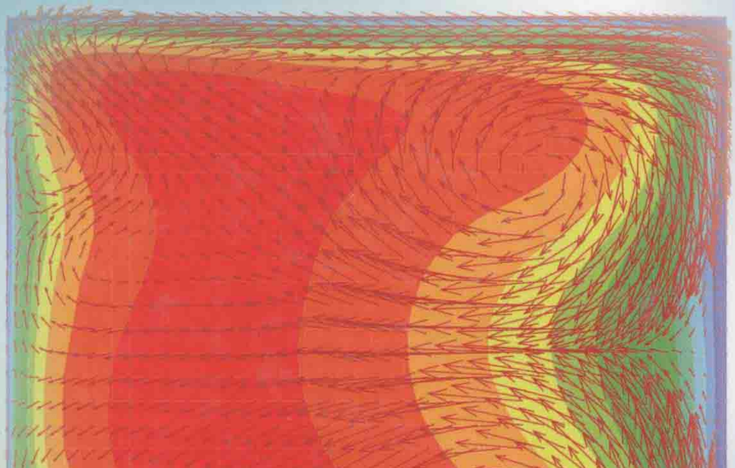
series in computational and physical processes in mechanics and thermal sciences

Computational Fluid Mechanics and Heat Transfer

THIRD EDITION



Richard H. Pletcher
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Computational Fluid Mechanics and Heat Transfer

THIRD EDITION

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Preface to the Third Edition

Another 15 years have gone by since the second edition of this text appeared. During this period, the rate of development in algorithms has slowed compared to any earlier period, but the increase in computational power has been astounding and shows no sign of slowing. Desktop computers can outperform the supercomputers of the early 1990s. The rate of improvement of computing power is such that a problem that required a year of computing time to solve 10 years ago can now be solved overnight. The increase in computing power has enabled engineers to solve more complete equations and complex geometries for aerodynamic flows, i.e., use less physical modeling and fewer approximations. It has also motivated efforts to compute more complex physical phenomena such as turbulence and multiphase flows.

Another clear trend is the increasing use of commercial software for computational fluid dynamics (CFD) applications. In the early days, CFD was mostly a do-it-yourself enterprise. It is more likely now that a CFD code is thought of as representing a large investment, and companies do not launch into writing a new one without considerable thought. It is more likely that CFD engineers will become involved in modifying or extending an existing code than in writing a new code from "scratch." However, even making modifications to CFD codes requires knowledge of algorithms, general numerical strategies, and programming skills. The text promotes programming skills by explaining algorithm details and including homework problems that require programming. Even those engineers that will utilize commercial codes and be responsible for interpreting the results will be better prepared as a result of the knowledge and insight gained from developing codes themselves. It is very important for engineers to know the limitations of codes and to recognize when the results are not plausible. This will not change in the future. The experience gained by writing and debugging codes will contribute toward the maturity needed to wisely use and interpret results from CFD codes.

It is essential that courses evolve as technology advances and new knowledge comes forth. However, not every new twist will have a permanent impact on the discipline. Fads die out, and some numerical approaches will become obsolete as computing power relentlessly advances. The authors have included a number of new developments in this edition while preserving the fundamental elements of the discipline covered in earlier editions. A number of ideas and algorithms that are now less frequently utilized due to advances in computer hardware or numerical algorithms are retained so that students and instructors can gain a historical perspective of the discipline. Such material can be utilized at the discretion of the instructor. Thirty-four new homework problems have been added bringing the total number of homework problems to 376.

