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# Mathematics of Uncertainty Modeling in the Analysis of Engineering and Science Problems



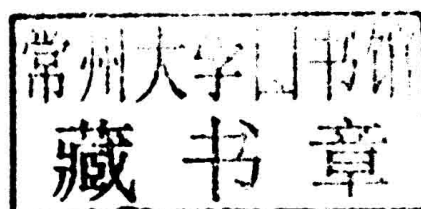
S. Chakraverty



# Mathematics of Uncertainty Modeling in the Analysis of Engineering and Science Problems

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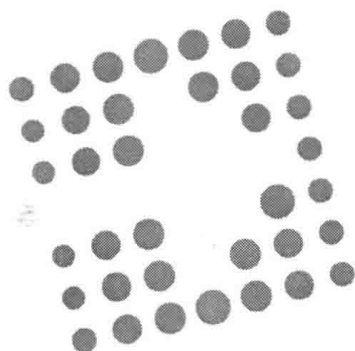
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# Preface

Uncertainty has been a concern to engineers, managers, and scientists for many decades in various practical problems. For a long time, uncertainty has been considered synonymous with random, stochastic, statistic, or probabilistic. Since the early sixties, views on uncertainty have become more heterogeneous and more tools that model uncertainty than statistics have been proposed by several engineers and scientists. The random or statistical modeling needs plenty of data or distributions to handle the uncertainty. This may be one of the problems in actual practice.

Every physical problem of engineering and science is inherently biased by uncertainty. There is often a need to model, solve and interpret the problems one encounters in the world of uncertainty. Models and parameters of practical application are usually established on the basis of plans, drawings, measurements, observations, experiences, expert knowledge, codes and standards and so on. In general, exact information and precise values do not exist. Various uncertainties may result from human mistakes, errors in the manufacture and construction and from the lack of information. Small samples, changing reproduction conditions and imprecise results of measurements are usual starting points for defining engineering and science models. In order to perform realistic science and engineering analysis and proper safety assessment, the uncertainty in both data and models must be appropriately taken into consideration.

Rather than the particular values of the material properties or parameters, we may have only the imprecise bounds of the values. These may be handled by taking the parameters in terms of interval and/or fuzzy. Recently investigations are carried out by various researchers throughout the globe by using the uncertainty of the material properties or parameters. The corresponding problem will become then uncertain and the analysis and solution would require then careful application of the methods. Recently the soft computing methods have come as a relief to handle such problems. In order to have the idea of handling the uncertainty in the physical problems, this book may give a new direction to take the challenge.

This book targets to handle the uncertainty mainly in term of interval, fuzzy and stochastic along with other methods of mathematics and machine intelligence. The book initially includes the basics of interval mathematics, evidence theory based uncertainty and probabilistic uncertainty modeling. General problems of various science, engineering and management are presented to handle the uncertainty with the use of soft and other computing techniques. This book is an attempt to bring together the faculties, scientists, engineers and technologists from various fields of science, engineering and management to discuss the recent trends, usefulness and challenges of mathematics of uncertainty in general and fuzzy, interval and stochastic based uncertainty modeling in particular.

As such, this book contains seventeen chapters covering various aspects of uncertainty from theoretical to application problems.

Accordingly, the first chapter by Hend Dawood includes an introduction about the theories of interval algebra and methods to uncertainty analysis in science and engineering problems. In view of this purpose, the author of this chapter introduces the key concepts of the algebraic theories of intervals that form the foundations of the interval techniques as they are now practised. It also provides a historical and epistemological background of interval mathematics and uncertainty. Finally this chapter describes some typical applications that clarify the need for interval computations to cope with uncertainty in a wide variety of scientific disciplines.

The second chapter is about uncertainty modeling using expert's knowledge as evidence written by D. Datta. Very often availability of data is incomplete in the sense that sufficient amount of data which is required may not be possible to collect. Therefore, uncertainty modeling in that case may not be possible using probability theory or Monte Carlo method. Fuzzy set theory or any other imprecision based theory may be applicable in that case. With a view to this, expert's knowledge is represented as the input data set. Belief and plausibility are the two bounds (lower and upper) of the uncertainty of this imprecision based system. The fundamental definitions and the mathematical structures of the belief and plausibility fuzzy measures are discussed in this chapter. Uncertainty modeling using this technique has been illustrated with a simple example of contaminant transport through groundwater.

