of the book. Each chapter tape is an

hour to an hour and a half in length and is divided into lessons that correspond to each section of the chapter.

For the Instructor

- **Instructor's Annotated Edition** Instructor versions of both worktexts are available to make for accessible and effective teaching. All answers to the problem sets are printed in a fifth color, under the problems, corresponding to the pages in the student's worktext.
- **Instructor's Solution Manual** The manual supplies instructor-appropriate solutions for half the odd and all the even exercises in the problem sets. Solutions for all other exercises comprise the Student's Solutions Manual.
- **Instructor's Drill and Test Manual** Included in the manual are 11 sets of ready-to-copy tests: one set for each chapter and one set for the entire book. Each set comprises two multiple-choice and four show-your-work tests. Items for half of the tests are ordered according to the sequence of topics in the book; items for the other half of the tests are in mixed-up order. Answers for every test item are provided, and are keyed to the book's section objectives. Completing the manual are over 900 extra drill exercises for selected section objectives. Answers for these exercises are provided as well.
- **Printed Test Bank** The test bank is comprised of multiple-choice test items organized by chapter, section, and objective.
- ***ExaMaster* + Computerized Test Bank** A flexible, powerful, computerized testing system, *ExaMaster* + offers teachers a wide range of integrated testing options and features. Available in either IBM or Macintosh format, teachers can select, edit, or create not only test items but algorithms for test items as well. Teachers can tailor tests according to a variety of criteria, scramble the order of test items, and administer tests on-line. *ExaMaster* + also includes full-function gradebook and graphing features.
- **Transparency Package** This package contains roughly 50 excerpts from the book, including graphs, worked examples, and key theorems, definitions, and properties, and are provided in a full-color transparency format suitable for overhead display.

For the Student

- **Student's Solutions Manual** This manual contains complete annotated solutions to every other odd problem in problem sets and all chapter review and chapter test exercises.
- **MathCue Tutorial** A computer software package of tutorials with problems that correspond to every section in the series. The software presents problems to solve and tutors students by displaying annotated, step-by-step solutions. Students may view partial solutions to get started on a problem, see continuous record of progress, and back up to review missed problems. Student scores can also be printed. Available for IBM and Macintosh.
- **MathCue Solution Finder** This software allows students to enter their own problems into the computer and get annotated, step-by-step solutions in return. This unique program simulates working with a tutor, tracks student progress, refers students to specific sections in the text when appropriate, and prints student scores. Available for IBM and Macintosh.
- **MathCue Practice** An algorithm-based software that allows students to generate large numbers of practice problems keyed to problem types from each section of the book. Practice scores students' performance, and saves students' scores session to session. Available for IBM and Macintosh.

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Preface to the Student

I often find my students asking themselves the question ''Why can't I understand this stuff the first time?'' The answer is ''You're not expected to.'' Learning a topic in algebra isn't always accomplished the first time around. There are many instances when you will find yourself reading over new material a number of times before you can begin to work problems. That's just the way things are in algebra. If you don't understand a topic the first time you see it, that doesn't mean there is something wrong with you. Understanding algebra takes time. The process of understanding requires reading the book, studying the examples, working problems, and getting your questions answered.

Here are some questions that are often asked by students starting a beginning algebra class.

How much math do I need to know before taking algebra? You should be able to do the four basic operations (addition, subtraction, multiplication, and division) with whole numbers, fractions, and decimals. Most important is your ability to work with whole numbers. If you are a bit weak at working with fractions because you haven't worked with them in a while, don't be too concerned; we will review fractions as we progress through the book. I have had students who eventually did very well in algebra, even though they were initially unsure of themselves when working with fractions.

What is the best way to study? The best way to study is to study consistently. You must work problems every day. A number of my students spend an hour or so in the morning working problems and reading over new material and then spend another hour in the evening working problems. The students of mine who are most successful in algebra are the ones who find a schedule that works for them and then stick to it. They work problems every day.

If I understand everything that goes on in class, can I take it easy on my homework? Not necessarily. There is a big difference between understanding a problem someone else is working and working the same problem yourself. There is no substitute for working problems yourself. The concepts and properties are understandable to you only if you yourself work problems involving them.

How to Be Successful in Algebra

If you have decided to be successful in algebra, then the following list will be important to you.