We have retained the two-part, ten-chapter format of the text. Additions and clarifications have been made in all chapters. Part I, consisting of Chapters 1 through 4, deals with the basic concepts and fundamentals of the finite-difference and finite-volume methods. The historical perspective in Chapter 1 has been expanded. The sections on the finite-volume method in Chapter 3 have been revised and expanded. The conjugate gradient and generalized minimal residual (GMRES) methods are now discussed in the section on Laplace's equation in Chapter 4. Part II, consisting of Chapters 5 through 10, covers applications to the equations of fluid mechanics and heat transfer. The governing equations are presented in Chapter 5. The equations for magnetohydrodynamic (MHD) flows and the quasi-one-dimensional form of the Euler equations are now included. Turbulence modeling has been updated. The coverage of large-eddy simulation (LES) has been expanded and detached eddy simulation (DES) has been introduced. In Chapter 8, the material on the parabolized Navier–Stokes (PNS) equations has been expanded to include methods for handling flow fields with significant upstream influences, including large streamwise separated regions. A number of updates and additions are found in Chapter 9. Coverage of Runge–Kutta schemes, residual smoothing, and the lower-upper symmetric Gauss–Seidel (LU-SGS) scheme have been expanded. Some recent variations in time-accurate implicit schemes are also included.

We continue to be grateful for the help received from many colleagues and past students while this material was developed and revised. We especially thank Zhaohui Qin for his help with several new figures and with updates to several appendices. Finally, we would like to thank our families for their patience and encouragement during the preparation of this third edition.

This text continues to be a collective work by the three of us. There is no junior or senior author. The order of the authors for the previous two editions was determined by coin flips. Anderson and Tannehill were named first author on the previous two editions, and Pletcher is first author on the current work.

Richard H. Pletcher

John C. Tannehill

Dale A. Anderson

Preface to the Second Edition

Almost 15 years have passed since the first edition of this book was written. During the intervening years, the literature in computational fluid dynamics (CFD) has expanded manyfold. Due, in part, to greatly enhanced computer power, the general understanding of the capabilities and limitations of algorithms has increased. A number of new ideas and methods have appeared. We have attempted to include new developments in this second edition while preserving those fundamental ideas covered in the first edition that remain important for mastery of the discipline. Ninety-five new homework problems have been added. The two-part, ten-chapter format of the book remains the same, although a shift in emphasis is evident in some of the chapters. The book is still intended to serve as an introductory text for advanced undergraduates and/or first-year graduate students. The major emphasis of the text is on finite-difference/finite-volume methods.

Part I, consisting of Chapters 1 through 4, presents basic concepts and introduces the reader to the fundamentals of finite-difference/finite-volume methods. Part II, consisting of Chapters 5 through 10, is devoted to applications involving the equations of fluid mechanics and heat transfer. Chapter 1 serves as an introduction and gives a historical perspective of the discipline. This chapter has been brought up to date by reflecting the many changes that have occurred since the introduction of the first edition. Chapter 2 presents a brief review of those aspects of partial differential equation theory that have important implications for numerical solution schemes. This chapter has been revised for improved clarity and completeness. Coverage of the basics of discretization methods begins in Chapter 3. The second edition provides a more thorough introduction to the finite-volume method in this chapter. Chapter 4 deals with the application of numerical methods to selected model equations. Several additions have been made to this chapter. Treatment of methods for solving the wave equation now includes a discussion of Runge–Kutta schemes. The Keller box and modified box methods for solving parabolic equations are now included in Chapter 4. The method of approximate factorization is explained and demonstrated. The material on solution strategies for Laplace's equation has been revised and now contains an introduction to the multigrid method for both linear and nonlinear equations. Coloring schemes that can take advantage of vectorization are introduced. The material on discretization methods for the inviscid Burgers equation has been substantially revised in order to reflect the many developments, particularly with regard to upwind methods, that have occurred since the material for the first edition was drafted. Schemes due to Godunov, Roe, and Enquist and Osher are introduced. Higher-order upwind and total variation diminishing (TVD) schemes are also discussed in the revised Chapter 4.