The next chapter is by Tazid Ali who has discussed about evidence based uncertainty modeling. Evidence is the essence of any decision making process. However in any situation the evidences that we come across are usually not complete. Absence of complete evidence results in uncertainty, and uncertainty leads to belief. The framework of Dempster-Shafer theory which is based on the notion of belief is overviewed in this chapter. Methods of combining different sources of evidences are surveyed. Relationship of probability theory and possibility theory to evidence theory is exhibited. Extension of the classical Dempster-Shafer Structure to fuzzy setting is discussed. Finally uncertainty measurement in the frame work of Dempster-Shafer structure is dealt with.

The fourth chapter presents hybrid set structures for soft computing authored by Sunil Jacob John and Babitha K. V. A Major problem in achieving an effective computational system is the presence of inherent uncertainty in the computational problem itself. Among various techniques proposed to address this, the technique of soft computing is of significant interest. Further, Generalized set structures like fuzzy sets, rough sets, multisets etc. have already proven their role in the context of soft computing. The computational techniques based on one of these structures alone will not always yield the best results but a fusion of two or more of them can often give better results. Such structures are regarded as hybrid set structures. This chapter surveys analysis of various hybrid set structures which are quite useful tools for soft computing and shows how this hybridization can help to improve the modeling of the real situations.

Then source and  $m$ -source distances of fuzzy numbers and their properties have been presented by Majid Amirfakhrian in Chapter 5. In many applications of fuzzy logic and fuzzy mathematics we need (or it is better) to work with the same fuzzy numbers. In this chapter the author presents source distance and  $m$ -source distance between two fuzzy numbers. Also some properties of parametric  $m$ -degree polynomial approximation operator of fuzzy numbers are also discussed. Numerical examples are solved related to the present analysis.

The sixth chapter is by Hemanta K. Baruah who has discussed construction of normal fuzzy numbers using the mathematics of partial presence. Every normal law of fuzziness can be expressed in terms of two laws of randomness defined in the measure theoretic sense. Indeed, two probability measures are necessary and sufficient to define a normal law of fuzziness. Hence, the measure theoretic matters with reference to fuzziness have to be studied accordingly. In this chapter, the author has discussed how to construct normal fuzzy numbers using mathematics of partial presence. Three case studies have been presented with reference to expressing stock prices in terms of fuzzy numbers.



The seventh chapter is on Numerical solution of fuzzy differential equations and its applications written by S. Chakraverty and Smita Tapaswini. Differential equations are extensively used in modelling different problems of science and engineering. For the sake of simplicity the parameters and variables involved in the systems are considered as crisp or defined exactly. The solution of the differential equation with the variables and parameters as crisp can be handled either by exact methods or by known numerical methods developed by many authors.

But rather than the particular (exact) value we may have only the vague or incomplete information about the variables and parameters as those are obtained by some experiment or experience. These are uncertain in nature. Now-a-days these uncertainties are modelled through convex normalised fuzzy sets. So, we need to solve differential equations accordingly with fuzzy variables and parameters.

Since, it is too complicated to obtain the exact solution of Fuzzy Differential Equations (FDEs), so the numerical methods are used to obtain the solution of fuzzy differential equations. As mentioned above, due to practical use of the realistic model of the problem, this actually turns in to FDEs. As such, investigation of the fuzzy differential equations (ordinary and partial) by taking initial or boundary conditions as fuzzy is an important area of research. In this chapter the authors have presented various numerical techniques viz. Euler and improved Euler type methods and Homotopy Perturbation Method (HPM) to solve fuzzy differential equations. Also application problems such as fuzzy continuum reaction diffusion model to analyse the dynamical behaviour of the fire with fuzzy initial condition is investigated. To analyse the fire propagation, the complex fuzzy arithmetic and computation are used to solve hyperbolic reaction diffusion equation. This analysis finds the rate of burning number of trees in bounds where wave variable/time are defined in terms of fuzzy. Obtained results are compared with the existing solution to show the efficiency of the applied methods.