1. **If you are in a lecture class, be sure to attend all class sessions on time** You cannot know exactly what goes on in class unless you are there. Missing class and then expecting to find out what went on from someone else is not the same as being there yourself.
2. **Read the book** It is best to read the section that will be covered in class beforehand. Reading in advance, even if you do not understand everything you read, is still better than going to class with no idea of what will be discussed. As you read through each section, be sure to work the practice problems in the margin of the text. Each practice problem is similar to the example with the same number. Look over the example and then try the corresponding practice problem. The answers to the practice problems are given on the same page as the problems. If you don't get the correct answer, see if you can rework the problem correctly. If you miss it a second time, check your solution with the solution to the practice problem in the back of the book.
3. **Work problems every day and check your answers** The key to success in mathematics is working problems. The more problems you work, the better you will become at working them. The answers to the odd-numbered problems are given in the back of the book. When you have finished an assignment, be sure to compare your answers with those in the book. If you have made a mistake, find out what it is, and correct it.
4. **Do it on your own** Don't be misled into thinking someone else's work is your own. Having someone else show you how to work a problem is not the same as working the same problem yourself. It is okay to get help when you are stuck. As a matter of fact, it is a good idea. Just be sure you do the work yourself.
5. **Review every day** After you have finished the problems your instructor has assigned, take another fifteen minutes and review a section you have already completed. The more you review, the longer you will retain the material you have learned.
6. **Don't expect to understand every new topic the first time you see it** Sometimes you will understand everything you are doing, and sometimes you won't. That's just the way things are in mathematics. Expecting to understand each new topic the first time you see it can lead to disappointment and frustration. The process of understanding algebra takes time. It requires that you read the book, work problems, and get your questions answered.
7. **Spend as much time as it takes for you to master the material** No set formula exists for the exact amount of time you need to spend on algebra to master it. You will find out as you go along what is or isn't enough time for you. If you end up spending two or more hours on each section in order to master the material there, then that's how much time it takes; trying to get by with less will not work.
8. **Relax** It's probably not as difficult as you think.

Contents

CHAPTER 1

The Basics

Introduction

Much of what we do in mathematics is concerned with recognizing patterns and classifying together groups of numbers that share a common characteristic. Two groups of numbers that we will study in this chapter are shown below. Each of the groups form a pattern. As you will see, the characteristic shared by the numbers in each of these sequences can be described with algebra.

Sequence of odd numbers = 1, 3, 5, 7, 9, . . .
Sequence of squares = 1, 4, 9, 16, 25, . . .

Once we have classified groups of numbers as to the characteristics they share, we sometimes discover that a relationship exists between the groups. Although it may not be obvious at first, there is a relationship that exists *between* the two sequences shown above. The relationship has been known for some time. In *The Book of Squares,* written by the mathematician known as Fibonacci in 1225, Fibonacci begins the introduction this way:

> I thought about the origin of all square numbers and discovered that they arise out of the increasing sequence of odd numbers.

The relationship that Fibonacci refers to (and that you will discover later in this chapter) can be described very concisely with algebra.

Overview

Chapter 1 contains some of the most important material in the book. Your goal is to master it. Your success in the following chapters depends upon how well you understand Chapter 1. Here is a list, in order of importance, of the ideas you must know after you complete this chapter.

1. You *must* know how to add, subtract, multiply, and divide positive and negative numbers.
2. You *must* understand and recognize when the commutative, associative, and distributive properties are being used.
3. You should know the major classifications of numbers. That is, you should know the difference between whole numbers, integers, rational numbers, and real numbers.

Study Skills

Some of the students enrolled in my beginning algebra classes develop difficulties early in the course. Their difficulties are not associated with their ability to learn mathematics; they all have the potential to pass the course. Students who get off to a poor start do so because they have not developed the study skills necessary to be successful in algebra; they do not put themselves on an effective

homework schedule, and when they work problems, they do it their way, not my way. Here is a list of things you can do to begin to develop effective study skills.

1. **Put Yourself on a Schedule** The general rule is that you spend two hours on homework for every hour you are in class. Make a schedule for yourself in which you set aside two hours each day to work on algebra. Once you make the schedule, stick to it. Don't just complete your assignments and stop. Use all the time you have set aside. If you complete an assignment and have time left over, read the next section in the book, and then work more problems.
2. **Find Your Mistakes and Correct Them** There is more to studying algebra than just working problems. You must always check your answers with the answers in the back of the book. When you have made a mistake, find out what it is, and correct it. Making mistakes is part of the process of learning mathematics. In the prologue to *The Book of Squares,* Fibonacci has this to say about the content of his book:

   > I have come to request indulgence if in any place it contains something more or less than right or necessary; for to remember everything and be mistaken in nothing is divine rather than human . . .