The governing equations of fluid mechanics and heat transfer are presented in Chapter 5. The coverage has been expanded in several ways. The equations necessary to treat chemically reacting flows are discussed. Introductory information on direct and large-eddy simulation of turbulent flows is included. The filtered equations used in large-eddy simulation as well as the Reynolds-averaged equations are presented. The material on turbulence modeling has been augmented and now includes more details on one- and two-equation and Reynolds stress models as well as an introduction to the subgrid-scale modeling required for large-eddy simulation. A section has been added on the finite-volume formulation, a discretization procedure that proceeds from conservation equations in integral form.

Chapter 6 on methods for the inviscid flow equations is probably the most extensively revised chapter in the second edition. The revised chapter contains major new sections on flux splitting schemes, flux difference splitting schemes, the multidimensional case in generalized coordinates, and boundary conditions for the Euler equations. The chapter includes a discussion on implementing the integral form of conservation statements for arbitrarily shaped control volumes, particularly triangular cells, for two-dimensional applications.

Chapter 7 on methods for solving the boundary-layer equations includes new example applications of the inverse method, new material on the use of generalized coordinates, and a useful coordinate transformation for internal flows. In Chapter 8, methods are presented for solving simplified forms of the Navier–Stokes equations including the thin-layer Navier–Stokes (TLNS) equations, the parabolized Navier–Stokes (PNS) equations, the reduced Navier–Stokes (RNS) equations, the partially parabolized Navier–Stokes (PPNS) equations, the viscous shock layer (VSL) equations, and the conical Navier–Stokes (CNS) equations. New material includes recent developments on pressure relaxation, upwind methods, coupled methods for solving the partially parabolized equations for subsonic flows, and applications.

Chapter 9 on methods for the "complete" Navier–Stokes equations has undergone substantial revision. This is appropriate because much of the research and development in CFD since the first edition appeared has been concentrated on solving these equations. Upwind methods that were first introduced in the context of model and Euler equations are described as they extend to the full Navier–Stokes equations. Methods to efficiently solve the compressible equations at very low Mach numbers through low Mach number preconditioning are described. New developments in methods based on derived variables, such as the dual potential method, are discussed. Modifications to the method of artificial compressibility required to achieve time accuracy are developed. The use of space-marching methods to solve the steady Navier–Stokes equations is described. Recent advances in pressure-correction (segregated) schemes for solving the Navier–Stokes equations such as the use of nonstaggered grids and the pressure-implicit with splitting of operators (PISO) method are included in the revised chapter.

Grid generation, addressed in Chapter 10, is another area in which much activity has occurred since the appearance of the first edition. The coverage has been broadened to include introductory material on both structured and unstructured approaches. Coverage now includes algebraic and differential equation methods for constructing structured grids and the point insertion and advancing front methods for obtaining unstructured grids composed of triangles. Concepts employed in constructing hybrid grids composed of both quadrilateral cells (structured) and triangles, solution adaptive grids, and domain decomposition schemes are discussed.

We are grateful for the help received from many colleagues, users of the first edition, and others while this revision was being developed. We especially thank our colleagues Ganesh Rajagopalan, Alric Rothmayer, and Ijaz Parpia. We also continue to be indebted to our students, both past and present, for their contributions. We would like to acknowledge the skillful preparation of several new figures by Lynn Ekblad. Finally, we would like to thank our families for their patience and continued encouragement during the preparation of this second edition.

This text continues to be a collective work by the three of us. There is no junior or senior author. A coin flip determined the order of authors for the first edition, and a new coin flip has determined the order of authors for this edition.

John C. Tannehill

Dale A. Anderson

Richard H. Pletcher

Preface to the First Edition

This book is intended to serve as a text for introductory courses in computational fluid mechanics and heat transfer (or, synonymously, computational fluid dynamics [CFD]) for advanced undergraduates and/or first-year graduate students. The text has been developed from notes prepared for a two-course sequence taught at Iowa State University for more than a decade. No pretense is made that every facet of the subject is covered, but it is hoped that this book will serve as an introduction to this field for the novice. The major emphasis of the text is on finite-difference methods.

