Graduate Texts in Physics

Eleftherios N. Economou

The Physics of Solids

Essentials and Beyond



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With 261 Figures



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Preface

In selecting the material and the presentation of this textbook I was aiming at the advanced undergraduate/first year graduate level, or so I was intending. Chapters 1–8 (with some possible omissions and additions, depending on the students' interests and the instructor's preferences) can serve as the core of a senior undergraduate course on Solid State Physics addressed to students in Physics, Chemistry, Materials Science, and Engineering. The book is designed to serve also as a textbook in a first-year graduate course on the Physics of Solids. Some familiarity with Electromagnetism, Quantum Mechanics, and Statistical Physics is assumed; anyway, extensive outlines of these subjects are presented in Appendices A, B, and C, respectively.

The emphasis in the book is on understanding the behavior of the various types of solids at a qualitative level and on being able to derive their properties at a quantitative level. To achieve this goal, concepts and theoretical tools are gradually introduced as needed and the results are continuously tested against the "touchstone" of experimental data. To be more specific, the book starts with Chap. 1 by reminding the readers of the basic ideas of physics: the atomic structure of matter, the wave-particle duality (which is distilled in the three principles of Quantum Mechanics), and the minimization of the free energy as the criterion of equilibrium. The latter is established through the competition of the squeezing electrostatic forces (characterized by the proton charge, e) and the expansion-driving quantum kinetic energy (characterized by Planck's constant, \hbar , the electronic mass, $m_{\rm e}$, and, to a lesser degree, the atomic mass, m_a). It is shown in Chap. 2 that the basic physical constants, ħ, m_e, e, through simple dimensional considerations and "a little thinking" allow us to derive most of the main properties of solids at a semiquantitative level. Chapter 3, which concludes the first introductory part of the book provides a first acquaintance with solids (their various types, their periodic crystal structures, and the periodicity-based Bloch's theorem).

In the second part of the book, the two simplest approaches to the real world of solids (with diametrically opposite starting points) are presented: The *Jellium Model* (JM), more appropriate for metals, and the *linear combination*

of atomic orbitals (LCAO) method, capable of handling semiconductors as well as other classes of solids. The JM is treated in more details than usual, because it offers the opportunity to introduce many important concepts, to calculate explicitly quantities of interest, to compare them with experimental data, and all these in the simplest possible way and with a minimum background. The importance of the LCAO method is emphasized because of its far reaching applicability. This part is concluded in Chap. 8 with an outline of the basic conceptual framework (leading to the independent electron approximation in an effective periodic potential) together with a list of important phenomena going beyond this framework.

In the third part, consisting of Chaps. 9–12, concepts and theoretical tools associated with the periodic order of crystalline solids are presented. These concepts and tools allow us to remedy the qualitative failures of the JM and quantitative inadequacies of the LCAO method.

Equipped with this calculational arsenal, we are in a position to study in the fourth part of the book (Chaps. 13–16) not only the specific classes of solids, such as simple metals, semiconductors, ionic solids, transition metals, and artificial structures, but also particular materials.

In the fifth part of the book (Chaps. 17–19), we are forced to go beyond the familiar ground of the periodic landscape in order to examine phenomena where the breakdown or the absence of periodic order is essential. These phenomena occur in surfaces and interfaces, in glasses, amorphous solids, and other disordered systems, and in finite structures, such as clusters, quantum dots, etc.

Finally, in the sixth part of the book (Chaps. 20–23), we make another excursion to the unknown territory in order to study two important phenomena (Magnetism and Superconductivity) associated with the breakdown of the independent electron approximation and the emergence of the crucial role of correlated electronic motion.

Many topics of current research interest (and hopefully of future importance) have found their way in the book: Graphene, Organic Semiconductors, Photonic and Phononic Crystals, Left-Handed Metamaterials, Plasmonics, Spintronics, etc. However, important subjects such as soft matter, nonequilibrium phenomena, and devices were left out (to say nothing about experimental techniques).

I tried to make the book self-contained by including 65 pages of appendices (marked by a gray stripe for easy identification). In these appendices, the basic concepts and formulas of Electromagnetism, Quantum Mechanics, Thermodynamics/Statistical Mechanics, and Theory of Elasticity, as well as the LCAO method as applied to molecules are presented. In order to make the book easy to use, I have put together, at the end of the book, the important tables (again marked by a gray stripe); moreover, tables of physical constants and the atomic system of units, as well as frequently used mathematical formulas were placed inside the front and the back hard cover of the book.

I would like to thank Drs. Maria Kafesaki, Stavroula Foteinopoulou, and George Kioseoglou for reading some chapters of the book and for making useful suggestions. I would like also to acknowledge many useful discussions with colleagues concerning the content of the book and its presentation. I am grateful to Mina Papadaki for her invaluable help in bringing to conclusion this text. Finally, I am greatly indebted to the reviewers for reading my manuscript from A to Z, very carefully; they pointed out several typos and made a great number of very valuable modifications for improving the text. I express my deep appreciation and thanks to them.

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