   Fibonacci knew, as you know, that human beings make mistakes. You cannot learn algebra without making mistakes.
3. **Imitate Success** Your work should look like the work you see in this book and the work your instructor shows. The steps shown in solving problems in this book were written by someone who has been successful in mathematics. The same is true of your instructor. Your work should imitate the work of people who have been successful in mathematics.

Content and Objectives

SECTION 1.1 Notation and Symbols

Objective 1 To translate between phrases written in English and expressions written in symbols
Objective 2 To simplify expressions containing exponents
Objective 3 To simplify expressions using the rule for order of operations
Objective 4 To recognize the pattern in a sequence of numbers

SECTION 1.2 Real Numbers

Objective 1 To locate numbers on the number line
Objective 2 To simplify expressions containing absolute value
Objective 3 To identify the opposite of a number
Objective 4 To identify the reciprocal of a number
Objective 5 To multiply fractions
Objective 6 To find the perimeter and area of squares, rectangles, and triangles

SECTION 1.3 Addition of Real Numbers

Objective 1 To add two or more real numbers
Objective 2 To simplify expressions containing absolute value
Objective 3 To extend an arithmetic sequence
Objective 4 To translate sentences from English into symbols and then simplify

SECTION 1.4 Subtraction of Real Numbers

Objective 1 To subtract two real numbers
Objective 2 To simplify expressions containing subtraction using the rule for order of operations
Objective 3 To translate sentences from English into symbols and then simplify
Objective 4 To find the complement and the supplement of an angle

SECTION 1.5 Properties of Real Numbers

Objective 1 To identify and apply the commutative and associative properties of addition and multiplication
Objective 2 To identify and apply the distributive property
Objective 3 To identify inverse and identity elements

SECTION 1.6 Multiplication of Real Numbers

Objective 1 To multiply two or more real numbers
Objective 2 To simplify expressions containing multiplication using the rule for order of operations
Objective 3 To multiply positive and negative fractions
Objective 4 To apply the distributive property
Objective 5 To translate sentences from English into symbols and then simplify
Objective 6 To extend a geometric sequence

SECTION 1.7 Division of Real Numbers

Objective 1 To divide two real numbers
Objective 2 To divide fractions
Objective 3 To simplify expressions containing division using the rule for order of operations
Objective 4 To translate sentences from English into symbols and then simplify

SECTION 1.8 Subsets of the Real Numbers

Objective 1 To associate numbers with subsets of the real numbers
Objective 2 To factor whole numbers into their prime factors
Objective 3 To reduce fractions to lowest terms

SECTION 1.9 Addition and Subtraction with Fractions

Objective 1 To add or subtract two or more fractions with the same denominator
Objective 2 To find the least common denominator for a set of fractions
Objective 3 To add or subtract fractions with different denominators.

SECTION 1.1 Notation and Symbols

Since much of what we do in algebra involves comparison of quantities, we will begin by listing some symbols used to compare mathematical quantities. The comparison symbols fall into two major groups: equality symbols and inequality symbols.

We will let the letters a and b stand for (represent) any two mathematical quantities. When we use letters to represent numbers, as we are doing here, we call the letters *variables.*

Comparison Symbols

Equality:	$a = b$	a is equal to b (a and b represent the same number)
	$a \neq b$	a is not equal to b
Inequality:	$a < b$	a is less than b
	$a \not< b$	a is not less than b
	$a > b$	a is greater than b
	$a \not> b$	a is not greater than b
	$a \geq b$	a is greater than or equal to b
	$a \leq b$	a is less than or equal to b

The symbols for inequality, $<$ and $>$, always point to the smaller of the two quantities being compared. For example, $3 < x$ means 3 is smaller than x. In this case, we can say "3 is less than x" or "x is greater than 3"; both statements are correct. Similarly, the expression $5 > y$ can be read as "5 is greater than y" or as "y is less than 5" because the inequality symbol is pointing to y, meaning y is the smaller of the two quantities.