The book has been divided into two parts. Part I, consisting of Chapters 1 through 4, presents basic concepts and introduces the reader to the fundamentals of finite-difference methods. Part II, consisting of Chapters 5 through 10, is devoted to applications involving the equations of fluid mechanics and heat transfer. Chapter 1 serves as an introduction, while a brief review of partial differential equations is given in Chapter 2. Finite-difference methods and the notions of stability, accuracy, and convergence are discussed in Chapter 3.

Chapter 4 contains what is perhaps the most important information in the book. Numerous finite-difference methods are applied to linear and nonlinear model partial differential equations. This provides a basis for understanding the results produced when different numerical methods are applied to the same problem with a known analytic solution.

Building on an assumed elementary background in fluid mechanics and heat transfer, Chapter 5 reviews the basic equations of these subjects, emphasizing forms most suitable for numerical formulations of problems. A section on turbulence modeling is included in this chapter. Methods for solving inviscid flows using both conservative and nonconservative forms are presented in Chapter 6. Techniques for solving the boundary-layer equations for both laminar and turbulent flows are discussed in Chapter 7. Chapter 8 deals with equations of a class known as the "parabolized" Navier–Stokes equations, which are useful for flows not adequately modeled by the boundary-layer equations, but not requiring the use of the full Navier–Stokes equations. Parabolized schemes for both subsonic and supersonic flows over external surfaces and in confined regions are included in this chapter. Chapter 9 is devoted to methods for the complete Navier–Stokes equations, including the Reynolds-averaged form. A brief introduction to methods for grid generation is presented in Chapter 10 to complete the text.

At Iowa State University, this material is taught to classes consisting primarily of aerospace and mechanical engineers, although the classes often include students from other branches of engineering and earth sciences. It is our experience that Part I (Chapters 1 through 4) can be adequately covered in a one-semester, three-credit-hour course. Part II contains more information than can be covered in great detail in most one-semester, three-credit-hour courses. This permits Part II to be used for courses with different objectives. Although we have found that the major thrust of each of Chapters 5 through 10 can be covered in one semester, it would also be possible to use only parts of this material for more specialized courses. Obvious modules would be Chapters 5, 6, and 10 for a course emphasizing inviscid flows or Chapters 5 and 7 through 9 (and perhaps 10) for a course emphasizing viscous flows. Other combinations are clearly possible. If only one course can be offered in the subject, choices also exist. Part I of the text can be covered in detail in the single course, or, alternatively, only selected material from Chapters 1 through 4 could be covered as well as some material on applications of particular interest from Part II. The material in the text is reasonably broad and should be appropriate for courses having a variety of objectives.

For background, students should have at least one basic course in fluid dynamics, one course in ordinary differential equations, and some familiarity with partial differential equations. Of course, some programming experience is also assumed.

The philosophy used throughout the CFD course sequence at Iowa State University and embodied in this text is to encourage students to construct their own computer programs. For this reason, "canned" programs for specific problems do not appear in the text. Use of such programs does not enhance basic understanding necessary for algorithm development. At the end of each chapter, numerous problems are listed that necessitate numerical implementation of the text material. It is assumed that students have access to a high-speed digital computer.

We wish to acknowledge the contributions of all of our students, both past and present. We are deeply indebted to F. Blottner, S. Chakravarthy, G. Christoph, J. Daywitt, T. Hoist, M. Hussaini, J. Ievalts, D. Jespersen, O. Kwon, M. Malik, J. Rakich, M. Salas, V. Shankar, R. Warming, and many others for helpful suggestions for improving the text. We would like to thank Pat Fox and her associates for skillfully preparing the illustrations. A special thanks to Shirley Riney for typing and editing the manuscript. Her efforts were a constant source of encouragement. To our wives and children, we owe a debt of gratitude for all of the hours stolen from them. Their forbearance is greatly appreciated.