The eighth chapter is that of Marek T. Malinowski who has discussed about modeling with Stochastic Fuzzy Differential Equations (SFDE). Applicability of ordinary differential equations in modeling dynamical systems is of great importance. However, modeling with ordinary differential equations is suitable in case of a perfect knowledge about considered dynamical system. In particular, parameters of the system and its initial state have to be known precisely. They should also be described using the precise, single values (singletons) of a phase space. However, a typical feature of behaviour of real dynamical systems is uncertain which contrasts with precision. The two most important kinds of uncertainties are randomness (stochastic uncertainty) and imprecision (sometimes called fuzziness, vagueness, ambiguity, softness). For a long time, an usual way of describing uncertainty in modeling was that of stochastic models which were based on probability theory and stochastic analysis. Stochastic models of dynamical systems in the form of random differential equations or stochastic differential equations do not include all kinds of uncertainties. They include stochastic uncertainty only. When data of the considered systems are imprecise due to lack of precision of measuring instruments there appears uncertainty which is not of stochastic type and then stochastic models are no longer applicable. Uncertain information which is not stochastic in its nature is appropriately treated by fuzzy set theory. These equations are new mathematical tools for modeling uncertain dynamical systems. Some qualitative properties of their solutions such as existence and uniqueness are recalled, and stability properties are shown. Here, the solutions are continuous adapted fuzzy stochastic processes. Some examples of applications of stochastic fuzzy differential equations in modeling real-world phenomena have been considered.

In chapter nine mathematics of probabilistic uncertainty modeling has been explained by D Datta. This chapter presents the uncertainty modeling using probabilistic methods. Probabilistic method of uncertainty analysis is due to randomness of the parameters of a model. Randomness of parameters is

characterized by specified probability distribution such as normal, log normal, exponential etc., and the corresponding samples are generated by various methods. Monte Carlo simulation is applied to explore the probabilistic uncertainty modeling. Monte Carlo simulation being a statistical process is based on the random number generation from the specified distribution of the uncertain random parameters. Sample size is generally very large in Monte Carlo simulation which is required to have small errors in the computation. Latin hypercube sampling and importance sampling are explored in brief. This chapter also presents polynomial chaos theory based probabilistic uncertainty modeling. Polynomial chaos theory is an efficient Monte Carlo simulation in the sense that sample size here is very small and dictated by the number of the uncertain parameters and also by choice of the order of the polynomial selected to represent the uncertain parameter.

Consequently, the next chapter describes reaction-diffusion problems with stochastic parameters using the generalized stochastic finite difference method by Marcin Kamiński and Rafał Leszek Ossowski. The main aim of this work is to demonstrate the new stochastic discrete computational methodology consisting of the generalized stochastic perturbation technique and of the classical Finite Difference Method (FDM) for the regular grids to model reaction-diffusion problems with random time series. The generalized stochastic perturbation approach is based on the given order Taylor expansion of all random variables. A numerical algorithm is implemented here using the direct differentiation method of the reaction-diffusion equation with respect to the height of a channel in 1D problem; further symbolic determination of the probabilistic moments and characteristics is completed by the computer algebra system MAPLE, v. 14. Computational illustration attached proves that it is possible to determine the fourth order probabilistic moments and coefficients as well as to consider time series with random coefficients for any dispersion of the input variables. Stochastic fluctuations of the input uncertainty source are defined here as the power time series with Gaussian random coefficients.

Chapter 11 is by C. Drossos and P. L. Theodoropoulos and includes MV-Partitions and MV-Powers. In this chapter, the Boolean partition to semisimple MV-algebras have been generalized. MV-partitions together with a notion of refinement is tantamount a construction of an MV-power, analogous to boolean power construction. Using this new notion the authors introduce the corresponding theory of MV-powers.

