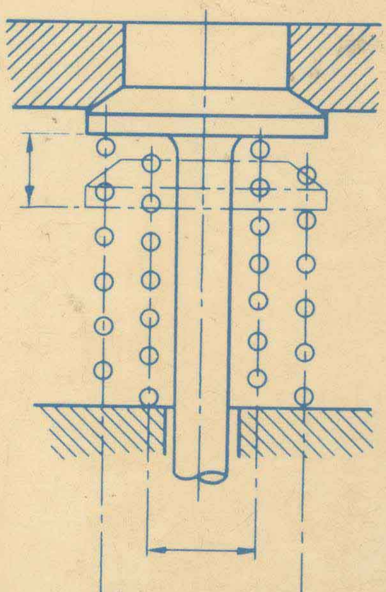
STRENGTH OF MATERIALS

Theory and Examples

R C STEPHENS

SI Units



STRENGTH OF MATERIALS
THEORY AND EXAMPLES

BY

R. C. STEPHENS

M.Sc.(Eng.), C.Eng., M.I.Mech.E.

FORMERLY PRINCIPAL LECTURER IN MECHANICAL ENGINEERING,
WEST HAM COLLEGE OF TECHNOLOGY

EDWARD ARNOLD

First published 1970
by Edward Arnold (Publishers) Ltd
41 Bedford Square
London WC1B 3DQ
Reprinted 1971, 1974, 1975, 1978, 1979, 1982

Paper edition ISBN: 0 7131 3211 6

Printed in Great Britain by
Butler and Tanner Ltd, Frome and London

STRENGTH OF MATERIALS

PREFACE

This book is intended to cover the basic Strength of Materials of the first two years of an engineering degree or diploma course; it does not attempt to deal with the more specialized topics which usually comprise the final year of such courses.

The work has been confined to the mathematical aspect of the subject and no descriptive matter relating to design or materials testing has been included.

Each chapter consists of a concise but thorough statement of the theory, followed by a number of worked examples in which the theory is amplified and extended. A large number of unworked examples, with answers, are also included.

The majority of examples have been taken, with permission, from examination papers set by the University of London and the Institutions of Mechanical and Civil Engineers; these have been designated U.Lond., I.Mech.E. and I.C.E. respectively. All questions were originally set in Imperial units; they have now been converted to equivalent S.I. units but are otherwise unchanged.

Over 500 questions have been solved and some errors in solutions are inevitable. Notification of these would be gratefully acknowledged.

R. C. STEPHENS

NOTE ON S.I. UNITS

The fundamental units in the *Système International d'Unités* are the metre, kilogramme and second, with the newton as the derived unit. Where mixed quantities are involved in a problem, the solution has generally been worked throughout in the basic units, e.g. for a given stress of 200 MN/m^2, the figure 200×10^6 N/m^2 has been substituted and for a density of 7·8 Mg/m^3, the figure $7{\cdot}8 \times 10^3$ kg/m^3 has been substituted.

In many examples of stress analysis or thick cylinders, however, it has been possible to work throughout in MN/m^2 (or the identical unit N/mm^2) and in the calculation of second moment of area of beam sections, etc, preliminary calculations have often been made in mm where this unit has been more appropriate.

The cm is not approved in S.I. units and has therefore not been used.

CONTENTS

CHAPTER 1

SIMPLE STRESS AND STRAIN

1.1 Introduction. When a load is applied to a member of a machine or structure, the material distorts. The *stress intensity* (usually abbreviated to *stress*) is the load transmitted per unit area of cross-section and the *strain* is a measure of the resulting distortion.

Assuming that the load is insufficient to cause rupture, it is resisted by the force of attraction between the molecules of the material and the deformation is the result of the slight re-orientation of the molecules.

If the material returns to its former shape when the load is removed, it is said to be *elastic*; if the strain is permanent, it is said to be *plastic*. Most engineering materials are elastic up to a certain stress (referred to as the *elastic limit*), after which they are partly elastic and partly plastic. The transition is not always abrupt, but for the purposes of calculation it is usually assumed to be so, an assumption which is reasonably justified for common mild steel.

In the simple theory of Strength of Materials, it is assumed that the material is *isotropic* (i.e. displays the same properties in all directions) and that it is equally rigid in tension and compression. It is further assumed that the stress is uniformly distributed over the area resisting the load; this is approximately true, except in the near vicinity of the point of application of the load or a sudden change of section (St Venant's Principle).

