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The Principle of Interaction between Mass Distribution and Deformation for Geotechnical Materials

岩土材料质量分布与变形之间的相互作用原理

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Preface

Rock and soil are the major constituent materials of the lithosphere of the earth. They are also the most widely used engineering materials. Rock and soil exhibit some remarkable characteristics of deformation and strength behavior, such as the pressure sensitivity(or pressure dependency), shear dilatancy, dependency of stress path etc. The two phenomena of pressure sensitivity and shear dilatancy have long been known by people. However, their origin of generation is not explained rationally yet.

Based on the Noll's principle of local action that asserts that the generation and transmission of stress in an object can be realized only through the deformation of that object, and the analysis of mechanism of mechanical response of geotechnical materials, the author (2006) proposed the principle of interaction between plastic volumetric and shear strains. The pressure sensitivity and shear dilatancy are the two kinds of manners of the interaction between plastic volumetric and shear strains. The effect of the shear strain on volumetric strains is direct, i.e., the shear dilatancy, whereas the effect of the volumetric strain on shear strains is realized only through changing the resistance to shearing. Thus, the two phenomena are not irrelevant to each other, or rather, they are linked up with each other by having a common origin of creation.

The principle of interaction between plastic volumetric and shear strains states that in the plastic deformation for geotechnical materials, there exist two relatively independent strains: volumetric and shear strains, and the highly complex and nonlinear interaction between them, which is the major origin of generation of the fundamental characteristics of plastic deformation behavior for geotechnical materials (Chapter 5).

According to the well-known formula proposed by Einstein, $E = mc^2$, the mass is also a measure of energy. Thus, the mass density of a material element represents a kind of energy storage per unit volume.

Herein, a form of energy that is a kind of energy storage associated only with the mass per unit volume was introduced, which is named the density of compaction energy that represents an ability of resistance to deforming.

In fact, the change in the plastic volumetric strain directly leads to that in the mass density. Thus, the effect of the plastic volumetric strain on shear strains is carried out essentially through changing the mass density. Therefore, the interaction between plastic volumetric and shear strains can be better understood as that between the mass density (or mass distribution) and deformation, which more deeply reflects the physical meaning of this interaction, that is, the ability of resistance to deforming depends upon the mass distribution, while the deformation can also make the mass redistribute.

Thus, the principle of interaction can also be called as the principle of interaction between mass distribution and deformation.

In the light of the principle of interaction, the three corollaries of the principle of interaction have been deduced.

Corollary 1 In the plastic deformation of geotechnical materials, the plastic volumetric strain controls the change in the resistance to shearing (Chapter 5).

It can also be stated as that in the plastic deformation of geotechnical materials, the mass density governs the change in the resistance to shearing.

Corollary 2 The dependency of stress path is a combined effect of the pressure sensitivity and shear dilatancy, that is, a comprehensive manifestation of the interaction between plastic volumetric and shear strains (Chapter 6).

Corollary 2 has theoretically been proved based on the principle of interaction (Chapter 6).

The large stress reversals, in fact, are a kind of special stress paths, so that the rotational hardening induced by them can be attributed to the dependency of stress path.

Corollary 3 The interaction between plastic volumetric and shear strains penetrates through the whole process of deformation until entering into the critical state in which this interaction disappears (Chapter 6).

According to the principle of interaction, it has theoretically been proved that the critical state line exists, and is unique and independent of the stress history (Chapter 6).

To reproduce the true behavior features of rock and soil it is necessary to capture what is behind the behavior of geotechnical materials. The principle of interaction tells us that the major fundamental deformation characteristics for geotechnical materials all stem from the interaction between plastic volumetric and shear strains. Therefore, in order to build such a constitutive model that it is able to more completely and accurately reproduce the mechanical response of geotechnical materials, the model must sufficiently reflect the interaction.

Under the direction of the principle of interaction, the constitutive equations for geotechnical materials have been derived within the framework of the thermodynamics of irreversible processes, in which four internal variables were introduced to quantitatively depict the interaction between plastic volumetric and shear strains. In addition, it has been proved that the second law of thermodynamics is satisfied during the process of plastic deformation (Chapter 7).