An algebraic study of the notion of independence of frames has been discussed by Fabio Cuzzolin in chapter 12. The theory of belief functions or “theory of evidence” allows the mathematical representation of uncertain piece of evidence on which decision can be based. Frequently, different piece of evidence belong to distinct, albeit related, domains or “frames.” For instance, audio and video clues can be combined to infer the identity of a person from a video. Evidence encoded by different belief functions on separate frames can be merged on a common frame, a combination which is guaranteed to exist if and only if the frames are “independent” in the sense of boolean algebras. In all other cases the evidence conflicts. Independence of frames and belief function combinability are then strictly related. In this chapter, the author discusses the notion of independence of frames in the theory of evidence from an algebraic point of view, starting from an analogy with standard linear independence. Final goal of this chapter is to search for a solution of the problem of conflicting belief function via a generalization of the classical Gram-Schmidt algorithm for vector orthogonalization. Families of frames can be given several algebraic interpretations in terms of semi-modular lattices, matroids, and geometric lattices. Each of those structures is endowed with a particular independence relation, which has been proved to be distinct albeit related to independence of frames.

In chapter 13, pricing and lot-sizing decisions in retail industry: A fuzzy chance constraint approach has been introduced by R. Ghasemy Yaghin, S.M.T. Fatemi Ghomi and S.A.Torabi. This integration of production, distribution pricing decisions in retail or even manufacturing environments is still in its

early stage in many companies. But it has the potential to radically improve supply chain efficiencies in the same way as Revenue Management (RM) has changed. Making decisions regarding production lot size and pricing have been proved to be very popular in the supply chain literature and is simultaneously known as the Joint Pricing and Lot Sizing Models (JPLM). These activities are often conducted either individually or sequentially with poor overall performance for the whole system resulting extra inventory and other deficiencies. In such environments, where decisions involve resources and data that are owned by different entities within the retailing industries, there are two paramount characteristics of the problems that a decision maker will be faced with: (1) Conflicting objectives that may arise from the nature of operations (e.g., maximize profit (or minimize cost) and at the same time increase customer service) and the structure of the operations where it is often difficult to align the goals of the companies. (2) Lack of precise data (e.g., cost) and/or presence of uncertainty with imprecise parameters (e.g., demand fuzziness). Thus, it is important that models addressing problems in this area should be designed to handle the foregoing two complexities. The importance of accounting for uncertainty in such environments spurs an interest to develop appropriate decision making tools to deal with uncertain and ill-defined parameters in joint pricing and lotsizing problems. The uncertainty of parameters can significantly affect the overall performance of retailing industry. Therefore, neglecting it, may impose high risks to firms afterwards. Generally, there are three distinct credibility-based fuzzy mathematical programming models (i.e., the expected value, the chance constrained programming, and the dependent-chance constrained programming), which have been used in parallel in most of published works applying this approach. Actually, the credibility measure could be defined as an average of the possibility and necessity measures. This chapter proposes a bi-objective credibility-based fuzzy chance constraint programming model to cope with these issues.

A new approach for suggesting takeover targets based on computational intelligence and information retrieval methods: a case study from the Indian software industry has been presented in chapter 14 by the authors Satyakama Paul; Andreas Janecek; Fernando Buarque de Lima Neto and Tshilidzi Marwala. In recent years researchers in financial management have shown considerable interest in predicting future takeover target companies in Merger and Acquisition (M&A) scenarios. However, most of these studies are based upon multiple instances of previous takeovers. The challenge now is to consider a company that is at the early stage of its acquisition spree and therefore has only limited data of (a single) previous takeover(s). Traditional studies on M&A, based upon statistical records of *multiple* instances of previous takeovers. This may not be suitable for suggesting future takeover targets for this company with single/few observation(s) since the lack of history data strongly limits the applicability of statistical techniques. Hence, as much knowledge as possible has to be extracted from the single/limited takeover history in order to guide this company during future takeover selections. In this chapter, the authors present a new algorithmic approach for suggesting future takeover targets for acquiring companies based on only one previous history of acquisition. The approach is based upon methods originating from Information Retrieval (IR) and Computational Intelligence (CI), and is exemplified as a case study using real and publicly available financial data of companies from the Indian software industry.