1.2 Tensile and compressive stress and strain. If a piece of material of cross-sectional area a is subjected to equal and opposite forces P, either tensile, as in Fig. 1.1(a) or compressive, as in Fig. 1.1(b), then

$$\text{stress} = \frac{\text{force}}{\text{cross-sectional area}}$$

i.e.

$$\sigma = \frac{P}{a} \qquad (1.1)$$

If the original length of the bar is l and under the effect of the force P it extends or compresses a distance x, then

$$\text{strain} = \frac{\text{change in length}}{\text{original length}}$$

i.e.

$$\varepsilon = \frac{x}{l} \qquad (1.2)$$

The deformed shapes of the bars are as shown dotted in Fig. 1.2; the strain in directions perpendicular to that of the load is proportional to that in the direction of the load and is of the opposite sign.

The ratio $\dfrac{\text{lateral strain}}{\text{axial strain}}$ is called *Poisson's Ratio* and is denoted by ν.

Thus if the axial strain is ε, the lateral strain is $-\nu\varepsilon$.

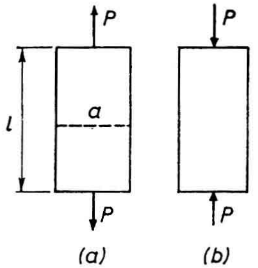


FIG. 1.1

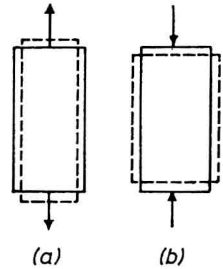


FIG. 1.2

1.3 Shear stress and strain. If a piece of material of cross-sectional area a is subjected to equal and opposite forces P which produce a state of shear, as shown in Fig. 1.3, then

$$\text{shear stress} = \frac{\text{force}}{\text{cross-sectional area}}$$

i.e.
$$\tau = \frac{P}{a} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1.3)$$

If the deformation in the direction of P is x and the perpendicular distance between the applied forces is l, then

$$\text{shear strain} = \frac{\text{deformation}}{\text{couple arm}}$$

i.e.
$$\phi = \frac{x}{l} \quad . \quad . \quad . \quad . \quad . \quad (1.4)$$

ϕ is the angular displacement in radians, since $\dfrac{x}{l}$ is very small.

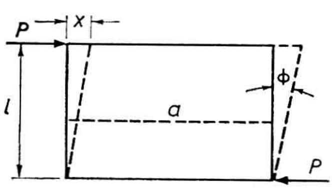


FIG. 1.3

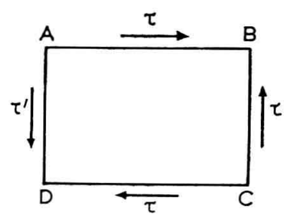


FIG. 1.4

When a shear stress τ is applied to the faces AB and CD of an element of the material, Fig. 1.4, a clockwise couple $(\tau \times AB \times t) \times BC$ is applied to the element, t being the thickness of the material. Since it does not rotate, however, an equal anticlockwise couple must be applied by means of shear stresses induced on faces AD and BC.

If the magnitude of these stresses is τ', then for equilibrium,

$$(\tau \times AB \times t) \times BC = (\tau' \times BC \times t) \times AB$$

$$\therefore \tau' = \tau$$

Thus a shear stress in one plane is always accompanied by an equal shear stress (called the *complementary shear stress*) in the perpendicular plane.

1.4 Hooke's Law. Hooke's Law states that when a load is applied to an elastic material, the deformation is directly proportional to the load producing it. Since the stress is proportional to the load and the strain is proportional to the deformation, it follows that the stress is proportional to the strain, i.e. the ratio stress/strain is a constant for any given material.

For tensile or compressive stresses, this constant is known as the *Modulus of Elasticity* (or *Young's Modulus*) and is denoted by E.

Thus
$$E = \frac{\sigma}{\varepsilon} = \frac{P/a}{x/l} = \frac{Pl}{ax} \qquad (1.5)$$

For shear stress, this constant is known as the *Modulus of Rigidity* and is denoted by G.

Thus
$$G = \frac{\tau}{\phi} = \frac{P/a}{x/l} = \frac{Pl}{ax} \qquad (1.6)$$

1.5 Factor of safety. The maximum stress used in the design of a machine or structure is considerably less than the ultimate stress (i.e. the stress at failure), to allow for possible overloading, non-uniformity of stress distribution, shock loading, faults in material and workmanship, corrosion, wear, etc.

The ratio $\dfrac{\text{breaking stress}}{\text{maximum design stress}}$ is called the *factor of safety*.

Instead of basing this factor on the stress at failure, it is sometimes based on the stress at the yield point (where the material suddenly becomes plastic) or, for materials which have no well-defined yield point, on the stress at which the extension is a certain percentage (e.g. 0·1 per cent) of the original length.