Two families of volumetric and shear yield loci which are taken as two sets of lines of constant plastic volumetric and shear strains, respectively, were adopted to depict the processes of strain hardening and strain softening evolutions. Fortunately, they can be plotted in the p-q stress plane based on the results of triaxial tests by using the method proposed by Huang et al. (1981). It should be pointed out that the directions of the outward normals to the volumetric and shear yield loci essentially represent those of plastic volumetric and shear strain increment vectors, respectively. Thus they can all be determined experimentally (Chapter 7).

In Chapter 8, the mechanism of damage of metals and some engineering materials has been expounded. Since the initiation, growth and coalescence of microcracks, and, eventually, the formation of macrocracks necessarily lead to the dilation of volume of material element and a corresponding drop in mass density of element, thereby reducing the resistance to deforming. Thus, for metals, it could be expected that the major mechanism of plastic deformation will be converted from the movement of dislocations to the interaction between plastic volumetric and shear strains when the volumes of elements expand to a certain extent. In addition, the damage of materials is actually a process of strain softening. Therefore, it can be described by means of the constitutive equations derived here, by which the elastic-plastic model describing the damage evolution has been built (Chapter 8).

The author (2002) proposed a numerical method of constitutive modeling for geotechnical materials. Based on the inverse problem theory, the constitutive modeling of materials, in fact, is an inverse problem that belongs to the problems of model identification in the inverse problem theory.

The problems of model identification are classified into two kinds: medium inverse problems and source inverse problems. The constitutive modeling for geotechnical materials belongs in the former, which is represented as an inversion of coefficients of differential equation. Thus, the constitutive modeling will become the inversion of coefficients of a set of constitutive equations. The constitutive equations of geotechnical materials are a set of field equations in the p-q stress plane, the coefficients of which are functions of stress state and are also dependent on the stress path, so their inversions require to be carried out in the whole p-q stress plane by means of some numerical techniques. Thus, it is called the numerical method of constitutive modeling.

Applying the numerical method of constitutive modeling the elastic-plastic models for clay and sand under five kinds of stress paths have been built. Through visualization, the surfaces of shear and volumetric strains were plotted in the p-q stress plane, respectively, which more completely show the features of strain hardening and strain softening evolutions, thereby confirming the abilities of these models to sufficiently reflect the interaction (Chapter 9).

In Chapter 3 and 4, the fundamental characteristics of deformation behavior for rock and soil, and the recent development of the study on constitutive modeling for geotechnical materials were briefly described, respectively.

The geotechnical materials are one kind of materials among the most complex materials in solid. The study on their behavior is closely related to many branches of physics, mechanics as well as chemistry, especially, continuum mechanics and thermodynamics.

Therefore, to conveniently consult the materials about their basic theories, the fundamentals of continuum mechanics and thermodynamics were briefly introduced in Chapter 1 and 2, respectively.

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Wuhan, China
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Contents

Chapter 1

Introduction to Continuum Mechanics

1.1 The definition of a continuum

The classical concept of a continuum originates from mathematics, in which the real number system is a continuum because there are infinitely many real numbers between any two distinct real numbers. The concept of a continuum of matter can be best explained by considering the concept of density. Let us assume a certain space S_0 permeated by a certain matter. Also consider a point P in S_0 and a sequence of subspaces S_1, S_2, ..., converging on P:

$$S_n \subset S_{n-1}, \quad P \in S_n \quad (n = 1, 2, \ldots)$$

Let the volume of S_n be V_n and the mass of the matter contained in S_n be M_n. If the limit of M_n/V_n exists as $n \to \infty$ and $V_n \to 0$, the limiting value is defined as the density of the mass distribution at the point P and is expressed as

$$\rho(P) = \lim_{\substack{n \to \infty \\ V_n \to 0}} \frac{M_n}{V_n} \tag{1.1.1}$$

This concept of the density of mass can also be extended to momentum, energy, etc. A material continuum is a material for which the densities of mass, momentum, and energy exist in the mathematical sense. The mechanics of such a material continuum is called continuum mechanics (e.g., see (Fung, 1994)).