Next we consider the symbols used to represent the four basic operations—addition, subtraction, multiplication, and division.

Operation Symbols

Addition:	$a + b$	The *sum* of a and b
Subtraction:	$a - b$	The *difference* of a and b
Multiplication:	$a \cdot b$, $(a)(b)$, $a(b)$, $(a)b$, ab	The *product* of a and b
Division:	$a \div b$, a/b, $\frac{a}{b}$, $b\overline{)a}$	The *quotient* of a and b

When we encounter the word *sum,* the implied operation is addition. To find the sum of two numbers, we simply add them. *Difference* implies subtraction, *product* implies multiplication, and *quotient* implies division. Notice also that there is more than one way to write the product or quotient of two numbers.

Grouping Symbols

Parentheses ( ) and brackets [ ] are the symbols used for grouping numbers together. (Occasionally braces { } are also used for grouping, although they are usually reserved for set notation, as we shall see.)

The following examples illustrate the relationship between the symbols for comparing, operating, and grouping and the English language.

Practice Problems

Write an equivalent expression in English.

1. $3 + 7 = 10$

2. $9 - 6 > 4$

3. $4(2 + 3) \neq 6$

4. $6(8 - 1) = 42$

5. $\frac{4}{x} = 8 - x$

Expand and multiply.

6. 7^2

7. 3^4

8. 10^5

Note: For each example in the text there is a corresponding practice problem in the margin. After you read through an example in the text, work the practice problem with the same number in the margin. The answers to the practice problems are given on the same page as the practice problems. Be sure to check your answers as you work these problems. The worked-out solutions for the practice problems are given in the back of the book. So if you find a practice problem that you cannot work correctly, you can look up the correct solution to that problem in the back of the book.

Answers

1. The sum of 3 and 7 is 10. **2.** The difference of 9 and 6 is greater than 4. **3.** 4 times the sum of 2 and 3 is not equal to 6. **4.** Six times the difference of 8 and 1 is 42. **5.** The quotient of 4 and x is equal to the difference of 8 and x. **6.** 49 **7.** 81 **8.** 100,000

▶ EXAMPLES

	Mathematical Expression	*English Equivalent*
1.	$4 + 1 = 5$	The sum of 4 and 1 is 5
2.	$8 - 1 < 10$	The difference of 8 and 1 is less than 10
3.	$2(3 + 4) = 14$	Twice the sum of 3 and 4 is 14
4.	$3x \geq 15$	The product of 3 and x is greater than or equal to 15
5.	$\frac{y}{2} = y - 2$	The quotient of y and 2 is equal to the difference of y and 2

◀

The last type of notation we need to discuss is the notation that allows us to write repeated multiplications in a more compact form—*exponents.* In the expression 2^3, the 2 is called the *base* and the 3 is called the *exponent.* The exponent 3 tells us the number of times the base appears in the product. That is,

$$2^3 = 2 \cdot 2 \cdot 2 = 8$$

The expression 2^3 is said to be in exponential form, while $2 \cdot 2 \cdot 2$ is said to be in expanded form. Here are some additional examples of expressions involving exponents.

▶ EXAMPLES Expand and multiply.

6.	$5^2 = 5 \cdot 5 = 25$	Base 5, exponent 2
7.	$2^5 = 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 = 32$	Base 2, exponent 5
8.	$10^3 = 10 \cdot 10 \cdot 10 = 1{,}000$	Base 10, exponent 3

◀

The symbols for comparing, operating, and grouping are to mathematics what punctuation symbols are to English. These symbols are the punctuation symbols for mathematics.

Consider the following sentence:

Paul said John is tall.

It can have two different meanings, depending on how it is punctuated.

1. "Paul," said John, "is tall."
2. Paul said, "John is tall."

Without the punctuation we do not know which meaning is intended. It is ambiguous without punctuation.

Let's take a look at a similar situation in mathematics. Consider the following mathematical statement:

$$5 + 2 \cdot 7$$

If we add the 5 and 2 first and then multiply by 7, we get an answer of 49. On the other hand, if we multiply the 2 and the 7 first and then add 5, we are left with 19. We have a problem that seems to have two different answers, depending on whether we add first or multiply first. We would like to avoid this type of situation. That is, every problem like $5 + 2 \cdot 7$ should have only one answer. Therefore, we have developed the following rule for the order of operations.