1.6 Stresses in thin cylindrical shells. When a thin cylinder is subjected to internal pressure, stresses are induced on the longitudinal section XX, Fig. 1.5, due to the force tending to separate the top and bottom halves, and on the circumferential section YY due to the force tending to separate the right- and left-hand ends of the cylinder.

The stress on the longitudinal section is termed the circumferential stress and that on the circumferential section is termed the longitudinal stress; the type of stress is determined by the direction of the arrows.

In determining the stresses induced, it is assumed that the thickness is small in comparison with the diameter so that the stress on a cross-section may be taken as uniform* and also that the ends give no support to the sides, an assumption which would be appropriate to a long cylinder such as a pipe.

Let the internal diameter and length be d and l respectively, the thickness of metal be t and the internal pressure be p.

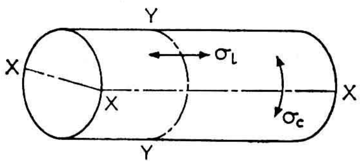


FIG. 1.5

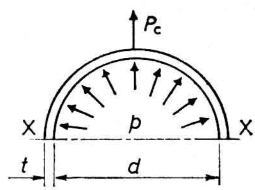


FIG. 1.6

Circumferential stress. The force tending to separate the top and bottom halves is the pressure multiplied by the projected area in a direction perpendicular to the diametral plane,† Fig. 1.6,

i.e. $$P_c = pdl$$

This is resisted by the stress acting on the longitudinal section, XX,

i.e. $$\sigma_c = \frac{pdl}{2tl} = \frac{pd}{2t} \quad . \quad . \quad . \quad . \quad . \quad (1.7)$$

If the cylinder is made up from riveted plates and the efficiency of the longitudinal joints is η_l then the average stress in the joint is given by

$$\sigma_c = \frac{pd}{2t\eta_l} \quad . \quad . \quad . \quad . \quad . \quad (1.8)$$

* See Chapter 14.

† The radial force on an element subtending an angle $d\theta$, Fig. 1.7, is $p \times \frac{d}{2} d\theta \times l$. The vertical component of this force is $\frac{pdl}{2} d\theta . \sin\theta$ so that the total force normal to XX is $\int_0^\pi \frac{pdl}{2} \sin\theta \, d\theta = pdl$.

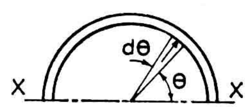


FIG. 1.7

Longitudinal stress. The force tending to separate the right- and left-hand halves is the pressure multiplied by the area of one end, Fig. 1.8,

i.e. $$P_l = p \times \frac{\pi}{4} d^2$$

This is resisted by the stress acting on the circumferential section, YY,

i.e. $$\sigma_l = \frac{p \times \frac{\pi}{4} d^2}{\pi d t} = \frac{pd}{4t} \quad . \quad . \quad . \quad . \quad (1.9)$$

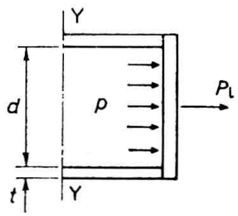


FIG. 1.8

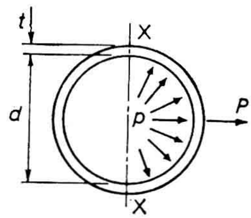


FIG. 1.9

If the cylinder is made up from riveted plates and the efficiency of the circumferential joints is η_c then the average stress in the joint is given by

$$\sigma_l = \frac{pd}{4t\eta_c} \quad . \quad . \quad . \quad . \quad . \quad (1.10)$$

It is evident from equations (1.8) and (1.10) that the efficiency of the circumferential joints need only be half that of the longitudinal joints.

1.7 Stress in thin spherical shells. Let the internal diameter be d, the thickness of metal be t and the internal pressure be p, Fig. 1.9. Then the force tending to separate the two halves on a section XX is the pressure multiplied by the projected area in the direction perpendicular to XX,

i.e. $$P = p \times \frac{\pi}{4} d^2$$

This is resisted by the stress acting on the section XX,

i.e. $$\sigma = \frac{p \times \frac{\pi}{4} d^2}{\pi d t} = \frac{pd}{4t} \quad . \quad . \quad . \quad . \quad (1.11)$$

If the shell is made up from riveted plates and the efficiency of the joints is η, then

$$\sigma = \frac{pd}{4t\eta} \quad . \quad . \quad . \quad . \quad . \quad (1.12)$$