However, in real-world systems, materials are usually composed of crystals and/or particles and there are cracks and voids between them. Thus, such a classical definition of material continuum given above could not be used in science and technology.

To fit the real world, an alternative definition of a continuum was proposed by Fung (1994). Let us consider a point P in a space S_0, also consider a sequence of subspaces $S_1, S_2, \ldots, S_n$ in S_0 with volumes $V_1, V_2, \ldots, V_n$ respectively, each enclosing the next one and all enclosing P. As $n \to \infty$, the limit of V_n tends to a finite positive number ω. Let the mass of the material enclosed in S_n be M_n. The sequence of the ratios M_n/V_n is said to have a limit ρ with an acceptable variability ε if

$$\left|\rho - \frac{M_n}{V_n}\right| < \varepsilon \tag{1.1.2}$$

as $n \to \infty$. The quantity ρ is then said to be the density of the material at P with an acceptable variability ε in a defining limit volume ω (Fung, 1994).

Similarly, the momentum of the materiel particles per unit volume and the energy per unit volume can also be defined. In addition, we can define the strain tensor or stress tensor with an acceptable variability in a defining limit length or area in the same way as given above.

If the density, momentum, energy, stress, and strain can be defined at every point in the space S_0, and if they are all continuous functions of spatial coordinates in S_0, then the material in S_0 is termed a continuum.

It should be pointed out that the defining limit volume should be the volume of the representative element of a material, which is the smallest one in the volumes of elements that are able to sufficiently reflect the physical and mechanical properties of that material, and the acceptable variability would be chosen as a proper value according to the structures of material at different scales of observation and the requirements of practical researches.

For experimental purposes and the numerical analysis, it is useful to consider the following orders of magnitude of the representative element, for example, as shown in Table 1.1 (Lemaitre, 1992).

Table 1.1 The orders of magnitude of the representative element

material	order of magnitude of element/mm^3
metals and ceramics	0.1^3
polymers and most composites	1^3
wood	10^3
concrete	100^3

Particle sizes of soils vary in a very large range: clay (finer than 0.005mm), silt (0.005∼0.06mm), sand (0.06∼2mm), grave (2∼60mm), cobbles (60∼200mm), and boulder (coarser than 200mm). Therefore, the orders of magnitude of the representative elements for soils with different particle sizes are quite different.

1.2 Deformation

Forces applied to a deformable body will cause the deformation of the body. In fact, the deformation of a body is a manner to resist the external forces.

The deformation of a material can be mathematically described when the material is assumed as a continuum.

Here, it should be emphasized that from the physical point of view, the deformation at a point should be understood as that of the immediate neighborhood of that point, or rather, the deformation of the representative element surrounding that point.

Let us introduce a rectangular Cartesian frame of reference, every element in the body that occupies a space S has a set of coordinates. When the body is deforming, a element P, located at a place with coordinates (a_1, a_2, a_3), moves to the point Q with coordinates (x_1, x_2, x_3). The vector $\overrightarrow{PQ}$ is then termed the displacement vector of the element.

Let the two set of variables (a_1, a_2, a_3) and (x_1, x_2, x_3) be the coordinates of any element in the body before and after deformation, respectively. Assume x_1, x_2, x_3 are single valued and continuous functions of a_1, a_2, a_3:

$$x_i = x_i(a_1, a_2, a_3) \tag{1.2.1}$$

In addition, assume the functions in Eq. (1.2.1) have the unique inverse

$$a_i = a_i(x_1, x_2, x_3) \tag{1.2.2}$$

The displacement vector $\boldsymbol{u}$ can be expressed in terms of its components

$$u_i = x_i - a_i \tag{1.2.3}$$

Let us consider three neighboring points P, P', P'' in the body as shown in Fig. 1.1.