Finally, a few words about the order in which the authors' names appear. This text is a collective work by the three of us. There is no junior or senior author. The final order was determined by a coin flip. Despite the emphasis of finite-difference methods in the text, we resorted to a "Monte Carlo" method for this determination.

Dale A. Anderson

John C. Tannehill

Richard H. Pletcher

Authors

Richard H. Pletcher received his BS from Purdue University, West Lafayette, Indiana, and his MS and PhD from Cornell University, Ithaca, New York. He is professor emeritus of mechanical engineering and past director of the Computational Fluid Dynamics Center at Iowa State University, Ames, Iowa. Prior to joining Iowa State University, he worked as a senior research engineer in the Propulsion Section at the United Aircraft Research Laboratories, East Hartford, Connecticut. Professor Pletcher has received awards from Iowa State University for both teaching and research. In 2009, he received the American Society of Mechanical Engineers Heat Transfer Memorial Award in Science. He has served as associate editor of the *Journal of Heat Transfer* and is on the editorial advisory board for *Numerical Heat Transfer*. Professor Pletcher has conducted basic and applied studies over a wide range of topics in fluid dynamics and heat transfer. He has served as principal investigator for numerous research grants from sponsors such as NSF, NASA, the Army Research Office, Allison Gas Turbines, and the Air Force Office of Scientific Research and has served as a consultant to industry and government. He has also served as major or co-major professor for 33 PhD and 17 MS students. He is a life fellow of the American Society of Mechanical Engineers and an associate fellow of the American Institute of Aeronautics and Astronautics.

John C. Tannehill began his career as an assistant professor of Aerospace Engineering at Iowa State University, Ames, Iowa, in 1969, was promoted to full professor in 1979, and is currently an emeritus professor. He is recognized as a pioneer in the field of computational fluid dynamics (CFD). Along with Dale Anderson and Richard Pletcher, he designed and implemented the first CFD courses at Iowa State University in 1972. These courses led to the first edition of this textbook in 1984. Professor Tannehill is internationally recognized for his research in computing high-speed flows using the complete or parabolized Navier–Stokes equations. He has been actively involved with NASA in developing CFD computer codes for many projects, including the Space Shuttle, the National Aerospace Plane (X-30), the High-Speed Civil Transport, and the Hyper-X Research Vehicle (X-43A). Professor Tannehill was the director of the CFD Center at Iowa State University from its establishment in 1984 to 2006. He is a fellow of the American Institute of Aeronautics and Astronautics and has received numerous other awards, including the Iowa General Assembly Excellence in Teaching Award and the Boylan Eminent Faculty Award for Research.

Dale A. Anderson received his BS from Parks College of Saint Louis University, St. Louis, Missouri, and his MS and PhD from Iowa State University, Ames, Iowa. He is currently professor emeritus at The University of Texas at Arlington. Prior to joining The University of Texas at Arlington, he was a professor of aerospace engineering and director of the Computational Fluid Dynamics Institute at Iowa State University. While at The University of Texas at Arlington, he served in various capacities, including professor of aerospace engineering, associate dean of the College of Engineering, dean of graduate studies, and vice president for research. Professor Anderson has served as a consultant or has held full-time positions with Lockheed-Martin, The Boeing Company, Aerospace Corporation, British Petroleum, U.S. Air Force, and Viskase Corporation. He has served as principal investigator on numerous grants and contracts and has published papers in diverse areas, including numerical methods, fluid dynamics, grid generation, reservoir simulation, and biomedical applications.

Professor Anderson is an associate fellow of the American Institute of Aeronautics and Astronautics and has received numerous awards for his contributions to teaching and research, including the Oliver L. Parks Award from Saint Louis University, the Professional Achievement Citation in Engineering from Iowa State University, the Haliburton Award for Excellence in Teaching, and the Haliburton Award for Outstanding Research from The University of Texas at Arlington.

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