1.8 Stress in thin rotating rims. Let a thin rim (one in which the radial depth is small in comparison with the mean radius) of cross-sectional area a, mean radius r and density m rotate at a tangential speed v, Fig. 1.10. Then the centrifugal force, F, on an element subtending an angle $d\theta$ is

$$m \times a \times r\, d\theta \frac{v^2}{r}$$

This is resisted by the radial components of the forces on the ends of the element. If the stress induced is σ, then

$$mav^2 d\theta = 2 \times \sigma a \times \frac{d\theta}{2}$$

from which $$\sigma = mv^2 \quad . \quad . \quad . \quad . \quad . \quad (1.13)$$

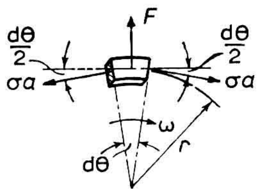


FIG. 1.10

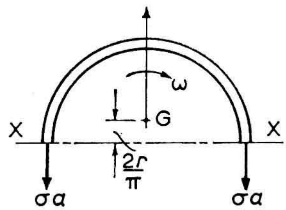


FIG. 1.11

Alternatively, the centre of gravity of the half of the rim above the diametral plane XX, Fig. 1.11, is at a distance $\frac{2r}{\pi}$ from XX. The centrifugal force on this part is then resisted by the stress σ acting on the section XX,

i.e. $$m \times a \times \pi r \times \omega^2 \times \frac{2r}{\pi} = 2\sigma a$$

from which $$\sigma = m\omega^2 r^2 = mv^2$$

1.9 Stresses in composite bars. A composite bar is a load-resisting member which is made up of two different materials.

Let the cross-sectional areas of the two materials be a_1 and a_2, the moduli of elasticity be E_1 and E_2 and the coefficients of expansion be α_1 and α_2.

Stresses due to external load. If the ends through which the load is applied are rigid, Fig. 1.12, the change in length of each part is the same,

i.e. $$x_1 = x_2$$

i.e. $$\frac{\sigma_1 l_1}{E_1} = \frac{\sigma_2 l_2}{E_2} \quad . \quad . \quad . \quad . \quad . \quad (1.14)$$

Also the sum of the loads carried by each part is equal to the applied load,

i.e.
$$P_1 + P_2 = P$$

or
$$\sigma_1 a_1 + \sigma_2 a_2 = P \qquad . \quad . \quad . \quad . \quad (1.15)$$

σ_1 and σ_2 can then be obtained from equations (1.14) and (1.15).

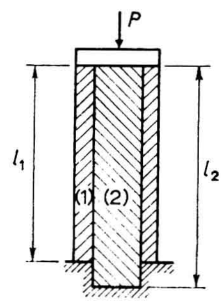


FIG. 1.12

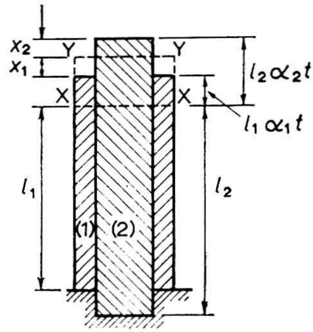


FIG. 1.13

Stresses due to change in temperature. Let XX, Fig. 1.13, be the initial level of the top of the composite bar and let YY be its final level after a temperature rise t. If both parts were free to expand, the extension of material (1) would be $l_1\alpha_1 t$ and that of material (2) would be $l_2\alpha_2 t$; if, however, the two materials are rigidly connected at the top, material (1) is forced to extend a distance x_1 and material (2) is forced to compress a distance x_2. It is only these forced changes in length, x_1 and x_2, which produce stresses in the materials.

From Fig. 1.13, it will be seen that

$$x_1 + x_2 = l_2\alpha_2 t - l_1\alpha_1 t$$

i.e.
$$\frac{\sigma_1 l_1}{E_1} + \frac{\sigma_2 l_2}{E_2} = (l_2\alpha_2 - l_1\alpha_1)t \qquad . \quad . \quad . \quad (1.16)$$

Also, since no external force is applied to the bar,

tensile force in material (1) = compressive force in material (2),

i.e.
$$\sigma_1 a_1 = \sigma_2 a_2 \qquad . \quad . \quad . \quad . \quad . \quad (1.17)$$

σ_1 and σ_2 can then be obtained from equations (1.16) and (1.17).

If the bar is subjected to an external load P, Fig. 1.14, as well as to a temperature rise t, then, from the equilibrium of the end plate,

$$P + P_1 = P_2$$

or
$$\sigma_1 a_1 - \sigma_2 a_2 = P \qquad . \quad . \quad . \quad (1.18)$$

assuming P to be compressive and $\alpha_2 > \alpha_1$.

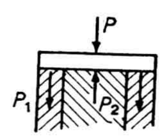


FIG. 1.14