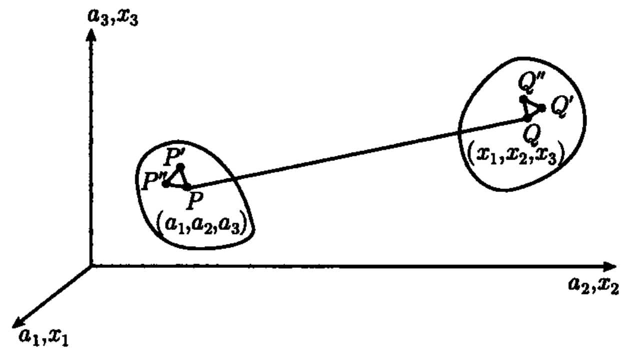

Fig. 1.1 Deformation of a body

The points Q, Q', Q'' denote the corresponding new positions of P, P', P'' in the deformed configuration.

Assume the distance between the point $P(a_1,\ a_2,\ a_3)$ and a neighboring point $P'(a_1 + \mathrm{d}a_1,\ a_2 + \mathrm{d}a_2,\ a_3 + \mathrm{d}a_3)$ is infinitesimal and its square can be expressed as

$$\mathrm{d}s_0^2 = \mathrm{d}a_1^2 + \mathrm{d}a_2^2 + \mathrm{d}a_3^2 \tag{1.2.4}$$

Similarly, the square of the distance between point $Q(x_1,\ x_2,\ x_3)$ and point $Q'(x_1 + \mathrm{d}x_1,\ x_2 + \mathrm{d}x_2,\ x_3 + \mathrm{d}x_3)$ can be written as

$$\mathrm{d}s^2 = \mathrm{d}x_1^2 + \mathrm{d}x_2^2 + \mathrm{d}x_3^2 \tag{1.2.5}$$

On account of Eqs. (1.2.1) and (1.2.2), we have

$$\mathrm{d}x_i = \frac{\partial x_i}{\partial a_j}\mathrm{d}a_j, \quad \mathrm{d}a_i = \frac{\partial a_i}{\partial x_j}\mathrm{d}x_j \tag{1.2.6}$$

Thus, we may obtain

$$\begin{aligned} \mathrm{d}s_0^2 &= \delta_{ij}\mathrm{d}a_i\mathrm{d}a_j = \delta_{ij}\frac{\partial a_i}{\partial x_l}\frac{\partial a_j}{\partial x_m}\mathrm{d}x_l\mathrm{d}x_m \\ \mathrm{d}s^2 &= \delta_{ij}\mathrm{d}x_i\mathrm{d}x_j = \delta_{ij}\frac{\partial x_i}{\partial a_l}\frac{\partial x_j}{\partial a_m}\mathrm{d}a_l\mathrm{d}a_m \end{aligned} \tag{1.2.7}$$

where δ_{ij} is the Kronecker delta.

Thus, the difference between $\mathrm{d}s^2$ and $\mathrm{d}s_0^2$ may be written as

$$\mathrm{d}s^2 - \mathrm{d}s_0^2 = \left(\delta_{\alpha\beta}\frac{\partial x_\alpha}{\partial a_i}\frac{\partial x_\beta}{\partial a_j} - \delta_{ij}\right)\mathrm{d}a_i\mathrm{d}a_j \tag{1.2.8}$$

or

$$\mathrm{d}s^2 - \mathrm{d}s_0^2 = \left(\delta_{ij} - \delta_{\alpha\beta}\frac{\partial a_\alpha}{\partial x_i}\frac{\partial a_\beta}{\partial x_j}\right)\mathrm{d}x_i\mathrm{d}x_j \tag{1.2.9}$$

We define the strain tensors

$$E_{ij} = \frac{1}{2}\left(\delta_{\alpha\beta}\frac{\partial x_\alpha}{\partial a_i}\frac{\partial x_\beta}{\partial a_j} - \delta_{ij}\right) \tag{1.2.10}$$

$$e_{ij} = \frac{1}{2}\left(\delta_{ij} - \delta_{\alpha\beta}\frac{\partial a_\alpha}{\partial x_i}\frac{\partial a_\beta}{\partial x_j}\right) \tag{1.2.11}$$

The strain tensor E_{ij} was introduced by Green and St.-Venan and is called Green's strain tensor. The strain tensor e_{ij} was introduced by Cauchy for infinitesimal strains, and by Almansi and Hamel for finite strains and is known as Almansi's strain tensor (e.g., see (Fung, 1994)).