Next chapter (chapter 15) discusses the Fuzzy Finite Element Method in diffusion problems by S. Chakraverty and S. Nayak. Diffusion is an important phenomenon in various fields of science and engineering. It may arise in a variety of problems viz. in heat transfer, fluid flow and atomic reactors etc. Corresponding problems may be modelled by different types of differential equations. The type of differential equation depends upon the problem at hand, the parameters, coefficients involved and on other operating conditions. The domain and dimension of the problem also make it complex. As such

these diffusion equations are being solved throughout the globe by various methods. Exact method is used when the problem may be modelled as simple whereas one can use numerical methods such as Finite Difference Method (FDM), Finite Element Method (FEM) and Boundary Element Method (BEM) etc. when the problem cannot be solved by exact/analytical method. It has been seen from literature that researchers have investigated these problems when the material properties, geometry (domain) etc. are in crisp (exact) form which is easier to solve. But in real practice the parameters used in the modelled physical problems are not crisp because of the experimental error, mechanical defect, measurement error etc. In that case the problem has to be defined with uncertain parameters and this makes the problem complex. In this chapter the related uncertain differential equation of various diffusion problems will be investigated using FEM, which may be called fuzzy or interval finite element method. The uncertainties have been handled by various authors using probability density functions or statistical methods. But the main difficulty in these methods is to have plenty of data and this also can not consider the vague or imprecise parameters. As such here the uncertainty in the differential equation will be modelled in term of interval and/or fuzzy then one has to use interval and fuzzy computation in the analysis of the problem. As regards, the solution problem with uncertain parameters turns into a fuzzy/interval FEM. This chapter has undertaken the challenge of uncertainty in term of fuzzy/interval. Accordingly a method is proposed and validated by considering some example problems. The given system is solved by using proposed fuzzy finite element method. Main advantage of this proposed fuzzy finite element method is that if the parameters are taken uncertain viz. fuzzy then we may predict the possibility of solution set at any nodal points of the domain. This concept may be generalised for more number of element discretization through computer program and one may get better distribution for the solution set. Here an alternative non probabilistic method as discussed above has been proposed to manage various engineering and science problems. The traditional interval arithmetic is modified for the said problem and a simpler method is proposed to compute interval arithmetic. The idea of modified interval arithmetic is then extended for uncertain fuzzy numbers also. As such uncertain parameters are taken as fuzzy. Then the fuzzy numbers are converted into interval using  $\alpha$ -cut techniques. These fuzzy numbers contain left monotonically increasing and right monotonically decreasing functions respectively. Two types of fuzzy numbers viz. TFN and TRFN have been considered for the investigation. A sample example problem of uncertain system of equations is first solved. The proposed fuzzy finite element method is then used here to investigate three different physical problems. Variation of temperatures in a circular rod and the quantification of uncertain temperature distribution in a tapered fin are the first two applied problems that are investigated. Further the proposed fuzzy finite element method is used to solve one group neutron diffusion equation for a bare square homogeneous reactor. It is found that the fuzzy finite element method with the proposed interval computation is simpler to handle and also efficient. Hence it may be used as a tool for various other diffusion problems.

Chapter 16 has been written by Georgia Georgiou, Hamed Haddad Khodaparast, and Jonathan E. Cooper. The application of uncertainty analysis for the prediction of aeroelastic stability, using probabilistic and non-probabilistic methodologies, is considered in this chapter. Initially, a background to aeroelasticity and possible instabilities, in particular “flutter,” that can occur in aircraft is given along with the consideration of why Uncertainty Quantification (UQ) is becoming an important issue to the aerospace industry. The polynomial chaos expansion method and the fuzzy analysis for UQ are then introduced and a range of different random and quasi-random sampling techniques as well as methods for surrogate modeling are discussed. A numerical model of the aircraft is investigated using an eigenvalue analysis and a series of linear flutter analyses for a range of subsonic and supersonic speeds. It is shown



how the Probability Density Functions (PDF) of the resulting critical flutter speeds can be determined efficiently using UQ approaches and how the membership functions of the aeroelastic system outputs can be obtained accurately using a Kriging predictor.