If the first derivatives of the components of displacement u_i are so small that the squares and products of the partial derivatives of u_i are negligible, then e_{ij} reduces to Cauchy's infinitesimal strain tensor,

$$e_{ij} = \frac{1}{2}\left[\frac{\partial u_j}{\partial x_i} + \frac{\partial u_i}{\partial x_j}\right] \tag{1.2.12}$$

The strain tensor e_{ij} is obviously symmetric. For symmetric strain tensors, a set of coordinates can be found, with respect to which the matrix of strain components can be reduced to a diagonal one,

$$\begin{pmatrix} e_1 & & \\ & e_2 & \\ & & e_3 \end{pmatrix}$$

The particular three coordinate axes are called the principal axes, and the corresponding strain components are called the principal strains.

In can be proved that three principal strains, e_1, e_2, e_3, are the roots of the following equation

$$|e_{ij} - e\delta_{ij}| = 0 \tag{1.2.13}$$

We define a strain deviation tensor $e'_{ij} = e_{ij} - \frac{1}{3}(e_{\alpha\alpha}\delta_{ij})$. Tensor e_{ij} and e'_{ij} have the following independent strain invariants:

$$I_1 = e_{ij}\delta_{ij}$$

$$I_2 = \frac{1}{2} e_{ik} e_{ik}$$
$$I_3 = \frac{1}{3} e_{ik} e_{km} e_{mi} \tag{1.2.14}$$

and

$$J_1 = e'_{ij} \delta_{ij} = 0$$
$$J_2 = \frac{1}{2} e'_{ik} e'_{ik}$$
$$J_3 = \frac{1}{3} e'_{ik} e'_{km} e'_{mi} \tag{1.2.15}$$

1.3 Stress

Let us consider a body B and a closed surface S within B. Let ΔS be a small surface element on S. Let $\boldsymbol{n}$ be the outward unit normal to ΔS. The material outside S exerts a force $\Delta \boldsymbol{F}$ on that inside S as shown in Fig. 1.2. The force $\Delta \boldsymbol{F}$ depends on the location, the area of ΔS and the orientation of the normal.

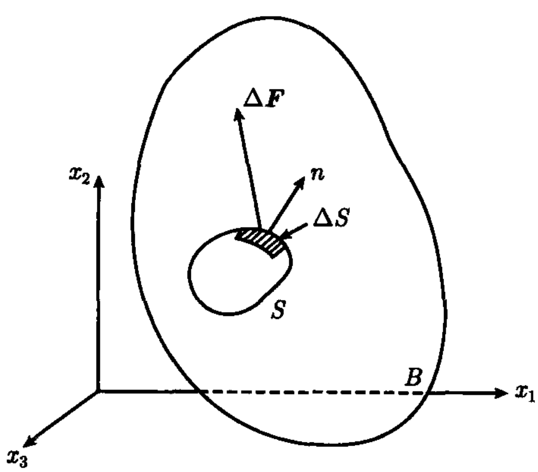

Fig. 1.2 A closed surface S in a body

Assume that as ΔS tends to a small but bounded size α, the ratio $\Delta \boldsymbol{F}/\Delta S$ tends to a definite limit $\mathrm{d}\boldsymbol{F}/\mathrm{d}S$ with an acceptable variability ε, and the moment of the force acting on the surface ΔS about any point within the area vanishes in the limit of small but bounded area α with an acceptable variability (Fung, 1994). The limiting vector can be expressed as

$$\boldsymbol{t}(\boldsymbol{n}) = \frac{\mathrm{d}\boldsymbol{F}}{\mathrm{d}S} \tag{1.3.1}$$