Finally, in Chapter 17, S. Chakraverty and Diptiranjana Behera investigated the uncertain static and dynamic analysis of imprecisely defined structural systems. This chapter targets to analyse the static and dynamic analysis of structures with fuzzy parameters using fuzzy finite element method. In finite element method the complicated structures are discretized into small finite elements, giving the element wise static or dynamic behavior. Assembling together for all the elements satisfying the boundary conditions, it gives the static response (displacements etc.) or the natural frequencies of the structures as the case may be. By using the finite element method, the static responses or natural frequencies can be computed by solving system of linear equations or generalized eigenvalue problems.

For easy computation in finite element method the geometrical and material properties are taken usually as crisp (exact), but, actually there are incomplete information about the variables and parameters being a result of errors in measurements, observations, applying different operating conditions or it may be maintenance induced error, etc. Rather than the particular values we may have only the vague, imprecise and incomplete information about the variables and parameters, which are uncertain in nature. It is an important issue in various scientific and engineering problems how to deal with variables and parameters of uncertain value. There are three classes of uncertain models, which are probabilistic, fuzzy theory and interval analysis.

In probabilistic approach, the uncertain variables are considered as random variables with a joint probability density function. Unfortunately, probabilistic methods may not deliver reliable results at the required precision without sufficient experimental data. It may be due to the assumptions made regarding the joint probability densities of the random variables or functions involved. As such, fuzzy theory and interval analysis are becoming powerful tools for many applications in the recent decades. In fuzzy theory and interval analysis, the uncertain parameters are expressed by fuzzy and interval variables, such as fuzzy and interval numbers, vectors or fuzzy and interval matrices.

Recently, few authors analyzed the problems by using finite element method with the fuzzy/interval uncertainty i.e. known as Fuzzy Finite Element Method (FFEM) or Interval Finite Element Method (IFEM). In this investigation, fuzzy finite element solution of static and dynamic problems of structural systems with uncertain/imprecise material or geometric properties will be targeted. As discussed above, the problem of static analysis leads to a system of simultaneous equations. Similarly, the problem of dynamic analysis turns into an eigenvalue problem.

In view of the above, if the parameters and variables are uncertain in nature (interval or fuzzy), then the corresponding system of simultaneous equations and the eigenvalue problems will also contain the uncertainty. Recently little efforts are done to handle this uncertainty for systems of simultaneous equations and eigenvalue problems. Few authors proposed methods for the solution of interval valued and fuzzy valued system of simultaneous equations and eigenvalue problems. However, sometimes those are not efficient and problem dependent. Those methods also fail when one consider fully fuzzy or interval systems.

As such, this chapter includes methods to solve fuzzy system of linear equations, fully fuzzy system of linear equations and fuzzy eigenvalue problems. These methods are applied to various structural problems to find the fuzzy static and dynamic responses of the structures. In addition, the chapter analyses the numerical solution of uncertain fractionally damped spring-mass system and the uncertainties have been considered in the initial condition of the problem.



In view of all the above chapters, there is often a need to model, solve, and interpret the problems one encounters by considering uncertain information. The aim of this book is therefore to provide the reader with basic concepts regarding soft computing with various methods of uncertainty handling solution, analysis and applications. Although the models, algorithms and interpretations are made here for some particular example problems but those may very well be used/extended to other areas of science and engineering too.

This book will certainly find an important source for graduate and postgraduate students, teachers and researchers in colleges, universities/institutes and industries in the field of various engineering such as mechanical, civil, aerospace, electrical, marine, chemical, mining and sciences such as mathematics/applied mathematics, physics, biotechnology etc. wherever one wants to model and analyze their uncertain physical problems. It is known that uncertainty is must in every field of engineering and science. Therefore, this book will be a handy and important rescue to handle their problems.

The book may represent a critical turning point because it may demonstrate how the most current, advanced and revolutionary mathematical and computational techniques can be put to effective use in uncertainty analysis. Participation of eminent experts and researchers from all over the world have hopefully given new directions for furthering research and developments in this subject. Finally, it is believed that the book will certainly be of immense help to the researchers in the field of uncertainty and related area.

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